

# The construction of [Alcazar, Cuarto del Principe](#)

September 16, 2016

These construction details have been generated automatically from the information used to produce the pattern [Alcazar, Cuarto del Principe](#). The encoded information use to construct the database is presented here.

## 1 References

Publication: [pc], Ref in that: Photo from Nick Crossling, Comment: Spain

## 2 Links

Link to patterninislamicart.com: [SPA 2721](#)

## 3 Database Geometry

The attributes are of three types:

1. Should be set (+).
2. Not relevant (-).
3. No user search (—).

### 3.1 Defined attributes

Star polygon with 12 points and angle of  $60^\circ$ ; – from a[24,1,3,1]  
Star polygon with 2 points and angle of  $80^\circ$ ; – from a[4,4,9,2]  
1 regular octagon – from p[8,1]  
Kites and darts: with 2 kites and 0 darts – from k[2,0]  
The symmetry \*442 ( $p4m$ )– from G[10]  
Number of polygons – from P[27]  
Number of edges – from B[186]  
Some edges not matched – from J[1]  
Satisfies two-colour property with straight cross-overs – from c[2]

Is edge-to-edge – from Z[1]  
Display available – from x[0]  
Does not conform to Two-Polygon condition – from U[0]  
No variants – from V[0]  
Interlace counts: Finite 1 Infinite 2 – from X[1,2]  
All angles a multiple of  $2.5/circ$  – from W[1,72]  
Irregular polygons counts: reflexive, pairs, singles – from S[9,1,0]  
Number polygons displayed – from N[397]

### 3.2 Undefined attributes

Star Polygon with kites (+) – from A[0,0,0,0]  
Class and 2 parameters (+) – from C[0,0,0]  
Revision text (—) – from R[0]  
Keywords (—) – from K[0]

## 4 Main documentation

The completed pattern is shown in the Figure 2 in a style which numbers each edge. In the details below, a point within the pattern is denoted by  $a \bullet b$  where  $a$  and  $b$  are two edges which meet at the denoted point. A polygon within the pattern is denoted by  $c-d$  where  $c$  is the first edge, and  $d$  is the last edge of the polygon. (The edges are numbered in an anti-clockwise order.) You will need to zoom in to see the edge numbers clearly.

This is based on some exact geometry, but has a central motif which is largely arbitrary, and is therefore subject to numerous variations in different locations. One variant is Wade SPA 2126x, and this variant is a ceiling in the Cuarto del Principe in the Alcázar. Another variant is [Alhambra, Baño Real](#) — upon which this graphic is based.

The variants have a common form based upon the 9 and 12 rosettes as in [Alhambra](#), [Mirador de Lindaraja](#) and [Tomb of Moulay Ishmael, Meknes](#). This *\*632 (p6m)* construction is used as part of the square grid. When used in this way, there is no longer room for the complete 9-rosettes.

Note that the 5-stars are not exact. The computation proceeds with L, the distance between adjacent 12-stars (centres).

## 5 Construction details

The following numerical calculations are performed:



Figure 1: Photo — © Nick Crossling

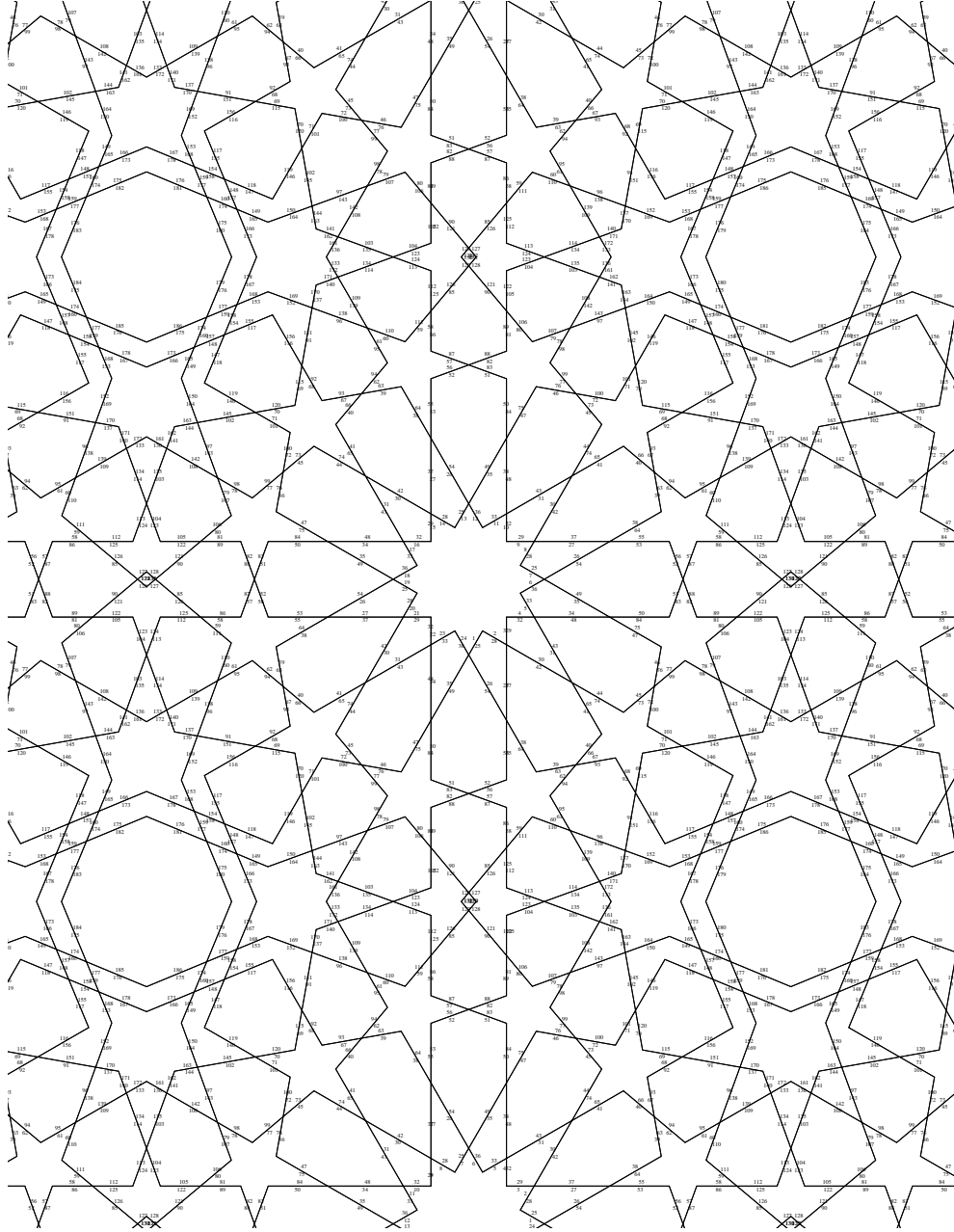


Figure 2: Alcazar, Cuarto del Principe — outline

<i>Formula</i>	<i>Value</i>
A = 1.0	+1.0000000000000000
B = A*sin(60)/sin(20)	+2.532088886237956
C = 2*B*sin(20)	+1.732050807568877
P = C*tan(55)	+2.473624908405561
Q = C*sin(20)/sin(35)	+1.032811370740278
D = Q*sin(40)/sin(15)	+2.565028983830200
W = C*cos(20)	+1.627595362698748
E = 2*W	+3.255190725397495
F = E*sin(15)/sin(45)	+1.191482499658978
J = 0.80*B	+2.025671108990365
G <i>implicit</i>	+2.245060870485210
M <i>implicit</i>	+0.502261266550639
L <i>implicit</i>	+1.405217971895420
K <i>implicit</i>	+1.448934600581475
N <i>implicit</i>	+2.223792357141243
R <i>implicit</i>	+2.805852699190740

The *implicit* values are calculated by using the fact that the  $x$  and  $y$  sums for each polygon is zero.

The individual polygons in the patterns are as follows:

1. The polygon 1-24 is a regular star (12pts) with a vertex angle of 60 degrees and an edge length of 1.19148249966.

2. The irregular polygon 25-28 has 4 sides. The edge vectors are:

$$\begin{aligned}
 F*\sin(45) \quad -F*\sin(45) &= +0.842505355173962 \quad -0.842505355173962 \\
 E*\cos(15) \quad E*\sin(15) &= +3.144272791158087 \quad +0.842505355173962 \\
 -E*\cos(15) \quad E*\sin(15) &= -3.144272791158087 \quad +0.842505355173962 \\
 -F*\sin(45) \quad -F*\sin(45) &= -0.842505355173962 \quad -0.842505355173962
 \end{aligned}$$

Similar polygons are: 29-32, 33-36.

3. The irregular polygon 37-42 has 6 sides. The edge vectors are:

$$\begin{aligned}
 E*\cos(30) \quad -E*\sin(30) &= +2.819077862357726 \quad -1.627595362698747 \\
 D \quad 0.0 &= +2.565028983830200 \quad +0.000000000000000 \\
 C*\sin(20) \quad C*\cos(20) &= +0.592396265452048 \quad +1.627595362698748 \\
 -C*\sin(20) \quad C*\cos(20) &= -0.592396265452048 \quad +1.627595362698748 \\
 -D \quad 0.0 &= -2.565028983830200 \quad +0.000000000000000 \\
 -E*\cos(30) \quad -E*\sin(30) &= -2.819077862357726 \quad -1.627595362698747
 \end{aligned}$$

Similar polygons are: 43-48, 49-54.

4. The irregular polygon 55-64 has 10 sides. The edge vectors are:

$$\begin{array}{lclcl}
D & 0.0 & = & +2.565028983830200 & +0.000000000000000 \\
C*\sin(20) & -C*\cos(20) & = & +0.592396265452048 & -1.627595362698748 \\
C*\sin(20) & C*\cos(20) & = & +0.592396265452048 & +1.627595362698748 \\
C & 0.0 & = & +1.732050807568877 & +0.000000000000000 \\
-C*\cos(40) & C*\sin(40) & = & -1.326827896337877 & +1.113340798452839 \\
C*\sin(20) & C*\cos(20) & = & +0.592396265452048 & +1.627595362698748 \\
-C*\cos(30) & -C*\sin(30) & = & -1.500000000000000 & -0.866025403784439 \\
-C*\cos(40) & C*\sin(40) & = & -1.326827896337877 & +1.113340798452839 \\
C*\sin(10) & -C*\cos(10) & = & +0.300767466360871 & -1.705737063904887 \\
-D*\cos(30) & -D*\sin(30) & = & -2.221380261440337 & -1.282514491915100
\end{array}$$

Similar polygons are: 65-74, 75-84.

5. The irregular polygon 85-90 has 6 sides. The edge vectors are:
- $$\begin{array}{lclcl}
B*\cos(40) & -B*\sin(40) & = & +1.939692620785909 & -1.627595362698747 \\
C & 0.0 & = & +1.732050807568877 & +0.000000000000000 \\
C*\sin(20) & C*\cos(20) & = & +0.592396265452048 & +1.627595362698748 \\
-C*\sin(20) & C*\cos(20) & = & -0.592396265452048 & +1.627595362698748 \\
-C & 0.0 & = & -1.732050807568877 & +0.000000000000000 \\
-B*\cos(40) & -B*\sin(40) & = & -1.939692620785909 & -1.627595362698747
\end{array}$$
- Similar polygons are: 91-96, 97-102, 103-108, 109-114.
6. The irregular polygon 115-120 has 6 sides. The edge vectors are:
- $$\begin{array}{lclcl}
C*\cos(55) & -C*\sin(55) & = & +0.993463529784308 & -1.418812959832445 \\
B*\cos(15) & -B*\sin(15) & = & +2.445810049676765 & -0.655352827650813 \\
G*\sin(22.5) & G*\cos(22.5) & = & +0.859147599785836 & +2.074165787483258 \\
-G*\sin(22.5) & G*\cos(22.5) & = & -0.859147599785836 & +2.074165787483258 \\
-B*\cos(15) & -B*\sin(15) & = & -2.445810049676765 & -0.655352827650813 \\
-C*\cos(55) & -C*\sin(55) & = & -0.993463529784308 & -1.418812959832445
\end{array}$$
7. The irregular polygon 121-128 has 8 sides. The edge vectors are:
- $$\begin{array}{lclcl}
-B*\cos(40) & -B*\sin(40) & = & -1.939692620785909 & -1.627595362698747 \\
C & 0.0 & = & +1.732050807568877 & +0.000000000000000 \\
C*\sin(20) & -C*\cos(20) & = & +0.592396265452048 & -1.627595362698748 \\
C*\sin(20) & C*\cos(20) & = & +0.592396265452048 & +1.627595362698748 \\
C & 0.0 & = & +1.732050807568877 & +0.000000000000000 \\
-B*\cos(40) & B*\sin(40) & = & -1.939692620785909 & +1.627595362698747 \\
-M*\sin(50) & -M*\cos(50) & = & -0.384754452235016 & -0.322847318964219 \\
-M*\sin(50) & M*\cos(50) & = & -0.384754452235016 & +0.322847318964219
\end{array}$$
8. The polygon 129-132 is a regular star (2pts) with a vertex angle of 80 degrees and an edge length of 0.502261266551.
9. The irregular polygon 133-136 has 4 sides. The edge vectors are:
- $$\begin{array}{lclcl}
A*\cos(60) & -A*\sin(60) & = & +0.500000000000000 & -0.866025403784439 \\
B*\cos(20) & B*\sin(20) & = & +2.379385241571817 & +0.866025403784439 \\
-B*\cos(20) & B*\sin(20) & = & -2.379385241571817 & +0.866025403784439 \\
-A*\cos(60) & -A*\sin(60) & = & -0.500000000000000 & -0.866025403784439
\end{array}$$
- Similar polygons are: 137-140, 141-144.

10. The irregular polygon 145-150 has 6 sides. The edge vectors are:
- |              |              |   |                    |                    |
|--------------|--------------|---|--------------------|--------------------|
| B            | 0.0          | = | +2.532088886237956 | +0.000000000000000 |
| -B*cos(40)   | B*sin(40)    | = | -1.939692620785909 | +1.627595362698747 |
| G*cos(57.5)  | G*sin(57.5)  | = | +1.206270326426483 | +1.893465133496457 |
| -L*cos(12.5) | -L*sin(12.5) | = | -1.371908695094669 | -0.304144835335929 |
| -K*cos(57.5) | -K*sin(57.5) | = | -0.778511993412588 | -1.222019047672726 |
| J*sin(10)    | -J*cos(10)   | = | +0.351754096628727 | -1.994896613186549 |
11. The irregular polygon 151-156 has 6 sides. The edge vectors are:
- |              |              |   |                    |                    |
|--------------|--------------|---|--------------------|--------------------|
| B            | 0.0          | = | +2.532088886237956 | +0.000000000000000 |
| J*sin(10)    | J*cos(10)    | = | +0.351754096628727 | +1.994896613186549 |
| -K*cos(57.5) | K*sin(57.5)  | = | -0.778511993412588 | +1.222019047672726 |
| -L*cos(12.5) | L*sin(12.5)  | = | -1.371908695094669 | +0.304144835335929 |
| G*cos(57.5)  | -G*sin(57.5) | = | +1.206270326426483 | -1.893465133496457 |
| -B*cos(40)   | -B*sin(40)   | = | -1.939692620785909 | -1.627595362698747 |
12. The polygon 157-160 is a regular star (2pts) with a vertex angle of 45 degrees and an edge length of 1.4052179719.
13. The irregular polygon 161-172 has 12 sides. The edge vectors are:
- |              |              |   |                    |                    |
|--------------|--------------|---|--------------------|--------------------|
| -A*cos(60)   | -A*sin(60)   | = | -0.500000000000000 | -0.866025403784439 |
| A*cos(20)    | -A*sin(20)   | = | +0.939692620785908 | -0.342020143325669 |
| A*sin(10)    | -A*cos(10)   | = | +0.173648177666930 | -0.984807753012208 |
| J*cos(20)    | J*sin(20)    | = | +1.903508193257454 | +0.692820323027551 |
| K*cos(22.5)  | -K*sin(22.5) | = | +1.338641021424641 | -0.554483266223059 |
| -N*sin(22.5) | N*cos(22.5)  | = | -0.851008492098064 | +2.054516243317824 |
| N*sin(22.5)  | N*cos(22.5)  | = | +0.851008492098064 | +2.054516243317824 |
| -K*cos(22.5) | -K*sin(22.5) | = | -1.338641021424641 | -0.554483266223059 |
| -J*cos(20)   | J*sin(20)    | = | -1.903508193257454 | +0.692820323027551 |
| -A*sin(10)   | -A*cos(10)   | = | -0.173648177666930 | -0.984807753012208 |
| -A*cos(20)   | -A*sin(20)   | = | -0.939692620785908 | -0.342020143325669 |
| A*cos(60)    | -A*sin(60)   | = | +0.500000000000000 | -0.866025403784439 |
14. The irregular polygon 173-178 has 6 sides. The edge vectors are:
- |              |              |   |                    |                    |
|--------------|--------------|---|--------------------|--------------------|
| N*sin(22.5)  | -N*cos(22.5) | = | +0.851008492098064 | -2.054516243317824 |
| L*cos(22.5)  | -L*sin(22.5) | = | +1.298252122951199 | -0.537753636706049 |
| -R*sin(22.5) | R*cos(22.5)  | = | -1.073753341637164 | +2.592269880023873 |
| R*sin(22.5)  | R*cos(22.5)  | = | +1.073753341637164 | +2.592269880023873 |
| -L*cos(22.5) | -L*sin(22.5) | = | -1.298252122951199 | -0.537753636706049 |
| -N*sin(22.5) | -N*cos(22.5) | = | -0.851008492098064 | -2.054516243317824 |
15. The polygon 179-186 is regular with 8 sides and an edge length of 2.80585269919.

The final form of the pattern includes the colours and the form of the lines. This is in Figure 3.

Format is symbolic.

The outline form of the pattern with edges and angles annotated is in Figure 4.

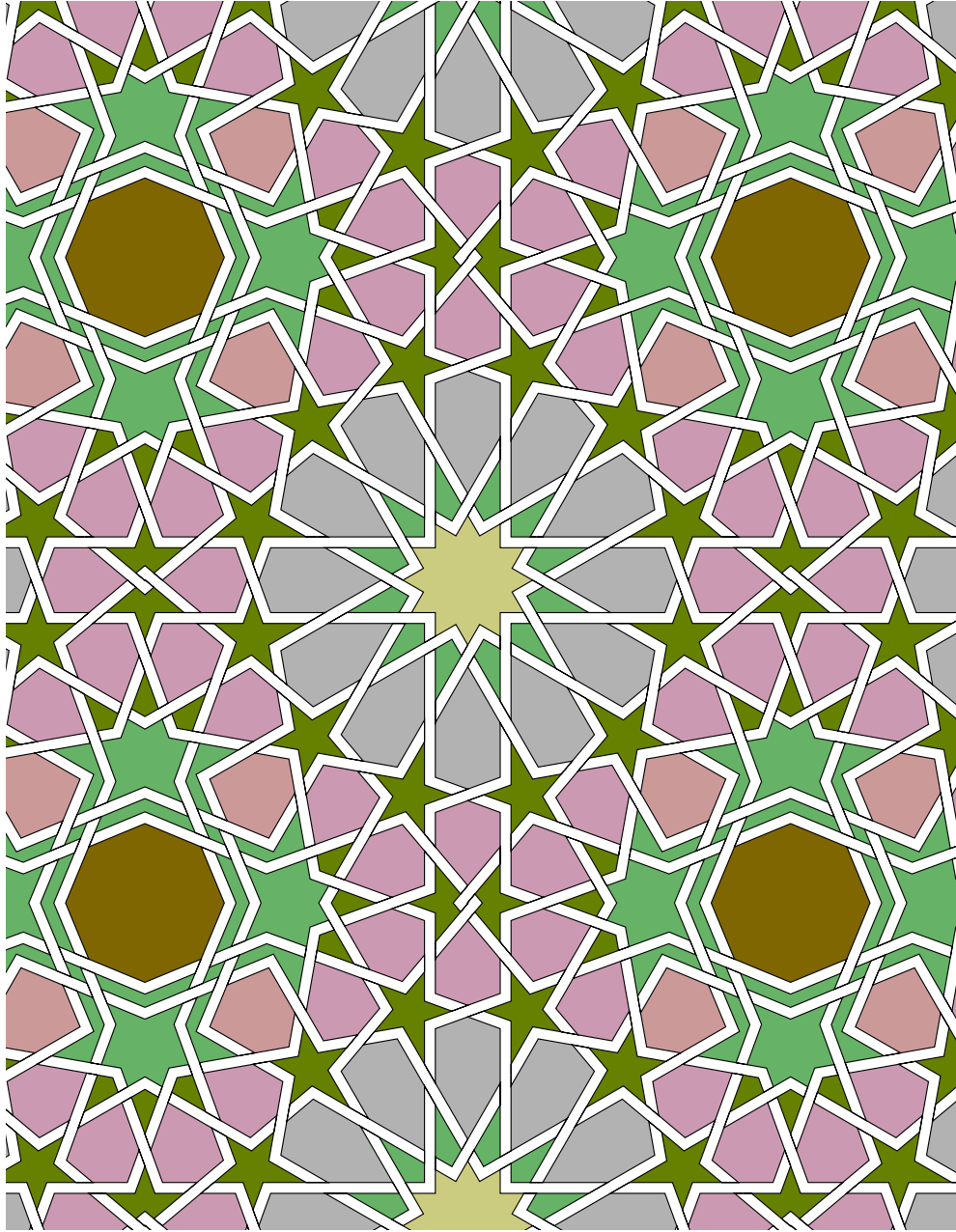


Figure 3: Alcazar, Cuarto del Principe — final



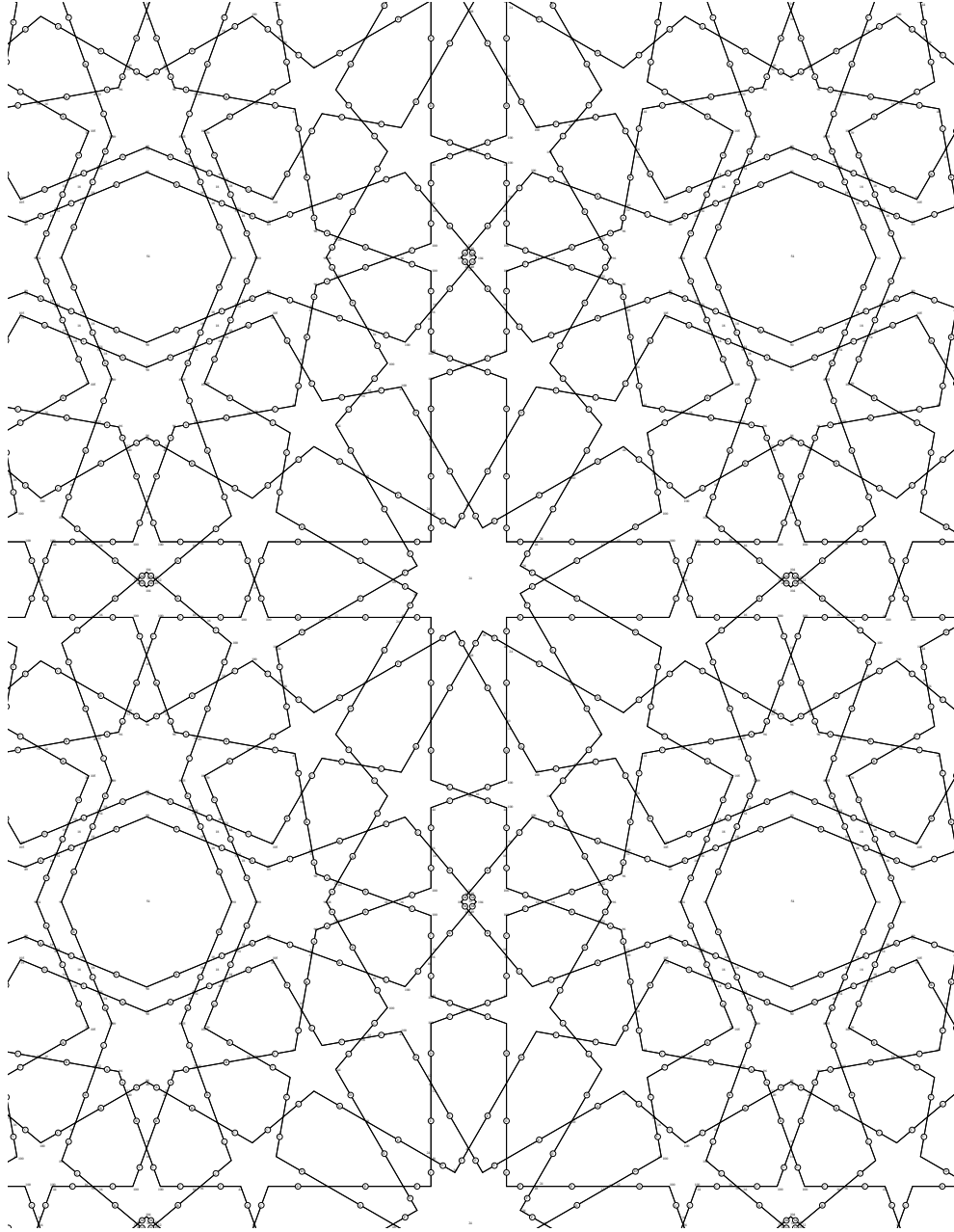


Figure 4: Alcazar, Cuarto del Principe — outline with edges and angles — angles a multiple of  $\pi/72$

This pattern has interlacing with straight cross-overs.

1. Interlace edges are : [1, 2, 30, 44, 45, 77, 78, 108, 136, 133, 109, 61, 62, 40, 41, 31, 5, 6, 26, 38, 39, 67, 68, 116, 117, 148, 166, 167, 154, 118, 119, 71, 72, 46, 47, 35]

The number of edges in this section is 36

This is an infinite interlace with repeated sections.

Area too small - aborted.

2. Interlace edges are : [3, 4, 34, 50, 51, 57, 58, 112, 113, 135, 162, 163, 145, 120, 115, 151, 170, 171, 134, 104, 105, 81, 82, 52, 53, 27]

The number of edges in this section is 26

This is an infinite interlace with repeated sections.

Area too small - aborted.

3. Interlace edges are : [59, 60, 96, 152, 153, 159, 175, 176, 160, 149, 150, 97, 79, 80, 90, 127, 128, 85]

The number of edges in this section is 18

This is a finite interlace with repeated sections.

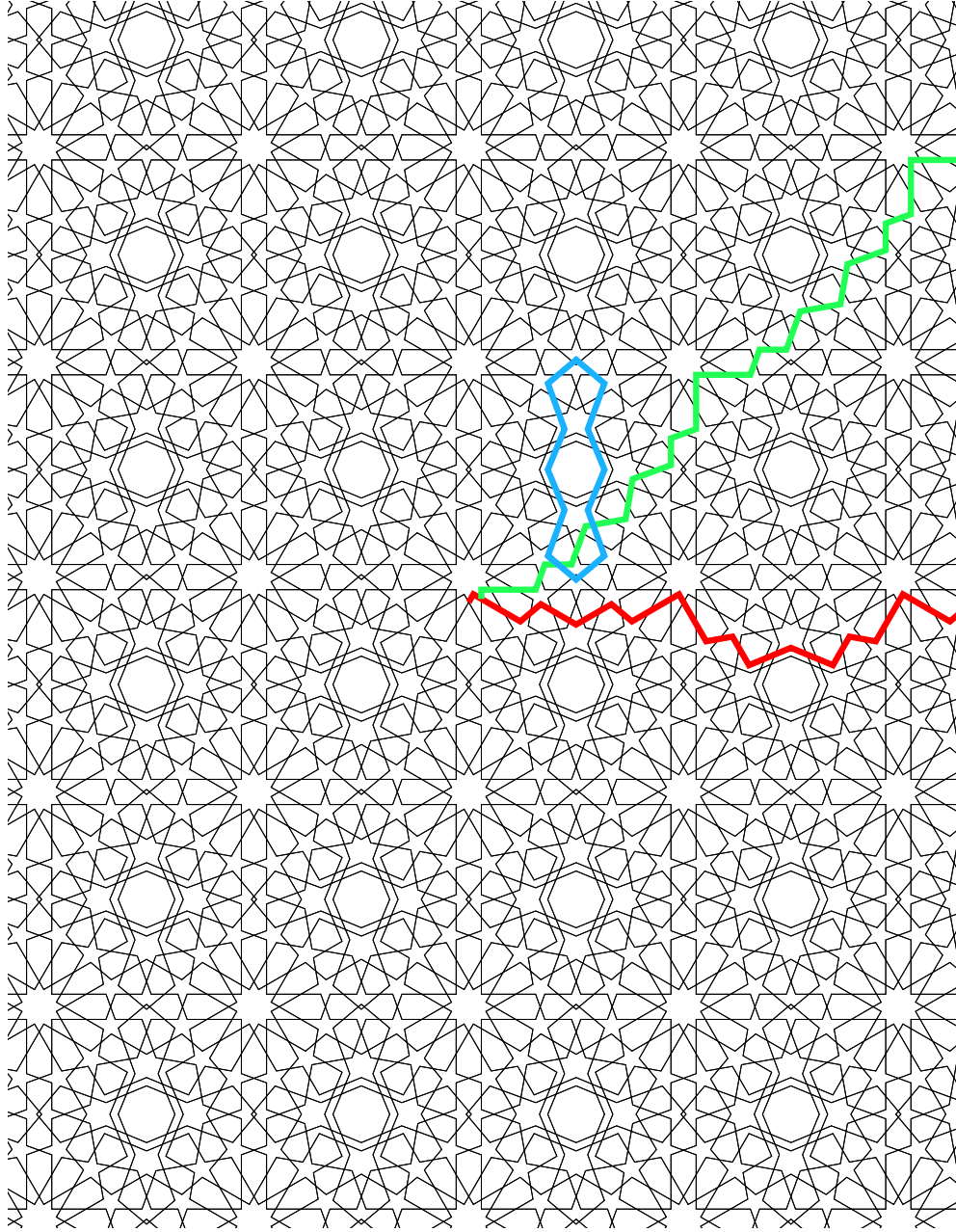


Figure 5: Coloured interlaces scale:3

Approximate value:  $J = 0.80 \cdot B$

v41