

The Alhambra Symmetry Groups

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1 Introduction

At least three people have documented their observations of the symmetry groups present on the Alhambra Palace, Granada [3, 1, 2]. This is yet another attempt, but uses the record of patterns available at tilingsearch.org.

2 Searching

If you use the comprehensive search facility ([at](#)), you can type in the text search for ‘Alhambra’ and find 61 tilings. One can then inspect each one of these to determine the symmetry group — tedious, but possible. There are two problems here:

1. One pattern is actually from the Alhambra Palace Hotel and is a modern construction and therefore should not be included, see [John Rigby’s photo no 26](#). (You can select the blue boxed text to locate the item on the Internet.)
2. The tedious process of recording the result of each of the other 53 patterns is error-prone.

We have a solution to the second problem. The system is actually built using SQL, a computer language to handle databases. Although the users of the web site cannot use the SQL, I can and do so to develop the site and explore new possibilities.

The actual SQL is:

```
SELECT T.Symmetry, T.Weblink
FROM Tiling T, Tile_Ref TR
WHERE
    T.Title LIKE "%Alhambra%" OR
    T.Comment LIKE "%Alhambra%" OR
    (TR.TileID = T.TileID AND
     (TR.Comment LIKE "%Alhambra%"))
)
GROUP BY T.Symmetry, T.Weblink;
```

The result lists the symmetry groups in an internal code; the text above shows that SQL is not feasible for a user interface!

In order of frequency, the results are as follows:

***442** (*p4m*). 39 being:

- Bourgoin, Plate 42,
- Cahier and Martin, page 14
- Alhambra tilings, page 345, Figure 169,
- Alhambra tilings, page 346, Figure 170,
- Alhambra tilings, page 347, Figure 171,
- Alhambra tilings, page 348, Figure 172,
- Alhambra tilings, page 416, Figure 218 (16),
- Alhambra tilings, page 417, Figure 219 (18),
- Alhambra tilings, page 417, Figure 219 (19),
- Alhambra tilings, page 317, Plate 18,
- Alhambra tilings, page 384, Plate 27,
- Castera, Fes, Royal Palace, page 120 -1,
- John Rigby's photo no 90,
- Alhambra, Comares Palace,
- Alhambra, Fleurent,
- Calvert, Ornament in the Alhambra,
- Jones, Plate XLII+,
- John Rigby's photograph 38a,
- John Rigby's photograph 40,
- Alhambra ornament,
- Alhambra, Portico, Court of the Myrtles,
- Alhambra, Sala de Lindaraja,
- DeGeorge, page 57, The Alhambra,
- DeGeorge, page 63, one motif, The Alhambra,
- DeGeorge, page 63, one motif, The Alhambra,
- Alhambra tiling from M C Escher, page 53,
- Alhambra, Granada
- Alhambra, Granada
- Cahier and Martin, page 21,
- Paccard, vol 1, page 198, bottom left
- Wade, page 97

- Alhambra tiling from M C Escher, page 41,
- Owen Jones, Plate XLII(3),
- Owen Jones, Plate XLIII(2),
- Owen Jones, Plate XLIII(3),
- Patio de la Alberca, The Alhambra,
- Abas and Salman, Plate 3 (a),
- J C Murphy, Plate L and
- J C Murphy, Plate LX. (The hotel one was excluded here.)

***632** ($p6m$). There are 8 of these:

- Bourgoin, Plate 69
- Alhambra tilings, page 423, Figure 224,
- Alhambra tilings, page 426, Figure 227,
- Alhambra tilings, page 427, Figure 228,
- Owen Jones, Plate XLIII(8),
- Version of Bourgoin, Plate 120 from Tony Lee
- Critchlow, frontispiece and
- Mosaic wall tiles, 13th century Seljuk, Turkey.

4*2 ($p4g$). There are 7 as follows:

- Alhambra tilings, page 329, Figure 161,
- Alhambra tilings, page 432, Figure 232,
- Critchlow, page 126,
- Figure 1.3(d), An Alhambra tiling,
- Figure 1.3(b), An Alhambra tiling,
- Wade, page 22 and
- C Humbert, Plate 153.

442 ($p4$). There are three:

- John Rigby's photo no 51,
- Figure from Alhambra and
- Abas and Salman, page 365.

333 ($p3$). There is one: Patio de la Alberca, The Alhambra.

632 ($p6$). There is one: Alhambra, Lion Courtyard.

***2222** (pmm). There is one: John Rigby's photo no 77.

No examples. $*\times$ (cm), $2*22$ (cmm), O ($p1$), 2222 ($p2$), $3*3$ ($p3lm$), $*333$ ($p3ml$), $\times\times$ (pg), $22\times$ (pgg), $**$ (pm) and $22*$ (pmg).

3 Analysis

The 10 symmetry groups that I am apparently missing is rather disappointing! I suspect the main reason is that my system ignores the colour in determining the symmetry group. This reduction in symmetry if colour is taken into account is well illustrated in [4]. It also seems that the distribution of the symmetry groups in the Alhambra is not that different from the entire collection on the website (which has a high proportion of Islamic patterns).

The search facility can be used directly on the Internet by requiring the text ‘Alhambra’ and also a specific symmetry group. The actual numbers above are likely to change as further patterns are added, and perhaps existing ones noted as being present at the Alhambra. (Version 10 used for the above search results.)

This area is made more difficult that one might imagine since no comprehensive digital library has been located of images from the Alhambra. The best collection appears to be that of David Wade, [see](#). Links to this site of individual photographs have already been added to my site. However, I am now in touch with Nick Crossling who has his own comprehensive photos of the Alhambra patterns.

Surveying the other papers on this topic, it does not appear that I have missed any tiling patterns. Marus Du Sautoy has kindly sent me his collection of 17 patterns. There is no inconsistency with the collection here, since the differences are mainly due to colour and a lesser extent due to his inclusion of non-geometric patterns.

References

- [1] B. Grünbaum. What groups are present in the Alhambra? Available on the Internet.
- [2] Marcus Du Sautoy. Finding Moonshine. Chapter 3. ISBN 978-0-00-721467-2. 2008.
- [3] J. Jaworski, A Mathematician’s Guide to the Alhambra. 2006. Available on the Internet.
- [4] B. Grünbaum, Z. Grünbaum and G C Shephard. Symmetry in Moorish and other Ornamants. *Comp. & Maths. with Appls.* Vol 12B, pp641-653. 1986.