A very simple introduction to the geometry of Islamic patterns

Brian Wichmann

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1

We start by looking at one quite simple pattern, Figure 1.

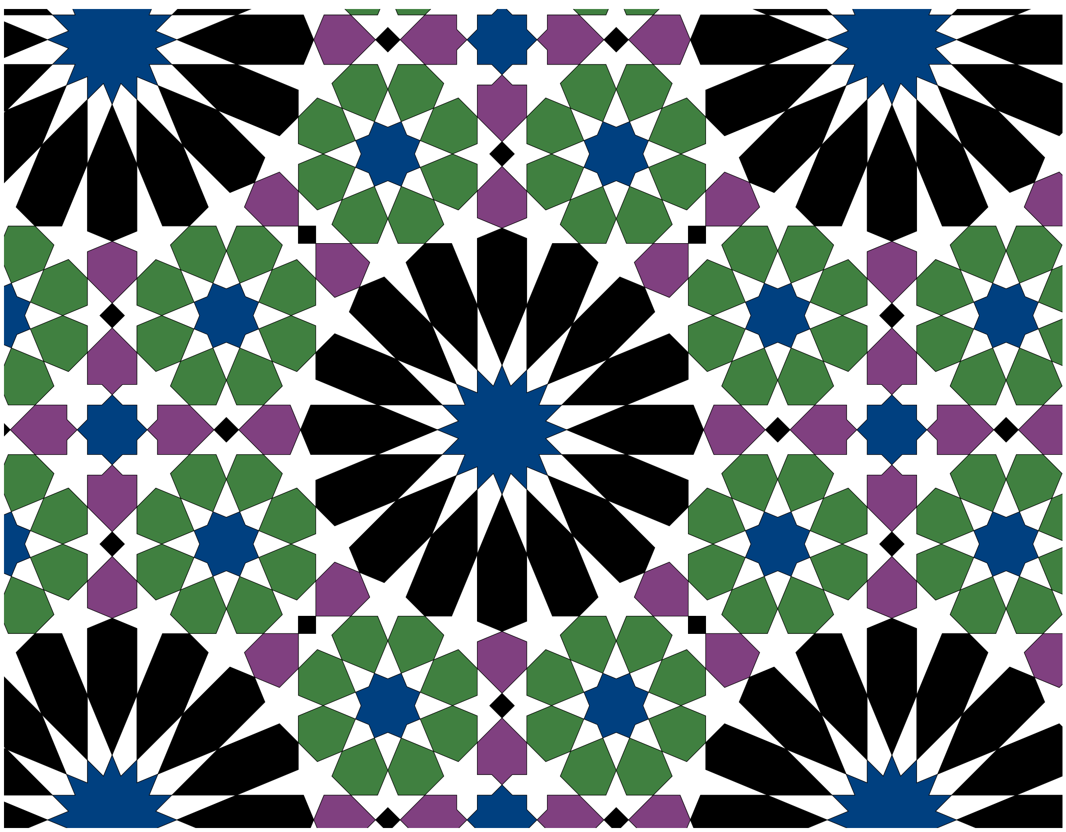

Figure 1: The Alhambra, Sala de las dos Hermanas

This pattern was drawn by M C Escher on a visit to the Alhambra — he had a good eye for an interesting pattern. The most important aspect of this pattern is the presence of two stars (with their surrounds): one with 8 points and the large one with 16 points.

These two stars are regular: the 16 sides of the 8-pointed star are all the same length and the angle at each vertex is 90°, while the 16-pointed star has 32 edges with a vertex angle of 45°. The 8-pointed star is called a *khatem*.

Another aspect of this pattern is about the lines. Where two lines cross, the lines do not change in direction. The lines themselves do change direction, so one can follow these lines which either continue indefinitely or go back on themselves. We show the result of doing this in Figure 2.

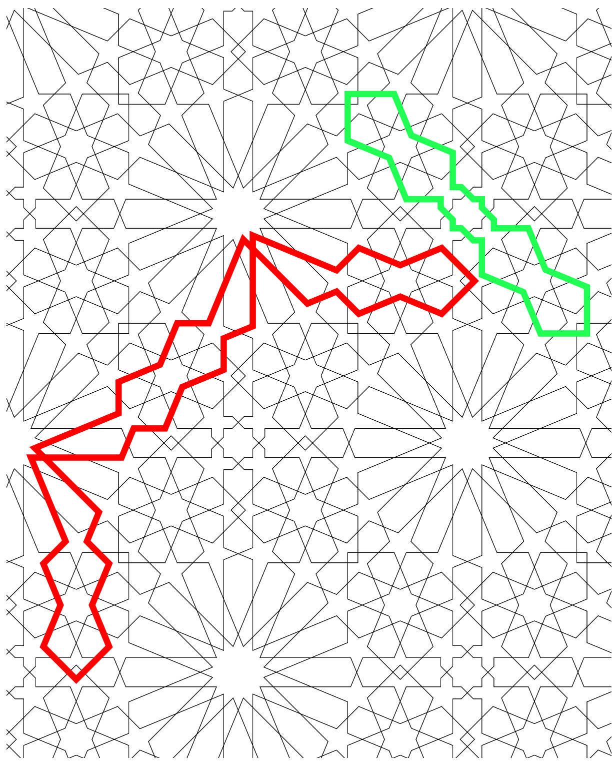

Figure 2: Following the two types of lines

Only two examples of tracing the lines have been shown since all the other ones that could be drawn are similar. The fact that the cross-overs are straight is important, but following the interlaces is just fun!

2

Looking at Figure 1 again, there is another aspect to consider. Each star is contained within a *rosette*. The additions to the star are a four-sided tiles called a *kite* and some six-sided tiles called a *petal*. The entire pattern consists of the two different sized rosettes fitted together with some other small additions. The art of Islamic design is therefore to place the rosettes with some additions to make an integrated whole. Note that there is one petal which is not part of a rosette—coloured differently.

For another example with rosettes, see Figure 3.

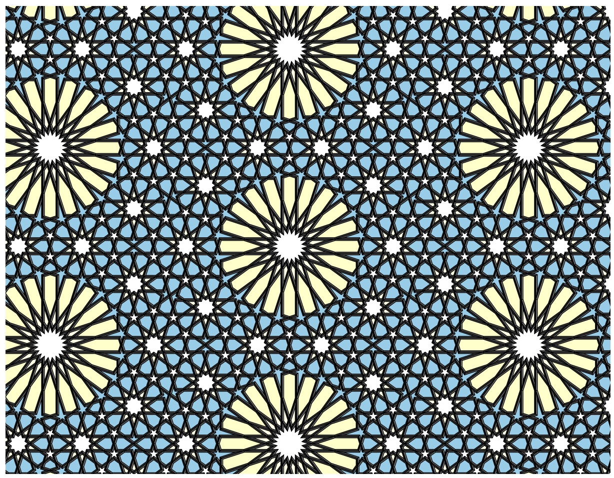

Figure 3: Two rosettes

There are just 2 types of rosette — with 10 and 20-pointed stars. There are also many small 5-pointed stars (you may need to zoom in to see these properly). There is an interesting link to Figure 1 which has the smaller petal repeated next to the small black square. In the pattern above the blue petal also appears outside the rosette.

3

Return to our favourite pattern in Figure 1, we want to consider another aspect. Consider a line drawn from the centre of the central large star to another large star. A reflection along this line leaves the pattern unchanged. Also, a rotation by 90° also leaves the pattern unchanged.

If one draws a square with the corners at the four outer large stars, then this square could be repeated indefinitely. This arrangement is the most common method a repeating Islamic patterns. There are in total 17 methods of repeating a pattern, but for simplicity we only consider the rotations which can be 2, 3, 4 or 6.

For an example of a pattern with a rotation of order 2 consider figure 4.

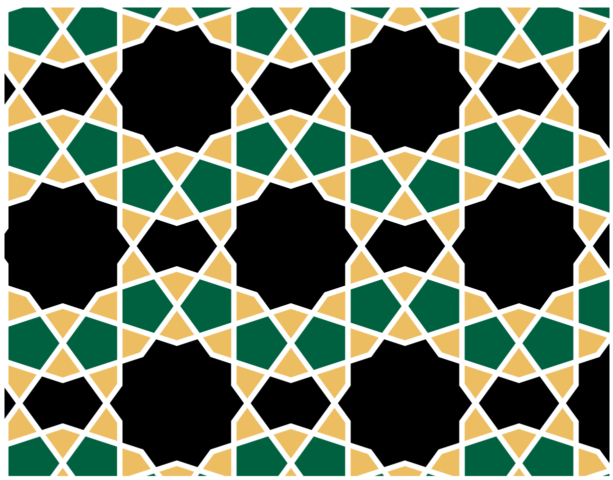

Figure 4: Rotation of order 2

In this case, we mean that a rotation of order 4 is not present in the pattern. Another example of this type of arrangement was shown in Figure 4. Both these example also have lines of reflection.

For an example of a pattern with a rotation of order 3 consider figure 4.

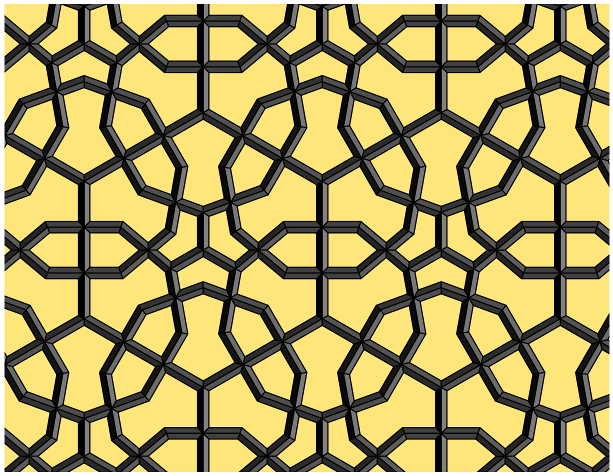

Figure 5: Rotation of order 3

Note that this example does *not* have straight cross-overs unlike the previous ones. Indeed, patterns with rotations of order 3 look different and are less common in Islamic art.

As we have already noted, our first example, Figure 1 had rotations of order 4, but we give another more elaborate example here, Figure 6.

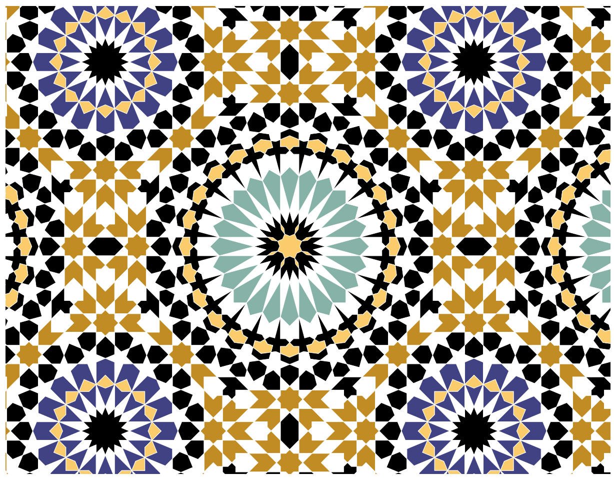

Figure 6: Rotation of order 4

Apart from the 24-pointed star, the rest is similar to our first Figure 1 using a khatem with kites is many places.

Our final example for rotations, is an example with a rotation of order 6, Figure 7

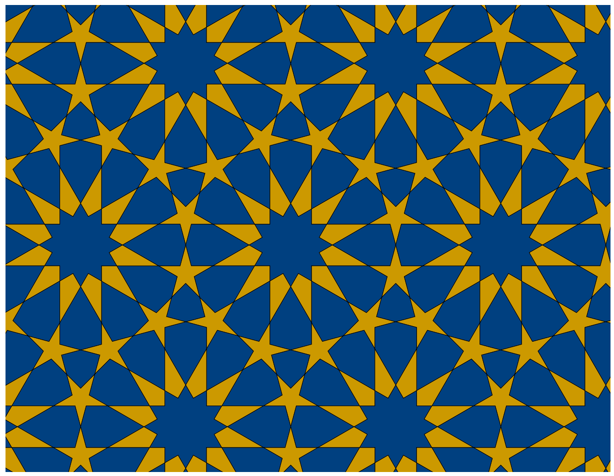

Figure 7: Rotation of order 6

This is a particularly elegant construction since the rosettes fit together so well in a hexagonal formation. The 5-pointed stars look regular, but are not quite.

4

In this section we look in more detail at patterns with a 10-pointed star. Our first example shows why such patterns can be elegantly designed, see Figure 8.

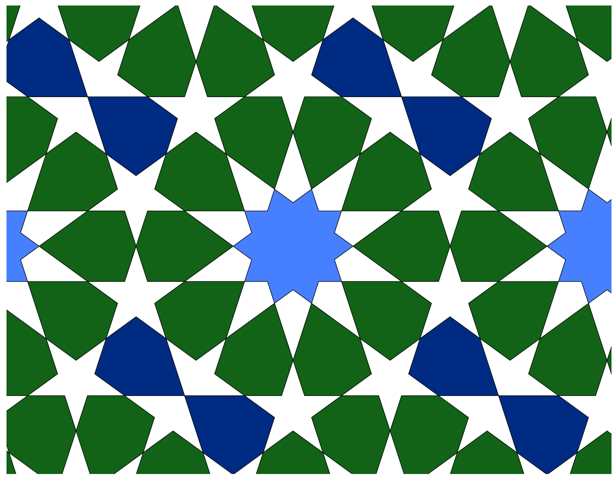

Figure 8: Exact 5-pointed star

Note that this design was contained within the more complex pattern in Figure 3. This significance of this design, unlike the previous pattern is that the 5-pointed star is exact. It is interesting to note that a rotation of order 5 is not possible within a repeating pattern.

We show some of the variety that is possible with decagonal patterns, see Figure 9.

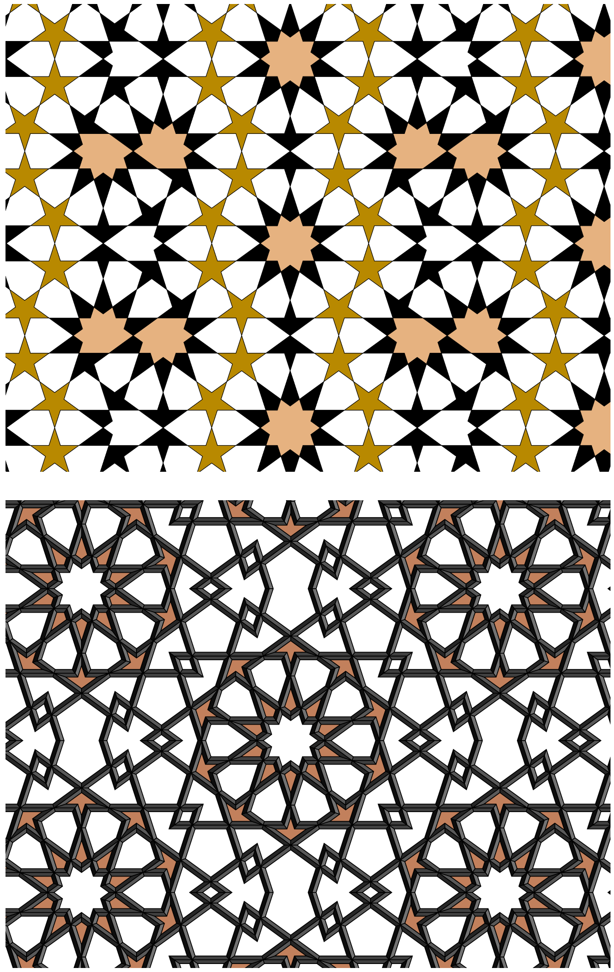

Figure 9: Variety of decagonal patterns

The top graphic uses many of the tiles of Figure 8, including the regular 5-pointed star. The bottom graphic completes the rosette in a different way allowing the inclusion of regular pentagons. Notice also that the diamonds can be varied in size. Both patterns give rise to many variations.

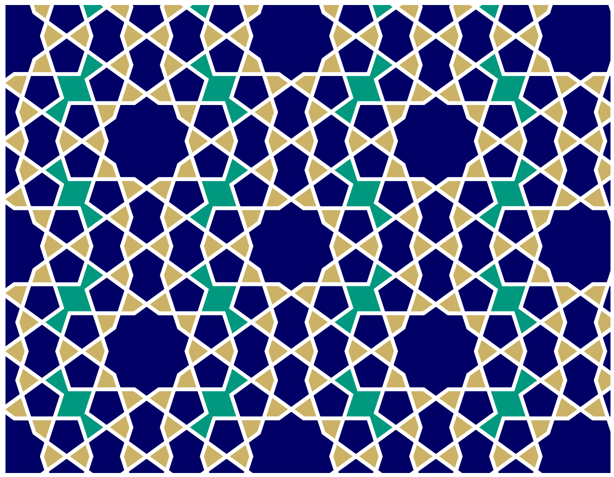

Figure 10: Different 10-pointed star

The rosette with the brown tiles in the lower pattern in Figure 9 can be replaced by the star which appears in Figure 10, The result builds upon the slightly simpler pattern in Figure 3. Again, many variations are possible.

5

Here we consider one aspect of ornamentation: how the lines are drawn. In Figure 10 we used thick white lines. Often this is done with *interlacing* in which the white lines cross each other with alternating up and down and shown in Figure 11. The top pattern is a small variant of Figure 1. The lower example of interlacing varies the colour which is less common. It is easier to trace the lines with this presentation style - one blue interlace and two white interlaces.

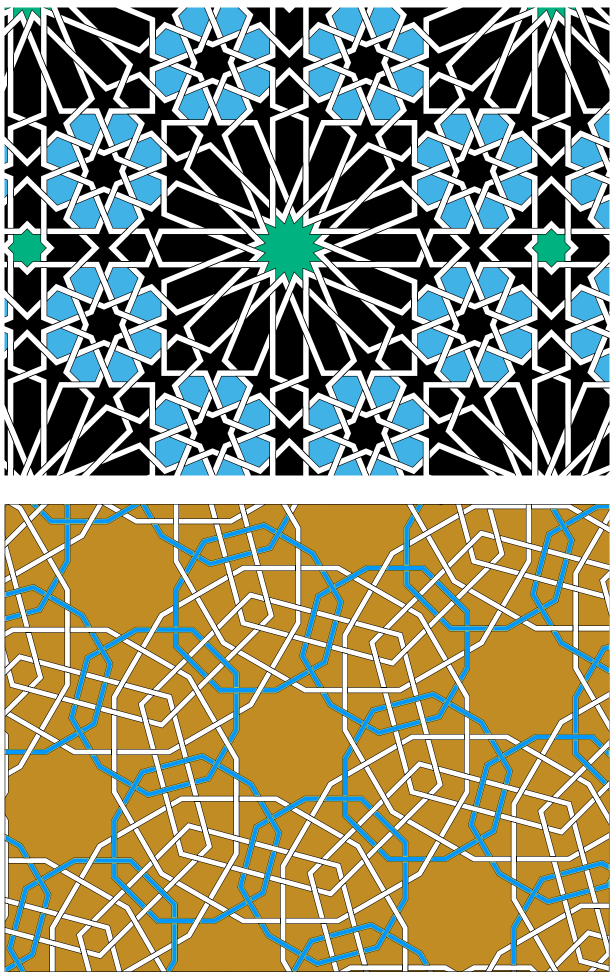

Figure 11: White nd coloured interlacing

Another method of drawing lines has been shown: Figure 8 bottom and Figure 5, which is designed to stand out by shading.

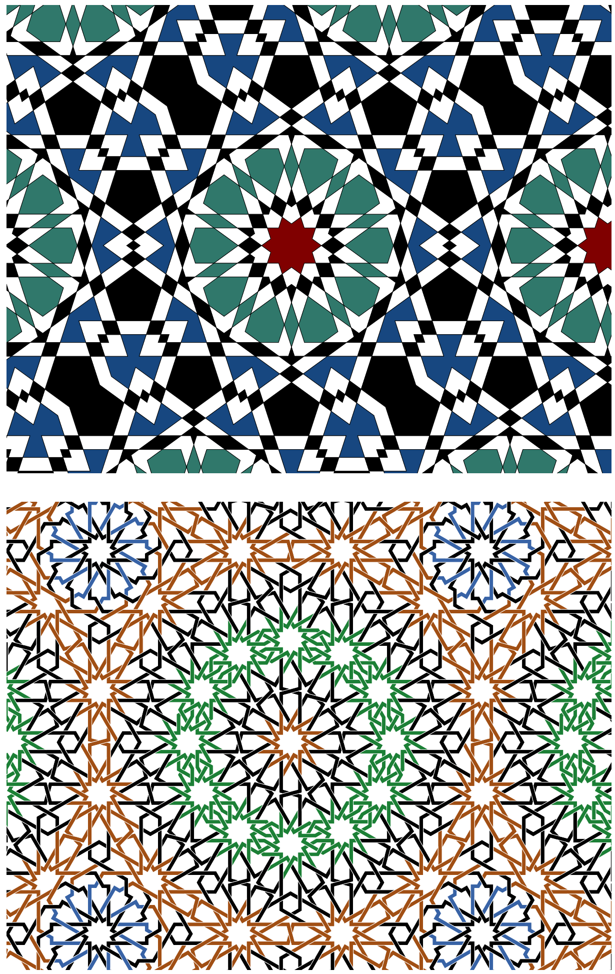

Figure 12: Variety of decagonal patterns

The upper pattern is actually the same basic design as lower pattern of Figure 11. To see this, visualise the basic design drawn with wide white lines. Then take the tiles and extend the edges of those tiles to extend the pattern within the white areas. The lower pattern has coloured interlacing, but the colour used in each interlace changes to give a dramatic effect.

6

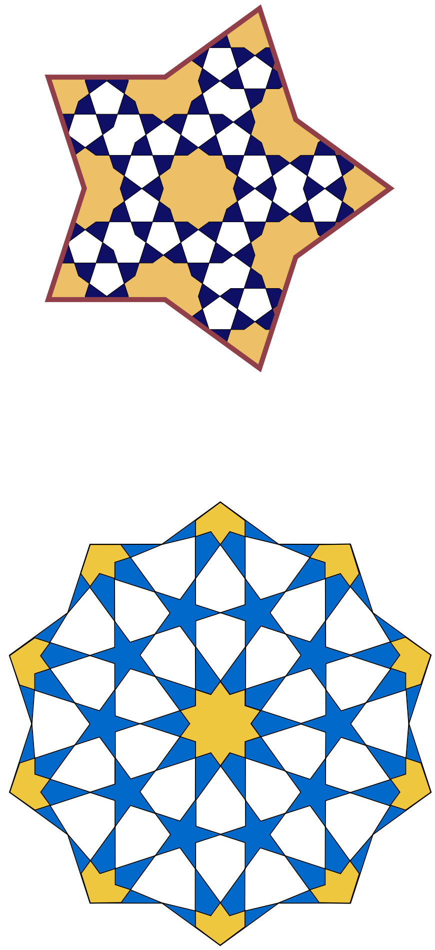

Figure 13: Decagonal motifs

These two small motifs are typically used in isolation or part of the larger design. They have five-fold and ten-fold symmetry.

7

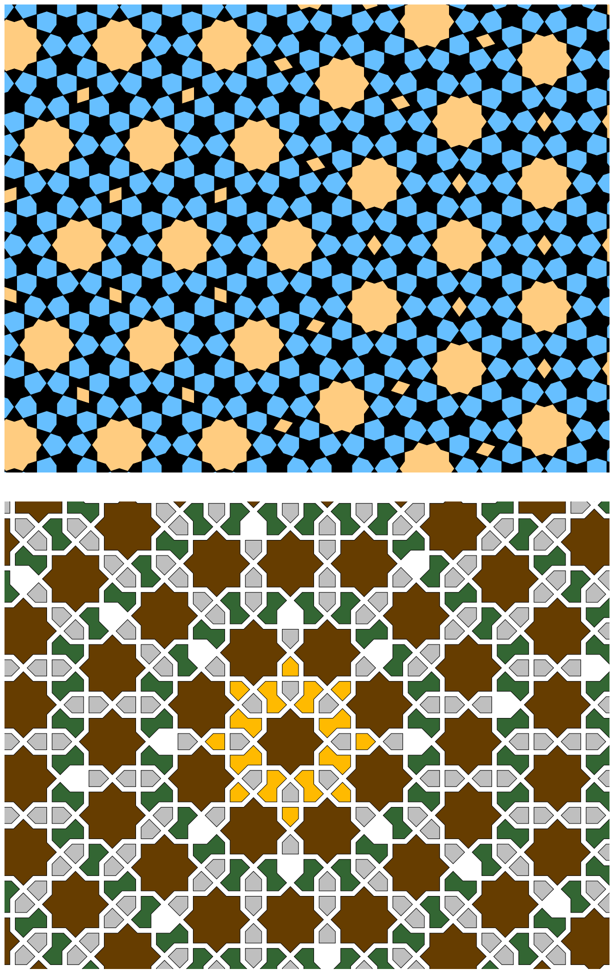

Figure 14: Centered patterns

These two patterns radiate out from a central point.

8 Further Reading

This introduction has not covered everything, so those wishing to explore the area further, more information appears below.

For further details about each of the figures see the list below.

1. Alhambra, Sala de las dos Hermanas.

2. For examples see Interlacing.
3. Bou Inania Madrasah, Fez.
4. Bourgoin, Plate 175.
5. Fatehpur Sikri, carved stone panel.
6. Ben Youssef Madrasah, Marrakesh.
7. Great Mosque, Thatta.
8. Bourgoin, Plate 171.
9. top: Itimad-ud-Daula's Tomb, bottom: Bourgoin, Plate 179.
10. Decagonal pattern.
11. top: Zellij dado, Sa'di tombes, Marrakesh, bottom: Gunbad-i Surkh, Maragha.
12. top: Royal Palace, Fez, bottom: Saadian Tombs, Marrakesh.
13. top: Roundel, Friday mosque, Isfahan, bottom: Friday mosque, Isfahan.
14. top: Kasehgaran Madrasah, Isfahan, bottom: Abdulla Khan madrasah, Bukhara.

The treatment of symmetry is incomplete in this introduction, but an excellent book to available for this important topic. This is: J H Conway, H Burgiel and C Goodman-Strauss. *The symmetry of things.* A K Peters Ltd. 2008. ISBN 978 1 56881 220 5

A book covering the material on the web site is available: *Islamic Design: a Mathematical Approach.* Birkhäuser. 2017. ISBN 978-3-319-699. The first part of the book covers the cultural background and is by David Wade. The second part is mathematical and is by the author.