

Symmetry in Islamic Geometric Art

Brian Wichmann*

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Abstract

This note considers the frequency of the 17 planar symmetry groups in Islamic Geometric Art. The collection used for this analysis contains over 1,500 patterns and is thought to be the largest available.

1 Introduction

At least five people have documented their observations of the symmetry groups present on the Alhambra Palace, Granada: Müller [12], Jaworski [9], Montesinos [13], Grünbaum[7] and Du Sautoy [5]. In this short note, we consider the symmetry groups used more generally in Islamic Geometric Art.

Around 1990, I decided to collect tiling patterns which led to the publication of a CD/booklet in 2001 [19]. Such publications are naturally limited in size and scope which can now be overcome by using a webserver on the Internet with dynamic searching facilities. Hence as an extension of the CD/booklet, an Internet service is now available at tilingsearch.org.

Subsequently, a book has been produced to ensure the long-term availability of the key information and design methodology, [20].

2 Finding the frequencies

The database which provides the searching facilities on the Internet service can be used to find the number of tilings for each symmetry group. Unfortunately, there is a technical problem in determining what pattern is Islamic. Here, the references are classified into those which are concerned only with Islamic art and those which are more general. Hence the criterion used to include a pattern in this count when it appears in a publication devoted to Islamic art, or when the pattern has been taken from the photograph of an Islamic site.

The other important point to note is that the count is of patterns themselves so that if a pattern appears at several different sites, it is counted only once.

Group		Percentage (out of 1545)	Example Figure
(orbital)	(standard)		
*442 ($p4m$)	$p4m$	47.8	1
*632 ($p6m$)	$p6m$	22.2	2
2*22 (cmm)	cmm	9.5	3
4*2 ($p4g$)	$p4g$	6.4	4
632 ($p6$)	$p6$	4.2	5
*2222 (pmm)	pmm	3.9	6
442 ($p4$)	$p4$	2.9	7
3*3 ($p31m$)	$p31m$	1.2	8
2222 ($p2$)	$p2$	0.6	9
333 ($p3$)	$p3$	0.3	10
** (pm)	pm	0.3	11
22* (pmg)	pmg	0.3	12
*333 ($p3m1$)	$p3m1$	—	13
* \times (cm)	cm	—	14
O ($p1$)	$p1$	—	15
22 \times (pgg)	pgg	—	16
$\times\times$ (pg)	pg	—	17

Table 1: Counts of Symmetries in Islamic Geometric Art

The results of this can be expressed in the following Table1¹.

The symmetry group is given in both the orbital notation of Conway-Thurston [3] and the conventional one. For each class, an example has been chosen by the author as being the best representative.

The last five entries needs some explaining. No examples were found using the criterion above. However, examples were found using a more relaxed criterion. The illustration of these examples gives the details with the reasons for thinking they *may* be an example of an Islamic design.

These counts are significantly different from those given in Abas [1, page 138] which has *632 ($p6m$) ($p6m$) as being more popular than *442 ($p4m$) ($p4m$).

3 Analysis

It is not possible to be very precise about what constitutes an Islamic design for the following reasons:

1. Many sources do not indicate the artefact on which it was based and hence making it impossible to verify the authenticity of a pattern. This is true of well-known sources like Bourgoin [2] (which we have taken as authentic).

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¹The counts have been produced from version 45 of the web site and will change slightly as further tilings are added.

<i>Source</i>	$p4m$	$p6m$
Bourgoin [2]	46	60
Paccard [15]	41	10
Wade [17]	17	20
Photographs	107	49

Table 2: Counts for various sources.

2. If an Islamic design is found in a pre-Islamic source, does this imply it should not be regarded as Islamic? This might be true of Figure 13 which we have arbitrarily taken as not authentic.
3. Some Victorian sources with re-drawn patterns are known to be inaccurate. Hence it would be best to have modern colour photographs as a source, but has only been possible with a minority of patterns.
4. In Morocco, there is an active community of craftsmen producing modern patterns. It is not clear if these should be included in this collection. (Ones which are known to be recent are typically not included.)

In spite of the above, the great majority of patterns are certainly Islamic being from Palaces like the Alhambra or from Mosques.

For the Alhambra, others have claimed to find all 17 groups, see Jaworski [9] and Du Sautoy [5, Chapter 3]. The main reason for fewer symmetries here is that I use the underlying symmetry ignoring colour. This reduction in symmetry if colour is taken into account is well illustrated by Grünbaum [8]. This is also shown in Figure 18. Marcus Du Sautoy kindly sent the author his 17 tilings as small pictures which showed that the differences between us were due to the colour issue above and also to the inclusion of non-geometric patterns (which typically have fewer symmetry properties). Müller [12] in analysing the Alhambra finds $p1$ and cm which are not found in strict Islamic sources in Table 1. On the other hand, Table 1 has instances of $p31m$, $p2$ and pm not found by Müller in the Alhambra.

One can get some insight into the proportion of $p4m$ to $p6m$ but looking at the counts from various sources recorded in Table 2.

These counts are taken from the web site. A tentative conclusion might be that ‘pattern books’ have a predominance of $p6m$ due to them being very eye-catching, but that in real buildings, especially floor and ceiling designs, $p4m$ is more common due to the need to fit the pattern into a rectilinear area.

4 Acknowledgments

Thanks to a referee who made many helpful suggestions and those who responded to queries from me, namely: Jan Abas ², Marcus Du Sautoy, Branko Grünbaum and Tony Lee.

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- [10] Owen Jones. The Grammar of Ornament. 1856. Dover (reprint). ISBN048625463 URL: <http://digidcoll.library.wisc.edu/cgi-bin/DLDecArts/DLDecArts-idx?type=header&id=DLDecArts.GramOrnJones&isize=M>
- [11] J. L. Locher (editor). *Escher — The Complete Graphic Work*. Thames and Hudson. 1992. ISBN 050027696,
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² It is with sorry that this footnote is added to this paper to report the death of Jan Abas on 9th May 2009.

- [13] José M. Montesinos. Classical Tessellations and Three-Manifolds. Springer-Verlag. 1985. ISBN 3-540-15291-1
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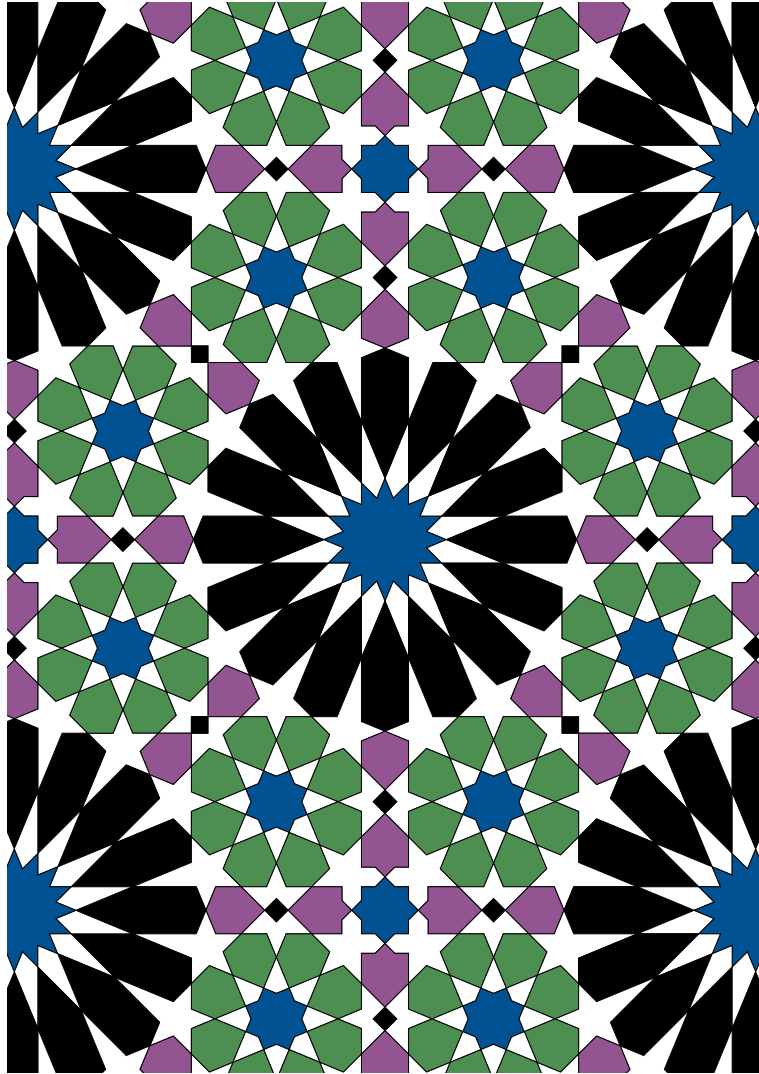


Figure 1: Alhambra tiling from M C Escher [11, Page 53]: $p4m$

This tiling is noted for having no degrees of freedom (given the angles and the topology, the edges lengths are in a fixed proportion). A good choice for Escher to draw when visiting the Alhambra.

URL: <http://www.tilingsearch.org/HTML/data172/E53.html>.

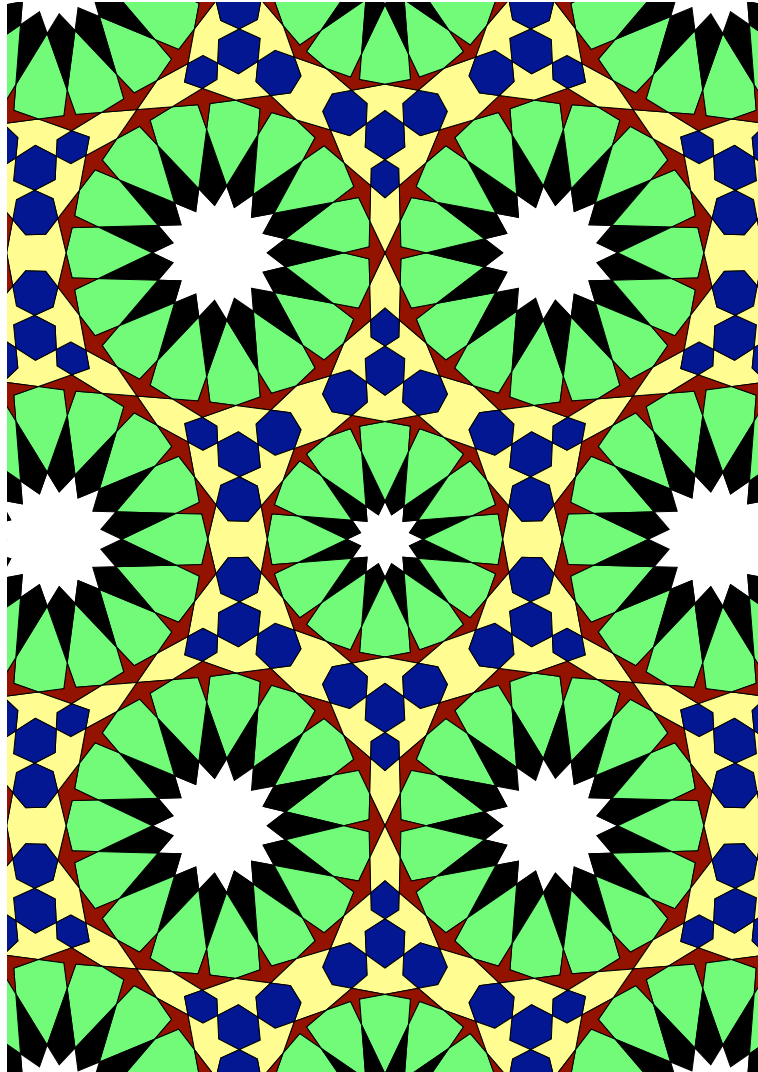


Figure 2: Bourgoin, Plate 128 [2]: $p6m$

The sources of many Bourgoin patterns are unknown — this one has not been located. Hence it may be that the original is somewhat different. The actual layout here is due to Tony Lee from an unknown Egyptian source.
URL: <http://www.tilingsearch.org/HTML/data173/B128.html>.

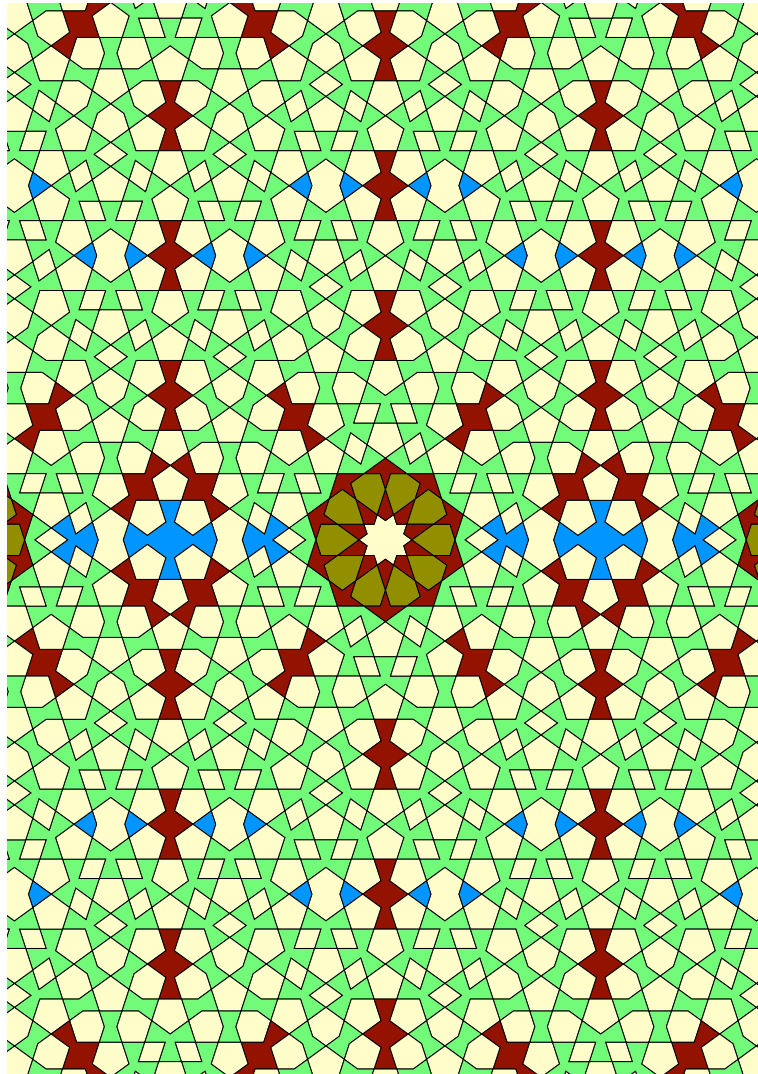


Figure 3: Bourgoin, Plate 190 [2]: *cmm*

Patterns with a ten-pointed star and regular pentagons can become quite complex as this example illustrates.

URL: <http://www.tilingsearch.org/HTML/data169/P190.html>.

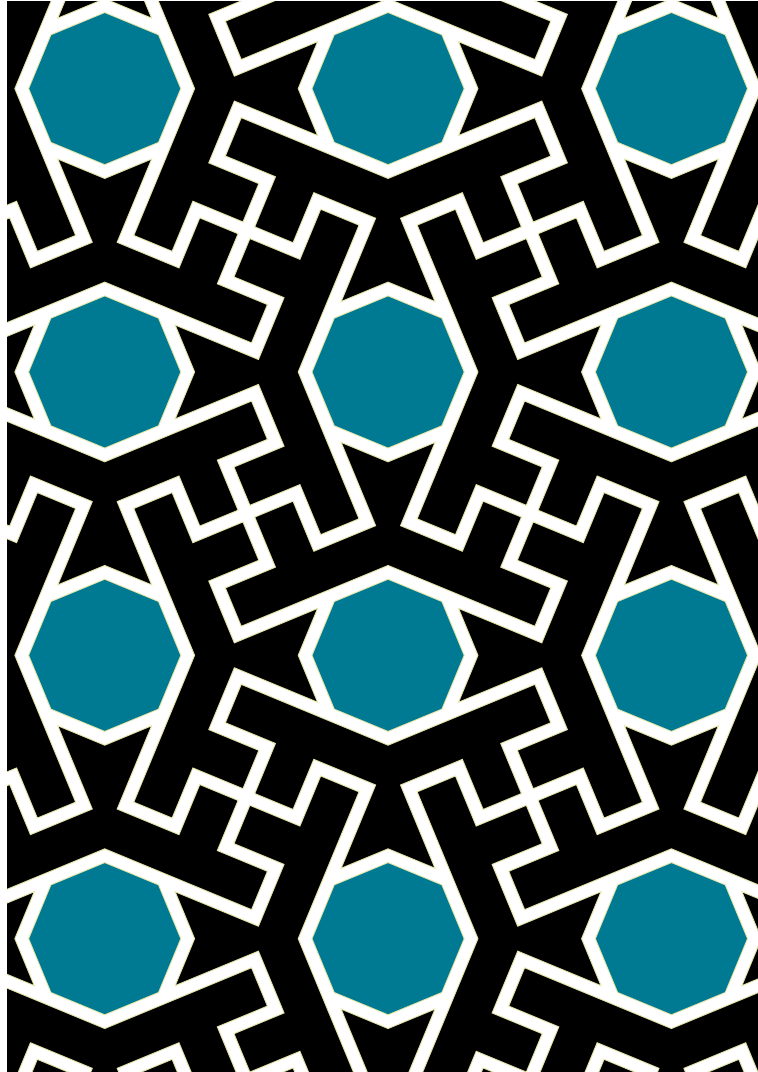


Figure 4: Topkapı Scroll, [14, page 324, Figure 69b]: *p4g*

The Topkapı scroll is one of the few contemporary written documents about Islamic design. This also looks a lot different from the other figures.

URL: <http://www.tilingsearch.org/HTML/data19/R14.html>.

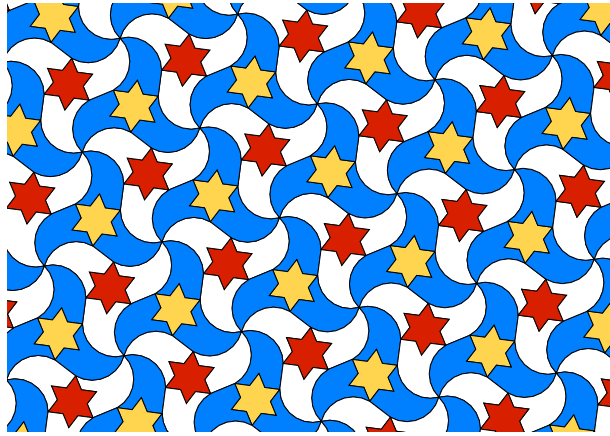


Figure 5: Paccard, [15, Voll1, page 181]: *p6*

This pattern is taken from a classic book about the art of Morocco which has active craftsmen producing such designs.

URL: <http://www.tilingsearch.org/HTML/data169/P181D3.html>.

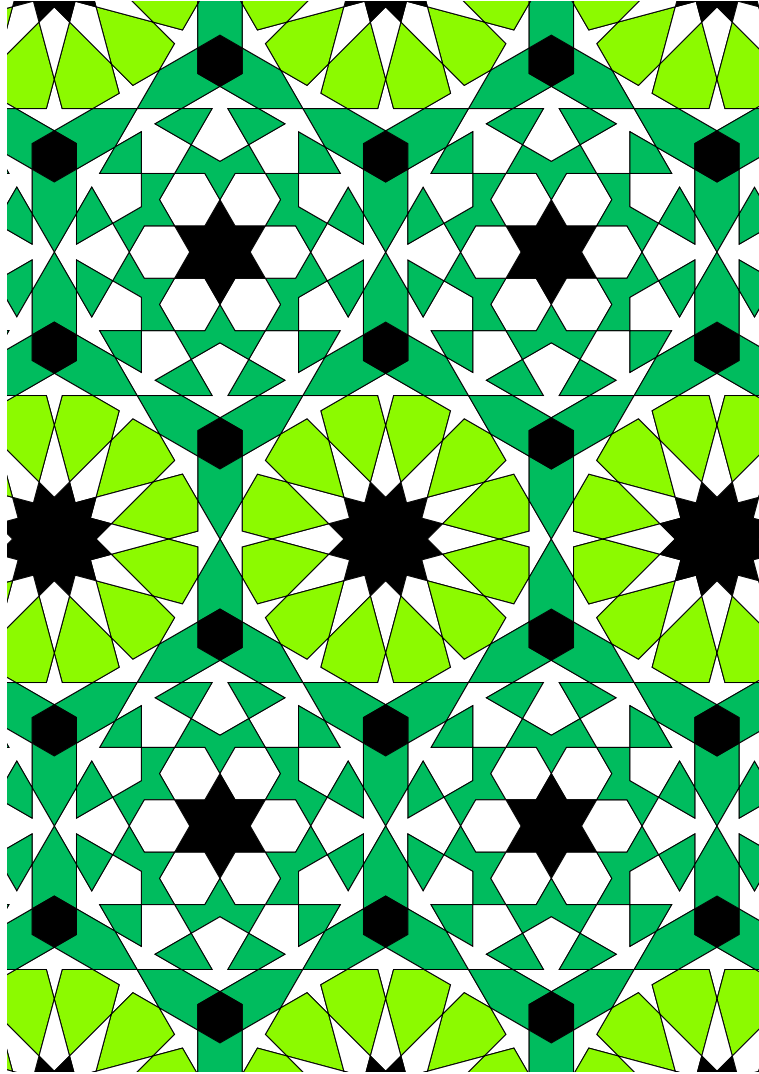


Figure 6: Bourgoin, Plate 112 [2]: *pmm*

This design is essentially two repeating horizontal strips put together. All the constituent tiles have at least one axis of symmetry.

URL: <http://www.tilingsearch.org/HTML/data169/P112.html>.

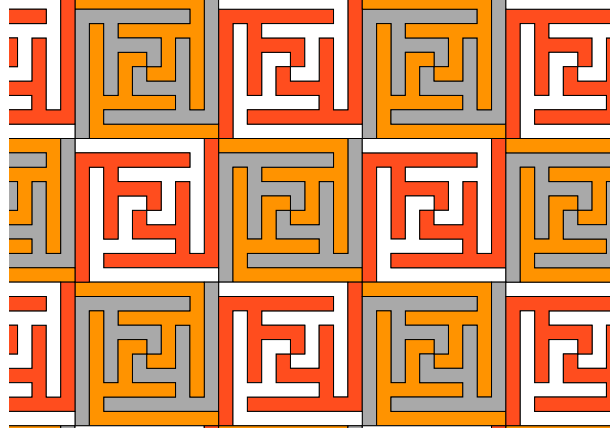


Figure 7: Yemeni Square, Ali: p_4

This pattern spells out 'Ali' in square Kufic script. See Wichmann [18] for details.

URL: <http://www.tilingsearch.org/HTML/data169/YEMEN.html>.

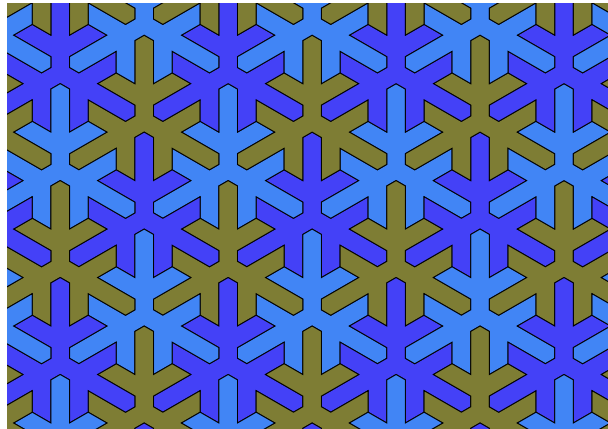


Figure 8: El-Said and Parman, [6, Figure 51]: p_{31m}

A snowflake design which comes in several different forms.

URL: <http://www.tilingsearch.org/HTML/data13/F51.html>.

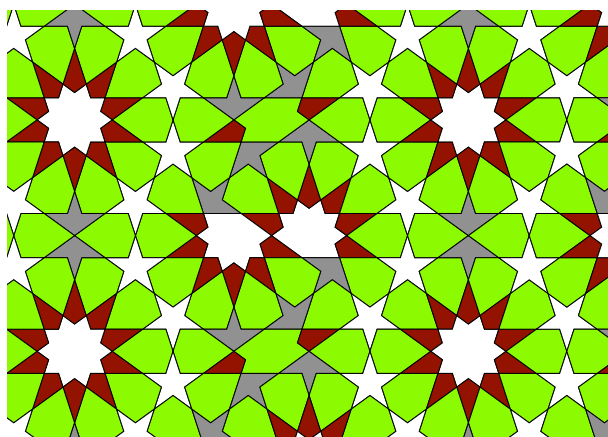


Figure 9: Bourgoin, Plate 188, bottom [2]: $p2$

This is another Bourgoin pattern for which the authentic source is not known.

URL: <http://www.tilingsearch.org/HTML/data169/P188B.html>.

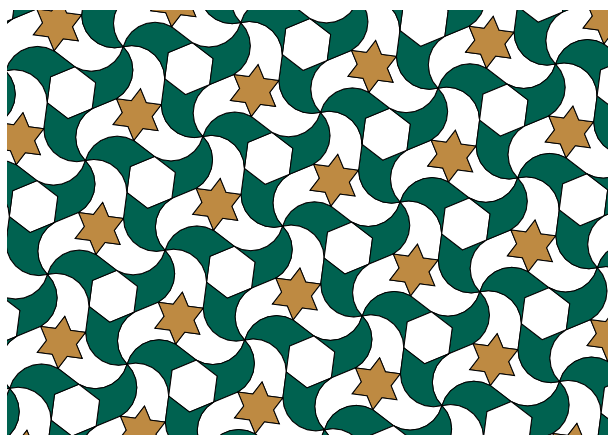


Figure 10: Patio de la Alberca, The Alhambra: $p3$

This graphic has been produced with the aid of a modern photograph (which shows the inconsistency of the colouring which allows for various interpretations of the symmetry group).

URL: <http://www.tilingsearch.org/HTML/data159/P052A.html>.

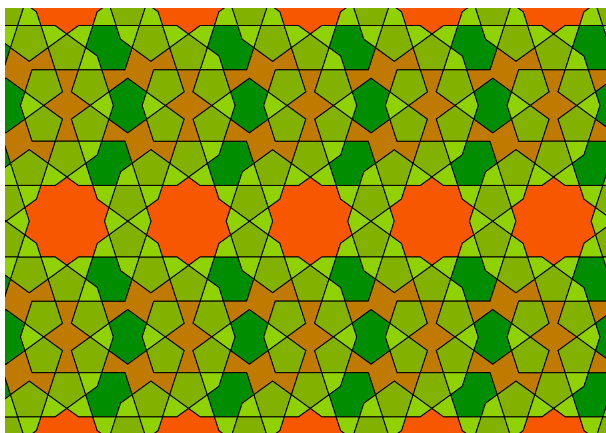


Figure 11: Topkapı Scroll, [14, page 313, Figure 50]: *pm*

URL: <http://www.tilingsearch.org/HTML/data19/R6.html>.

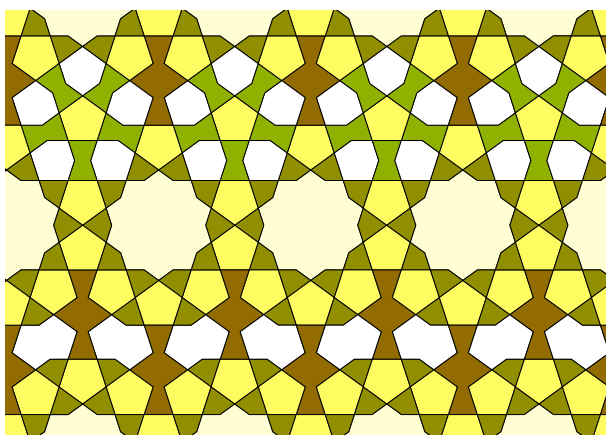


Figure 12: Mirza Akber, Sheet 31: *pmg*

The work of Mirza Akber, the architect to the court of Persia, was donated to the V & A in Victorian times — it has not been widely studied. See a recent book by Dutta [4].

URL: <http://www.tilingsearch.org/HTML/data9/S31A.html>.

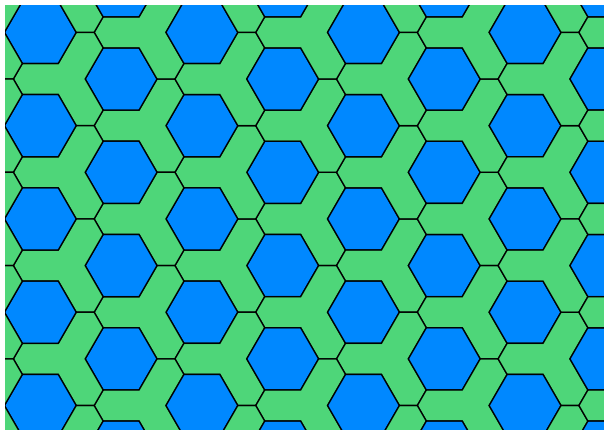


Figure 13: Owen Jones, Plate XLIV(15) [10]: $p3m1$

This is from a Persian manuscript in the British Museum and hence it is unclear if it is an authentic Islamic design.

URL: <http://www.tilingsearch.org/HTML/data5/J44A.html>.

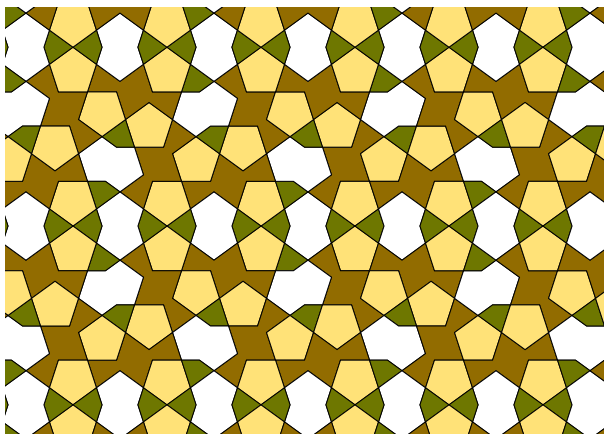


Figure 14: Some patterns using specific tiles, 34: cm

This pattern is a recent construction, see Rigby [16]. It uses tiles frequently used in Islamic designs, see Figure 12.

URL: <http://www.tilingsearch.org/HTML/data154/TRY37.html>.

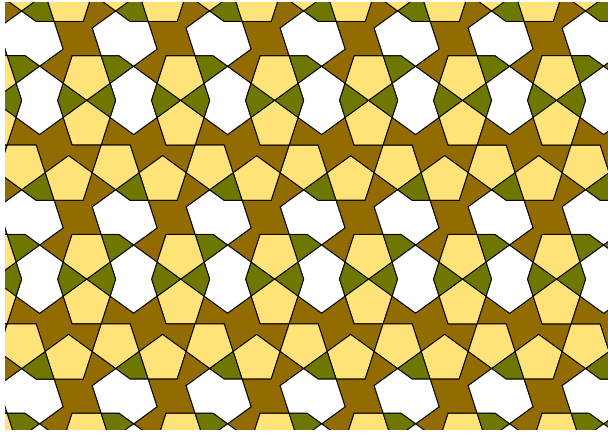


Figure 15: Some patterns using specific tiles, 33: *p1*

See note on the last figure.

URL: <http://www.tilingsearch.org/HTML/data154/TRY39.html>.

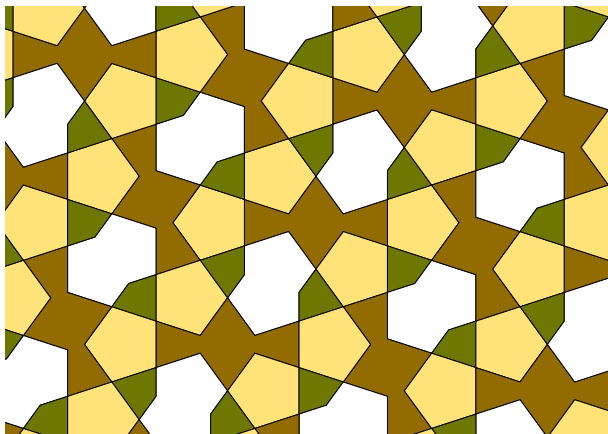


Figure 16: Some patterns using specific tiles, 53: *pgg*

See note on Figure 14.

URL: <http://www.tilingsearch.org/HTML/data154/TRY00.html>.

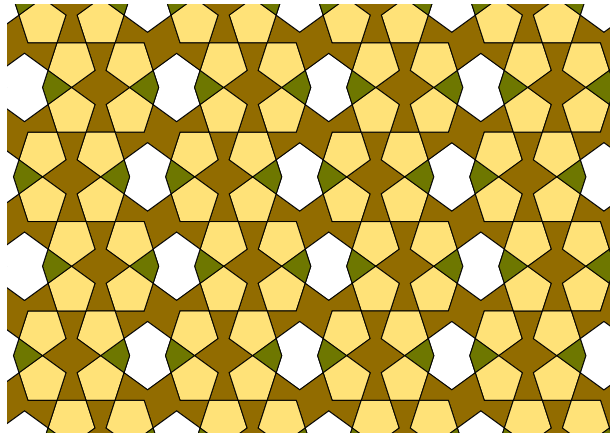


Figure 17: Some patterns using specific tiles, 5: *pg*

See note on Figure 14.

URL: <http://www.tilingsearch.org/HTML/data154/TRY77.html>.

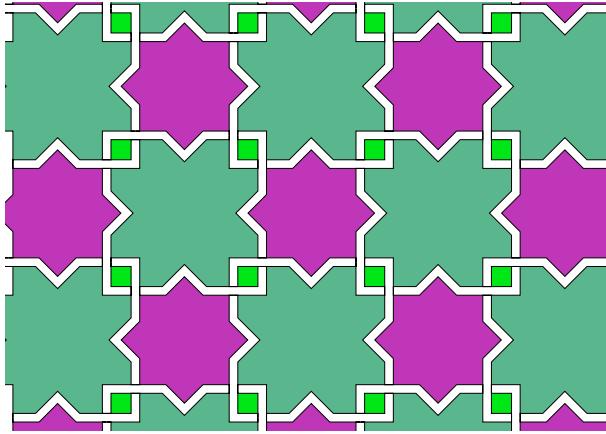


Figure 18: Presentation of symmetry: $p4m$ or $p4$

As shown, taking the interlacing into account, this tiling has symmetry $p4$ as recorded in Montesinos [13, Page 227]. However, if the interlacing is ignored, then there is reflective symmetry making the tiling symmetry $p4m$.

Also, the colours of the polygons can be changed to reduce the symmetry — for instance, if one tile was coloured with a unique colour, there would be no symmetry. (For technical reasons, the colour here is different from the original in the Alhambra.)

URL: <http://www.tilingsearch.org/HTML/data15/CM014.html>.