Ans Mon 3 Nov 1975 Ans Mon 3 Nov 1975

MEMORANDA

ISLAMIC STAR PATTERNS ~ NOTES

A. J. LEE, from November 1975

Summaries of main results of Researches into the Geometry of Islamic Star Patterns. during Nov 1964 —

These notes are fairly haphazard, so no detailed lists of contents are feasible, but a few major topics are listed below

Fri 8 Feb 1985

The pink underlinings on the left margins of pp. ii–iv are from earlier pencilled notes, and should be ignored.

MEMORANDA

Islamic Star Patterns – Notes

I first became seriously interested in studying Islamic geometric ornament, and especially the star patterns, in November 1964, after borrowing a copy of "Moorish Spain" by E. Sordo & W. Swaan (1963) from St. Albans public library.* I had always been interested in mathematics, particularly geometry and symmetry, and tilings of various plane surfaces – euclidean plane, sphere, hyperbolic plane, and multidimensional polytopes, so my mind was ripe for what was up until then a totally new field of inquiry for me. Although the book just referred to did not concentrate on geometric patterns there were nevertheless enough examples illustrated to give me the impression that the star patterns could be systematically studied, not only at the purely geometrical level, individually for each new pattern, but at a higher level, generalizing methods of motif construction, & of means of linking groups of motifs into units of repeating patterns. Indeed, many patterns could obviously be grouped into related series so clearly that often "missing" members of a given series could be reconstructed, which I did not at first realize were to found as authentic Islamic patterns outside Spain. My interest was thoroughly fired, so I began searching the shelves of the public library for any kind of books which might give photographs of Islamic ornament. These included travel books as well as more specialist books on Islamic art and architecture.

One book from the public library which particularly started me reading the more interesting publications on the subject was "The Legacy of Islam", edited by Sir Thomas Arnold & Alfred Guillaume (1931). From this book, and "Islamic Architecture and its Decoration" by D. Hill & Oleg Grabar (1964)* I was able

* I bought my own copy on 6 February 1968.

* My copy is dated 20 March 1965; I purchased a second copy on Feb 1976.

After Fri 8 Feb 1985

to borrow a number of works through The St. Albans public library; among These were
E.H. Hankin (1925) "The drawing of ~~Saracenic~~ Geometric Patterns in Saracenic Art";
M.J. Bourgoin (1879) "Le Trait des Entrelacs";
B.P. Denike (1939) "~~Uzbekistan~~ Central Asian Architectural Ornament"
L.I. Rempel (1961) "Uzbekistan Architect. Ornament"
The last two publications being in Russian.
All These works were examined in the first half of 1965, and by the end of 1965 I had thoroughly laid the foundations for a good deal of my subsequent studies into The geometry of the star patterns. Over the years numerous files of notes, sketches and detailed drawings began to bulge, notebooks were filled and my own bookshelves began to see an increasing number of works devoted to Islamic Art and culture.

Since I did not have access to a photocopier at first, I copied out by hand large chunks of the texts of various books I borrowed, tracing The illustrations ~~where~~ feasible. In This way I traced almost all of the 200 plates from Bourgoin's work, copying out all the French notes to each plate, and I copied whole chapters and Illustrations, in Russian, from Rempel's book in the same way. Most of The papers by Hankin I also copied, and I still have my original hand copies of these works. Dover Books subsequently (1973) reissued the plates from Bourgoin's book,* but I don't think my earlier effort was in vain, since it was an extremely useful exercise in industry and pattern drawing.

As well as examining a wide selection of books of all kinds, from public library and University College library,* I also borrowed a few colour slides from a number of people, including Dr. Jenny Parrington, Prof. Hans Grünebig, Mrs Jan Leece (then Mitchell) and my cousin Mrs Joyce Kraus (née Folds).

* London

* under The title "Arabic Geometrical Pattern and Design."

Fri 8 Feb 1985

However, it wasn't until fairly late into the 1970's that I began to think about setting down the results of my studies in some coherent fashion, with vague ideas of getting something published. It is curious that this quickening of my interest coincided with the Islamic Festival Year of 1976, held in various centres throughout Great Britain, and marked by the appearance of a number of new books, among them three on Islamic geometric patterns:

D. Wade (1976) "Pattern in Islamic Art".
I. El-Said & A. Parman (1976) "Geometric Concepts in Islamic Art".
K. Critchlow (1976) "Islamic Patterns. An Analytical and Cosmological Approach"

All three works were on a superficial level, suitable for members of the general public with some interest in pretty patterns, but none showed any deep understanding of Islamic patterns or of geometry or the symmetry of patterns in general.

In 1977 I obtained a borrower's ticket at the Library of the School of Oriental and African Studies, which greatly enlarged the range of books and journals available to me. In the same year I read an article in the December issue of "Art & Archaeology Research Papers" (ed. by Dalu Jones & George Michell) by William Betsch on "The Fountains of Fez" (pp. 33–46).* It wasn't until August 1978 that I wrote to Betsch, pointing out our mutual interests in Moroccan zellij ornament. Eventually we met; I showed him a number of my files of drawings and notes on Moroccan patterns and I saw many of his slides taken on his many journeys to Fez. Almost immediately we were both fired by an enthusiasm for producing a joint work on Moroccan zellij – he was to provide the primary data by photographing original examples and interviewing craftsmen, and I was to provide geometrical analyses, drawings and much of the analytical text. As a preliminary exercise

* I bought a copy of this number on 28 July 1978, so I obviously did not see it immediately it was published.

Fri 8 February 1985

MEMORANDA

I borrowed about a hundred of Betsch's slides of Fez fountains, and my analyses and drawings from these now fill at least two fat files, in addition to numerous black and white prints and many loose notes and drawings directly derived from these slides. Unfortunately, nothing came of this intended collaboration. I did write an introductory paper on "Islamic Star Patterns", which was to have been the first of two or perhaps three papers centred on Moroccan zellij. I submitted this to Art & Archaeology Research Papers (AARP) in April, 1980, but by that time the publishers were in financial difficulty and were unable to accept any new manuscripts. A further enquiry written in October 1982 showed no change in the situation. Meanwhile, a number of people in University College London saw the paper, expressed interest, and urged me to get it published. Among these were Prof. Hans Kalmus, and Prof. C.A. Rogers in the mathematics dept. U.C.L. Prof. Rogers in fact showed the manuscript to a number of other people and made some efforts towards getting some means of publication. I received some particularly favourable comments from Dr. Robert Hillenbrand in Edinburgh University (November 1983). Urged on by all this activity on my behalf I submitted the a revised manuscript of "Islamic Star Patterns" to Muqarnas, an annual devoted to Islamic art & architecture, and edited by Prof. Oleg Grabar, on 6th February 1984. It was accepted on the 19th of April, and if all proceeds well it is scheduled to appear in 1986,* according to my most recent information.

Regarding the previous paper as directed mainly at the "Arts" side, I have recently been making efforts at writing a similar paper emphasizing geometrical aspects, and which will be submitted to a scientific journal, but to date nothing definite has been worked out. I still have a continuing desire to summarize all of my researches as a larger work, to be published as a book, and notebooks such as this one, and completed preliminary papers are all directed towards the eventual realization of this aim.

* Muqarnas 4: 182–197, 34 figs. (1987) Mon 23 Nov '87

See revised nomenclature of geom. rosettes on p. 21 → Sat 3 Dec 1977

Mon 3 Nov 1975 — Saturday, JANUARY 1, 1966

<u>Geometrical Rosettes. – Nomenclature.</u> ("Geometrical arabesque" (Hankin, '25)

This motif is peculiar to Islamic ornament, although sporadically borrowed by other cultures, especially those temporarily conquered by Islam.

For descriptive purposes terms are required for all parts of the rosette which undergo variation from pattern to pattern. Preliminary suggestions are given on the opposite page, subject to later revision and improvement.

A rosette is named after the number of principal radii it contains (= number of outer cells, etc), i.e. an *n*-rayed rosette has *n* outer cells, etc., and *n* planes of symmetry through its centre, *e*. If *n* is even, these symmetry planes or axes occur in perpendicular pairs, and the rosette may be centred on at least diad axes in any given repeating pattern. It should be noted however that the usual representation of such a rosette as an interlacing band pattern will destroy the mirror axes and reduce its overall symmetry to simple rotational symmetry, of left- or right-handedness.

The most frequent form of Type I rosettes is as shown in Fig. 1, with the four points *a*, *b*, *c* and *d*. Occasionally the innermost segments *cd* are ~~[illegible]~~ omitted and *c* becomes the inner point. Less often segments *cd* are continued inwards to extra points *d'*.

It should be obvious that each rosette is drawn perfectly regularly

Sunday, JANUARY 2, 1966

by dividing the space round the centre *e* into 2n equal angles. All points at a given level, whether *a*, *b*, *c* etc are at the same distance from the centre, i.e. they are situated on circles of radii *ae*, *be*, *ce*, etc. Points *b*, *c* and *d* are collinear so that it is only necessary to locate radii for any two of them and the third will be automatically determined. It is usually sufficient to locate *b* and *c*. The slope of the <u>lateral segments</u> of the rosette rays is an important factor in determining the location of the points *b*, *c* and *d*. When the lateral segments are parallel to the principal radius, and hence to each other, they may be described as "<u>parallel</u>". If they converge peripherally, as in the rosette shown in Fig. 1, N.B. The labelling of points a, b .... e, f needs revision (2

Monday, JANUARY 3, 1966

sun 2 Nov 1975

Suggested Nomenclature for Type I Rosettes.

Fig. 1

Usual method of location of points b.

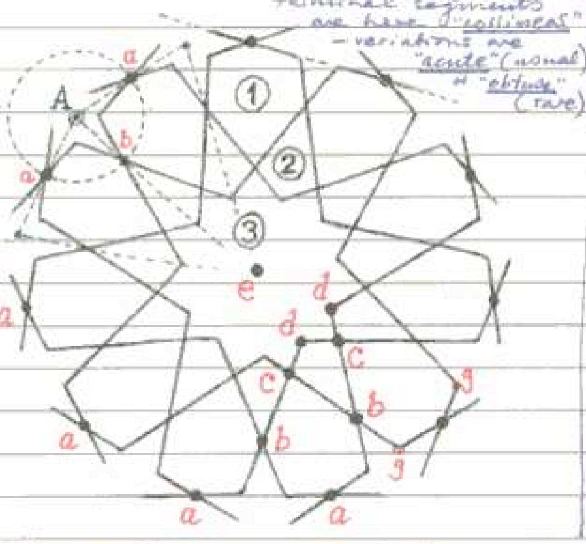
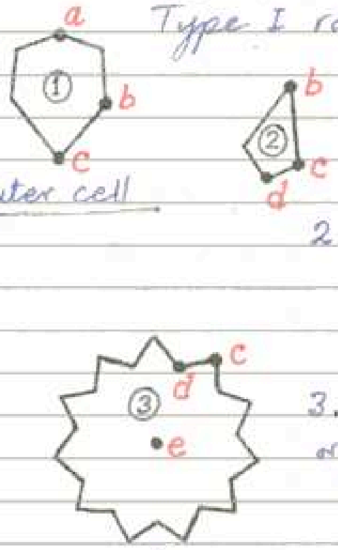


Fig. 2

Constituent cells of a Type I rosette.



1. Outer cell

2. Mid-cell

3. Inner cell, or Central Star

a = outer points, on circumcircle;
b = outer midpoints, on outer midcircle;
c = inner midpoints, on inner mid-circle;
d = inner points, on in-circle;
e = centre of rosette.

The terminal cross or cap cross is drawn through each point a.

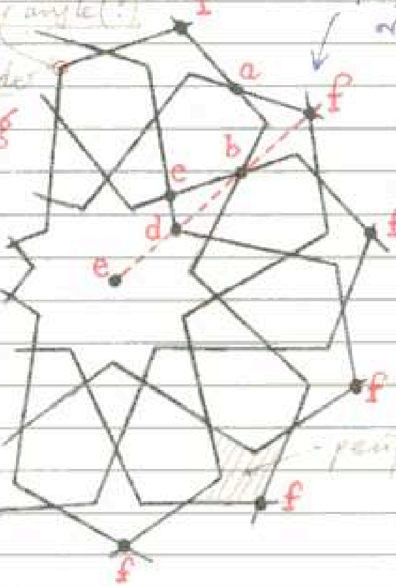
Fig. 3

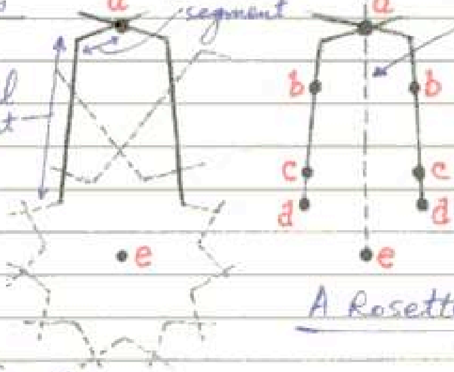
shoulder angle (?)

shoulder or point g ?

subterminal point

edbf = secondary radius, or interradius





Lateral segments are here "convergent" – variations are "parallel" (usual) or "divergent" (rare)

A Rosette Ray

Fig. 4

The repetition n times round the centre e in an n-rayed rosette results in the formation of the outer, mid- and inner cells by multiple intersections of the sides of all n rays.

Stellated Rosette of Type I

f = stellate points on stellate circle.

Simpler terminology:–
terminal segment = cap
lateral segment = side
(e.g. "rays with collinear caps, sides parallel" etc.

Mon 3 Nov 1975

they will be termed "convergent". If they diverge peripherally they are described as "divergent". Divergent lateral segments are rarely encountered; rosettes usually have either parallel lateral segments or they are slightly convergent.

Similar variations are seen in the slope of the terminal segments. If adjacent terminal segments form part of the straight line joining the two terminal points a, then they are termed "collinear". If they slope away from this line, towards the centre of the rosette, they are termed "acute"; if they slope on the other side of this line away from the centre, they are termed "obtuse".

Most terminal segments are collinear or acute. Obtuse terminal segments are rare and produce an unsatisfactory effect. It is obvious that in the case of collinear terminal segments, these form part of a regular ~~n-gon~~ polygon, with as many sides as the rosette has rays, whose vertices are point a.

When rosettes are combined in patterns, ~~they~~ two rosettes are most commonly linked by sharing a common terminal point a in such a manner that this shared point a is on the straight line joining their two centres c', c'', shown in Fig. 8 opposite. The most usual and most logical solution is that in which the terminal segments of the two rosettes form two lines crossing at point a, i.e. in Fig. 8, $\theta' = \theta''$. Since the terminal segments continue as two straight lines through the contact point a, this type of junction may be called "continuous". It is the normal method of joining rosettes; indeed, there is no reason to adopt any departure from this method particularly since such a departure produces an ugly effect.

Mon 3 Nov 1975

one really needs to refer to the shoulder, or point "g" here.

This is unsatisfactory, in that the two external segments are unequal. See p.

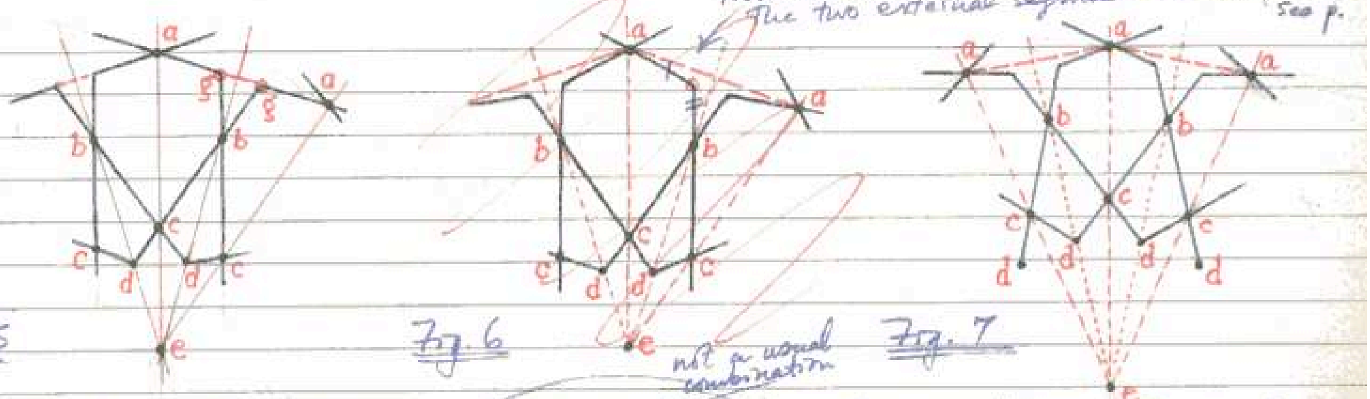


Fig. 5 Fig. 6 Fig. 7

Terminal segments collinear
Lateral segments parallel

T.ss. acute
L.ss. parallel

Terminal segments acute
Lateral segments convergent

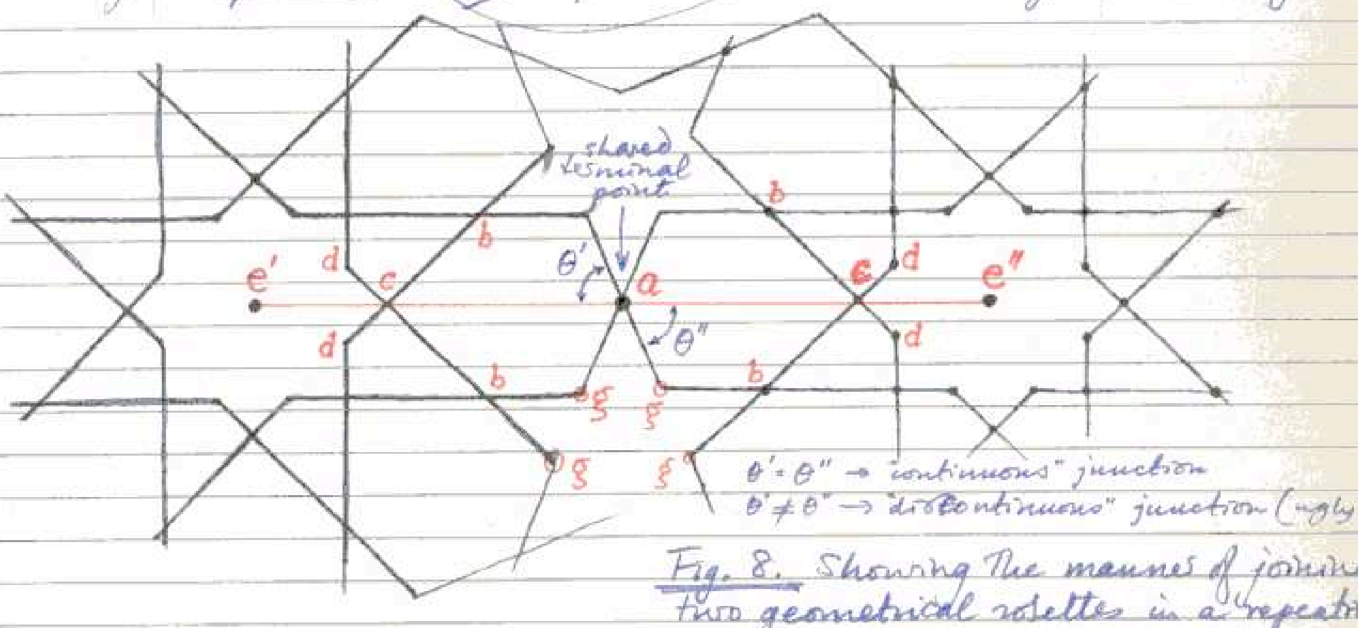


Fig. 8. Showing the manner of joinin[g] two geometrical rosettes in a "repeati[ng] pattern. e'ae" is a straight line.

This shows a simple type I junction, at the shared terminal point a. If the rosettes are identical, obviously θ' = θ"; the latter relation usual[ly] holds when the rosettes are dissimilar, i.e. the four terminal segment[s] form two lines crossing at point a. It is obvious that if the rosetta[s] are unequal they cannot both have collinear terminal segment[s]. If one of them is given collinear terminal segments it is usually t[he] smaller of the two, in which case those of the larger rosettes will automatically become acute. The junction of terminal segments sho[wn] above may be termed "continuous". If θ' ≠ θ" the junction may be ca[lled] "discontinuous" — this however produces a very ugly effect and is uncommon. (This becomes especially ugly when drawn as interlacing-bands).

5)

Mon 3 Nov 1975

Nomenclature of type II Rosettes.

N.B. One should perhaps not speak of Type II, III etc rosettes, but stars. Type II etc should be used with patterns.

The structure of Type II rosette is simpler than that of Type I. The inner points d of the first type are usually absent, and the terminal and lateral segments are confluent due to the fact that points a, b and c have become collinear.

Patterns involving Type II rosettes usually have a more rigidly geometrical appearance and a less decorative quality than type I patterns, and indeed a type II rosette bears a much closer resemblance to a star polygon than does a type I. In some cases, as in the 10-pointed rosette illustrated in Fig. 9, the rosette is simply a regular star polygon with the innermost segments omitted.

Where appropriate, the nomenclature of regular polygons could be adapted to Type II rosettes; e.g. the rosette of fig. 9 is directly derived from a $\{10/3\}$. However, not all type II rosettes have their lateral segments aligned between two a points — the precise angles between a pair of lateral segments usually depends on such factors as the relative sizes of the different kinds of rosette constituting the pattern.

Comparing the rosettes from Types I and II versions of the same pattern, it will often be noticed that the two types of rosette can be superimposed on their a and b points, which for this reason are termed nodal points. This is a feature in particular of those patterns using (3 x 2) rhombuses (see later). (fig. 11)

Type II rosettes are linked in a similar manner to those of Type I.

Type III Rosettes are formed directly from type II by enlarging the outer cells of the latter until they overlap slightly adjacent cells, forming small overlapping rhombs (fig. 12).

Type III rosettes are joined not at the newly created secondary terminal points a′, but by sharing the original terminal point a, which in effect means that the outer cells of two joined rosettes will themselves overlap.

Mon 3 Nov 1975

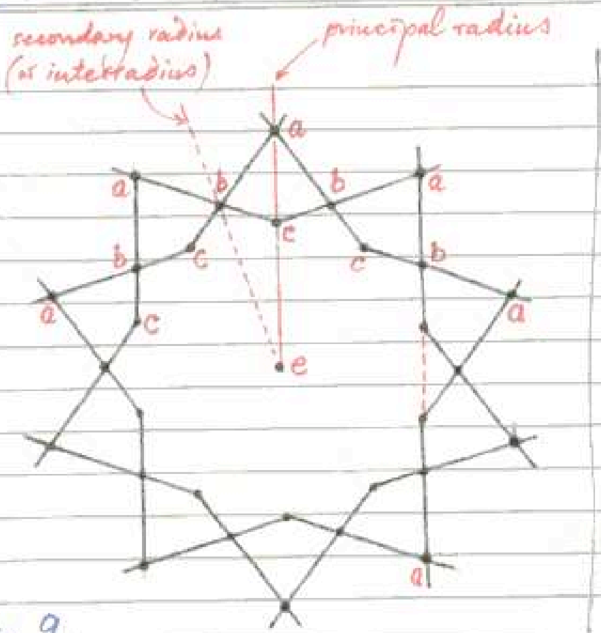


Fig. 9.
a = outer points b = mid-points
c = inner points. e = centre.

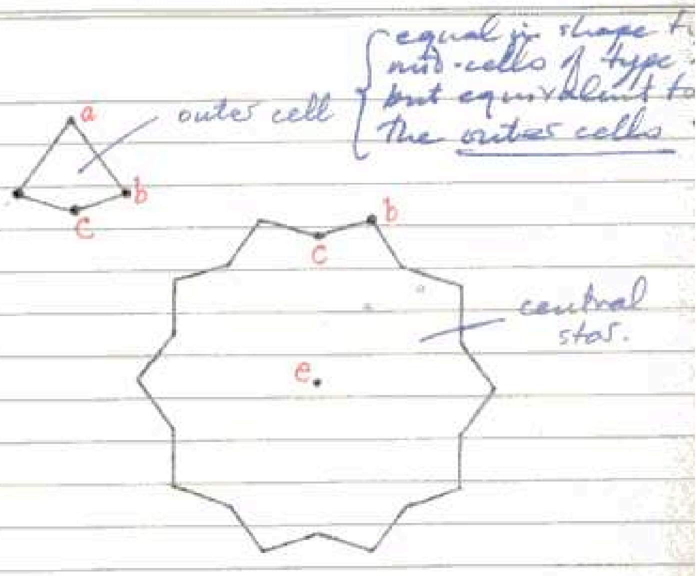


Fig 10. Constituent cells of a Type
Rosette.

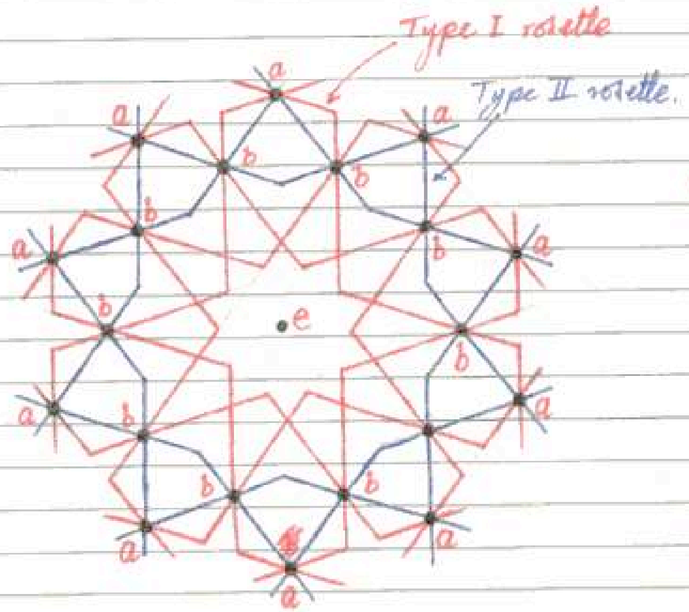


Fig. 11. Illustrating the relation between
Type I and II rosettes on the
same nodal points, a and b

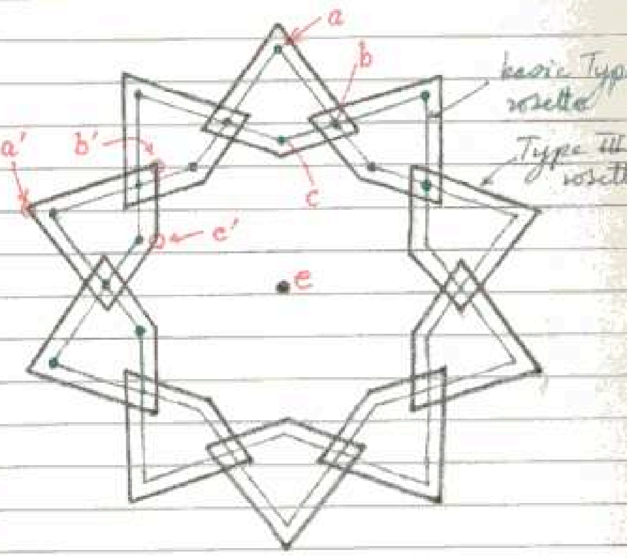


Fig. 12. Formation of a Type III rosette
by independently enlarging the
outer cells until they overlap
in small rhombuses at points b.

Wed 5 Nov 1975.

RHOMBIC PATTERNS – Introduction

One of the most common and widespread patterns using type 1 rosettes in contact at their outer points is that shown at fig. 13A. The pattern consists of an infinite extension of the basic unit shown, i.e. the rhombus ABCD is endlessly repeated by translation in two directions. An important point brought out by the figure is that the rhombus has definite and specific angles characteristic of 10-rayed rosettes, namely, a smaller angle, at A and C, of 72° (4 divisions of 18° each), and a larger angle, at B and D, of 108° (6 divisions of 18° each). For ease in future calculations it is convenient to label the size of rhombus by means of the acute angles of the right-angled triangle ADE, that at A having 2 divisions, that at D having 3 divisions. Since the fundamental angle size for any rosette of n rays is π/n the angle sizes are appropriately given as fractions of π (radians), or 180°. Thus the rhomb ABCD is a (3,2) rhomb with 10-rayed rosettes centred on its vertices, i.e. the acute angles in triangle ADE are respectively 3/10 and 2/10, expressed as fractions of 180°. Note that an angle ~~shown~~ expressed as 2/10 must not be reduced to the form 1/5.

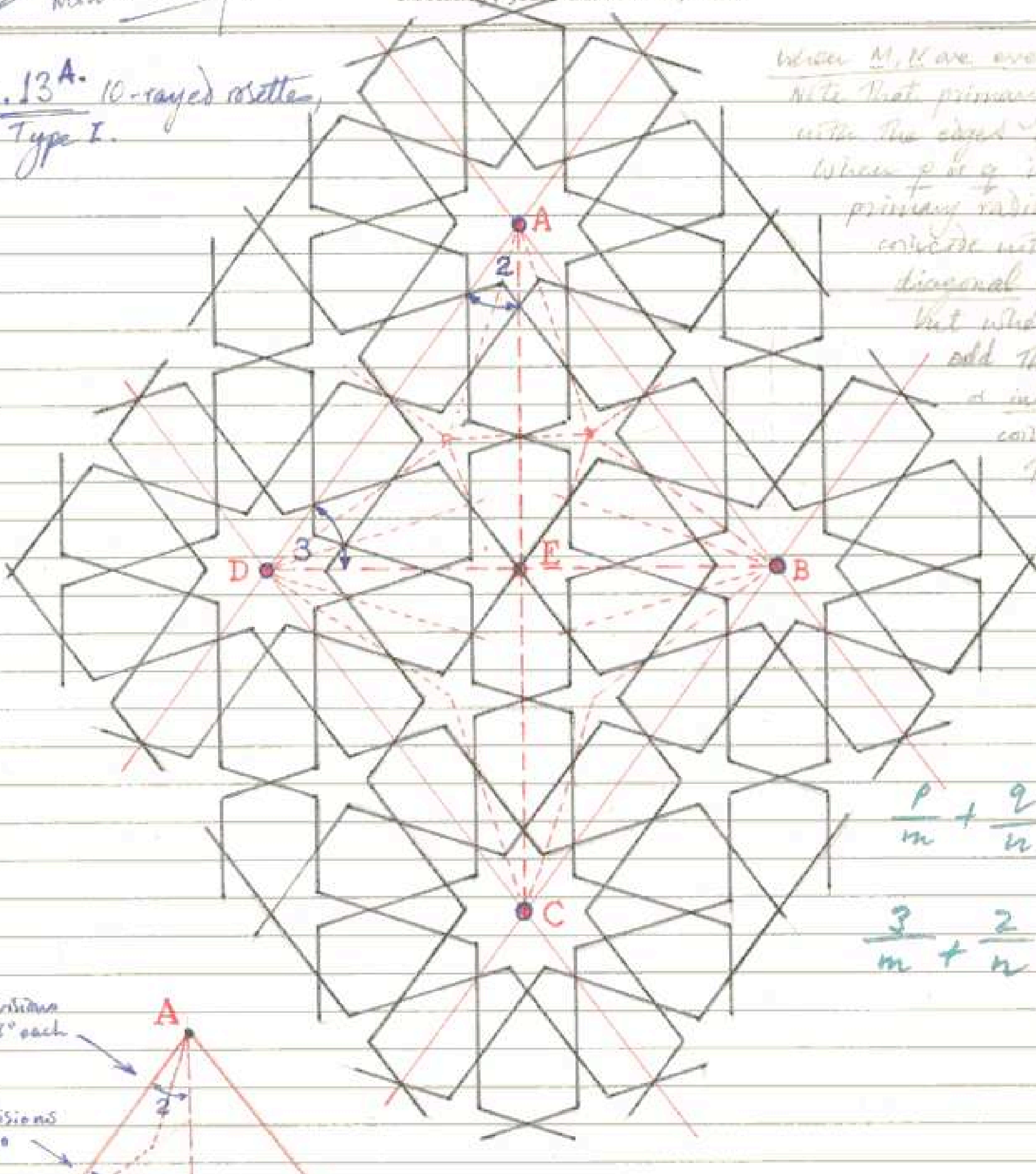
Previous authors have noted that Islamic patterns often use rhombs as repeating units in addition to other shapes. No one seems to have investigated this aspect systematically however. For example, are other ~~sizes~~ shapes of rhombs possible with 10-rayed rosettes, and if so, how many? different sizes are possible Can (3,2) rhombs be constructed using other kinds of rosettes, other than 10's, and again, if so, how many? Can basic patterns be constructed using rhombs of two or more different kinds sizes simultaneously, for given rosette numbers?

Using the appropriate methods these problems are easily solved, and the answers greatly enlarge the possibilities for variation among Islamic, and indeed other kinds of patterns. (see p. 11 ...)

Patterns using rhombs in the manner of Fig. 13A, with the sides of the rhomb coinciding with radii of the rosettes will have their angles integral multiples of the fundamental angle π/n. Other patterns, among which are those termed elsewhere "dislocations" use rhombs whose sides do not coincide with any radius of a rosette, and which have in consequence non-integral multiples of the angle π/n.

Nov 3 Nov 1975

Fig. 13^A. 10-rayed rosette, Type I.



When M, N are even-numbered — note that primary radii coincide with the edges of the rhombus. When p or q is even, a primary radius will also coincide with the rhombus diagonal from that vertex, but when p or q is odd the secondary, or inter-radius will coincide with the axis from the vertex.

$$\frac{p}{m} + \frac{q}{n} + \frac{1}{2} = 1$$

$$\frac{3}{m} + \frac{2}{n} + \frac{1}{2} = 1$$

2 divisions of 18° each

3 divisions of 18°

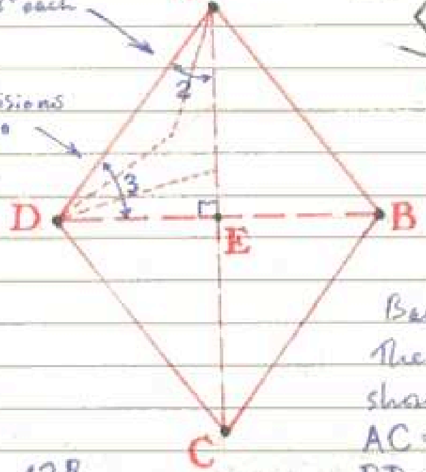


Fig. 13^B

Basic rhombus for the repeating pattern shown above.
AC = major axis
BD = minor axis E = centre

Rhombus = equilateral parallelogram; axes bisect one another at right angles, at the centre, E.

Thu 6 Nov 1975

Mon 3 Nov 1975

ah = bh

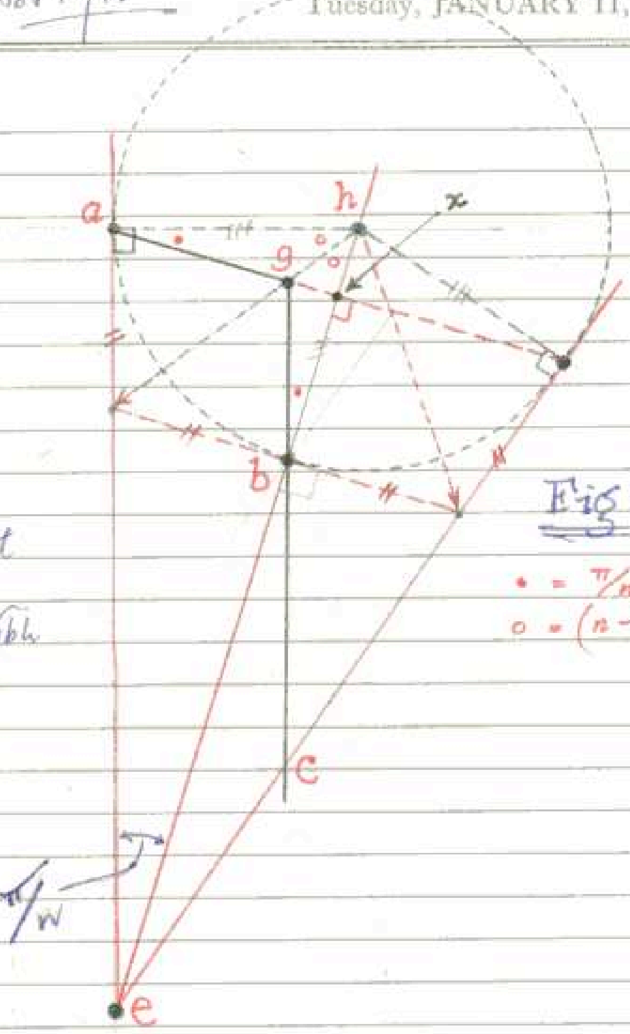
$\widehat{ahg} = \widehat{ghb}$

ag = bg

∴ triangles agh, bgh are congruent

$\widehat{gah} = \pi/n = \widehat{gbh}$

∴ gb ∥ ae.



When point **b** is chosen in this way, and g lies on the bisector of $\widehat{ahb}$, and terminal segments are collinear, then lateral segments will be parallel.

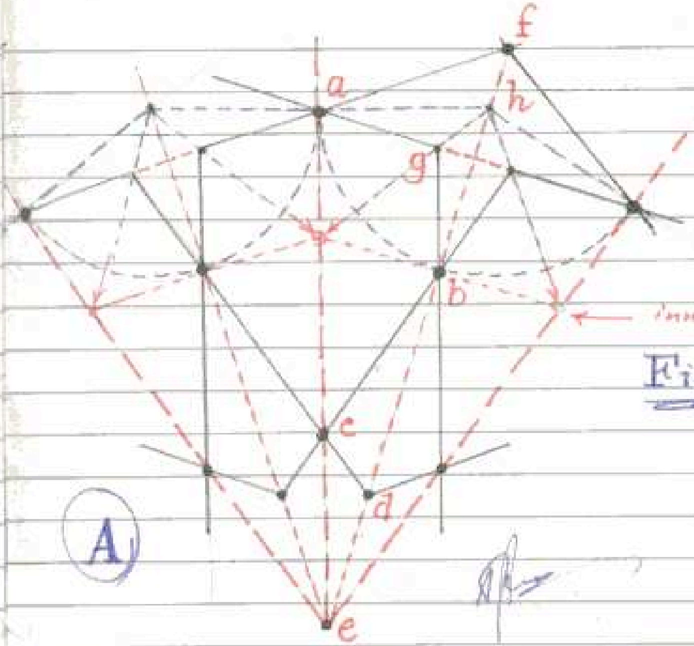
Fig 14.

$\bullet = \pi/n$

$\circ = (n-2/4n).\pi$

Note: The detailed construction for peripheral stars given here only needs to be done on one star in a pattern. After the points a, b and c are found, circles of radius ae, be and ce, respectively, will determined all the remaining points on other radii and interradii. The slope of the terminal segments of the rosette, if they are not collinear, can be constructed easily by continuing the terminal segment ag to the point x on interradius eh, then drawing a circle with radius ex through the remaining interradii. Draw lines with slope ax, which cut the lateral segments in points g, and the rosette is completed. Thus, the use of circles to find the majority of repeated points in a pattern greatly simplifies the task of construction, and avoids the tedious construction of complete nets of regular or non-reg. polygons as advocated by Hankin (1901, 1925) and other authors.

Mon 3 Nov 1975



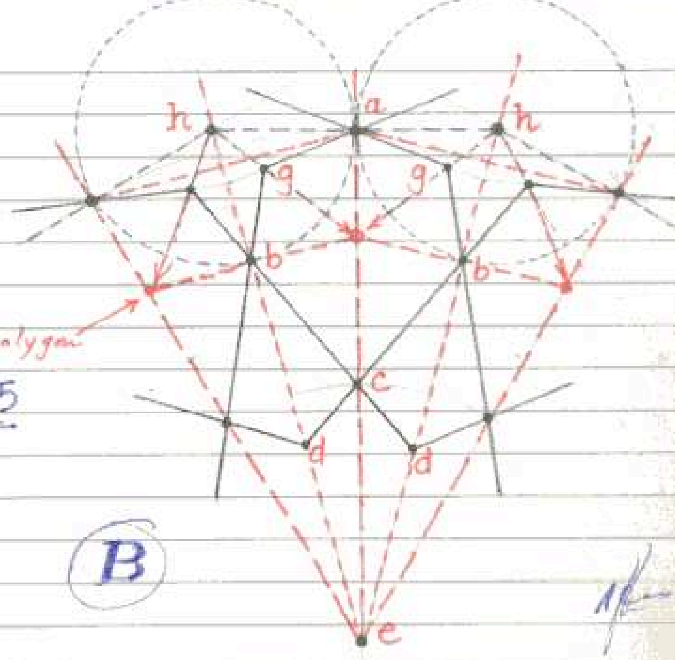


Fig 15

+ Terminal segments (ag) collinear
+ Lateral segments (gd) parallel

+ Terminal segments (ag) acute
+ Lateral segments (gd) converge

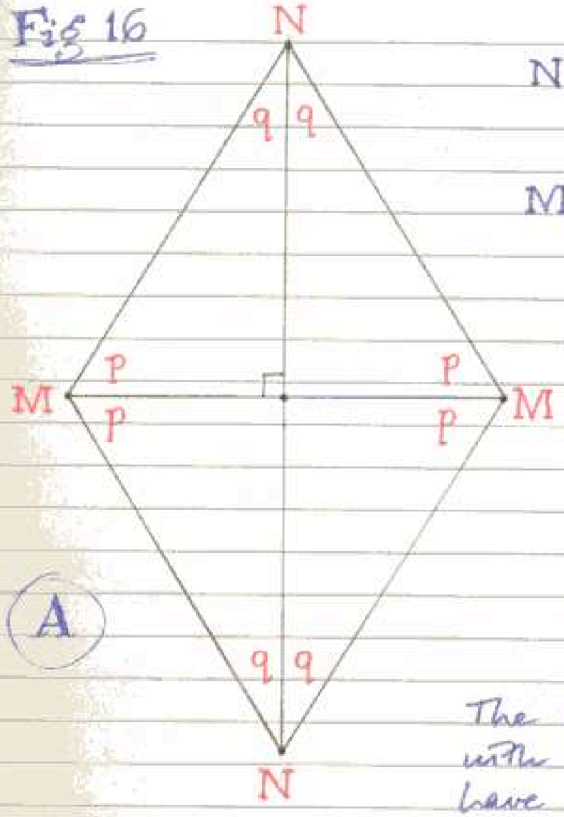
The types of terminal and lateral segments slopes cannot usually be chosen independently. It should be noted in both diagrams above that ah = bh and point g lies on the bisector of angle ahb, i.e. $a\hat{h}g = g\hat{h}b$. When the terminal segments are acute as in B, the predetermined position of point b necessitates converge lateral segments.

The main result achieved in the above figures is that ag = g
In cases where ag ≠ gb it is better to make ag < gb rather tha the other way round.

NOTE: When angle $h\hat{a}g$ is not fixed by the collinearity of the terminal segment, it may be determined by the angle θ (fig 8) of a dissimi rosette joined at a. If point b is determined by the circle shown above centred on h then the slope of the lateral segment is al determined. In the Maghreb the larger of two joined rosettes is usually given parallel sided rays of a width equal to those the smaller rosette (e.g. in the case of 8's and 16's in the same pattern).

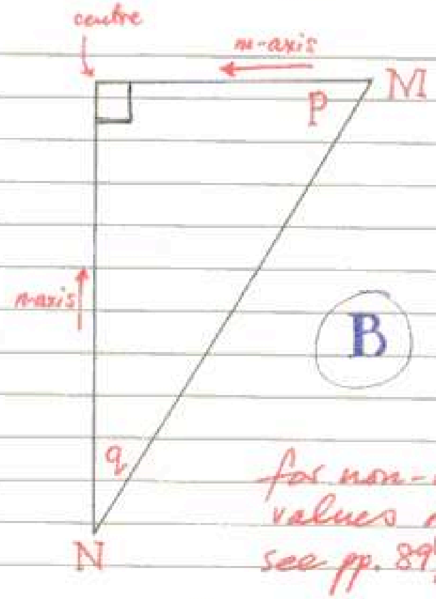
Thu 6 Nov 1975 → Thursday, JANUARY 13, 1966

Fig 16



$$N = \frac{2Mq}{M-2p}$$

$$M = \frac{2Np}{N-2q}$$



for non-integral values of p, q, see pp. 89, 90.

The rhombus has opposite pairs of rosettes or stars with M and N rays, respectively. M and N often have different values, but may be the same. Certain radii of the rosettes coincide with both the sides of the rhombus and its axes (that is, if the rosettes are even-rayed, i.e. if M, N are even numbers; if M N are odd complete coincidence is not obtained). In the right-angled triangle (B) which is one quarter of the rhombus, the angle at M has p equal divisions, each of which equals π/M, or $1/M$ of 180°. Similarly the angle at N has q equal division, each of size $1/N$. Thus the size of the angle at M is p/M, and that at N is q/N.

If we wish to express p/M in terms of N, we have

$$\frac{q}{N} + \frac{p}{M} = \frac{1}{2}, \qquad \frac{q}{N} = \frac{1}{2} - \frac{p}{M} = \frac{M-2p}{2M}$$

Therefore $N = \dfrac{2Mq}{M-2p}$

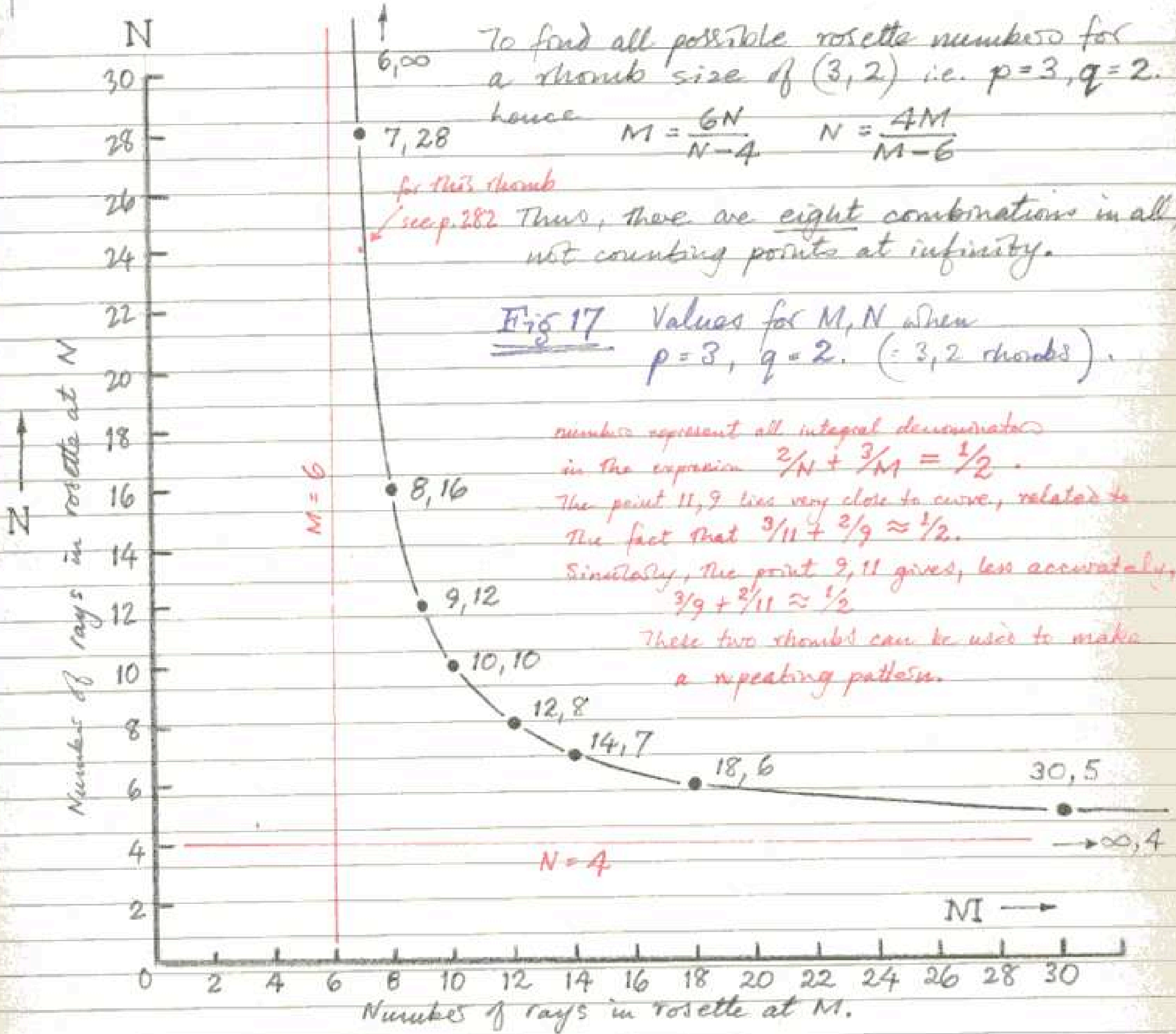
Similarly $M = \dfrac{2Np}{N-2q}$

For a given rhomb (p, q) these equations represent hyperbolae, whose positive integral solutions give pairs of values for possible combinations of rosette numbers. If M = N one can find the total number of different sizes of rhomb possible for that rosette number.

Asymptotes at $m = 2p$, $n = 2q$.

Asymptotes are at $m = 2p$, $n = 2q$ 12

Thu 6 Nov 1975



To find all possible rosette numbers for a rhomb size of (3, 2) i.e. $p = 3$, $q = 2$.

hence $M = \frac{6N}{N-4}$ $N = \frac{4M}{M-6}$

for this rhomb / see p. 282 Thus, there are eight combinations in all, not counting points at infinity.

<u>Fig 17</u> Values for M, N when $p = 3$, $q = 2$. (∴ 3, 2 rhombs).

numbers represent all integral denominators in the expression $2/N + 3/M = 1/2$.

The point 11, 9 lies very close to curve, related to the fact that $3/11 + 2/9 \approx 1/2$.

Similarly, the point 9, 11 gives, less accurately, $3/9 + 2/11 \approx 1/2$

These two rhombs can be used to make a repeating pattern.

M	7	8	9	10	12	14	18	30	$p = 3$
N	28	16	12	10	8	7	6	5	$q = 2$

Note: in the final text, only one such graph need be shown, the others given as numerical results only. The choice for the graph is obviously that for (3, 2) rhombs in view of the fundamental role the (3, 2) patterns play among rosette patterns.

Thu 6 Nov 1975

M = N

Tuesday, JANUARY 18, 1966

Thu 6 Nov 1975

Rosette Numbers	No. of rhombs	Different pairs of values for (p, q)							
4,4	1	1,1							
6,6	1	2,1							
8,8	2	3,1	2,2						
10,10	2	4,1	3,2						
12,12	3	5,1	4,2	3,3					
14,14	3	6,1	5,2	4,3					
16,16	4	7,1	6,2	5,3	4,4				
18,18	4	8,1	7,2	6,3	5,4				
20,20	5	9,1	8,2	7,3	6,4	5,5			
22,22	5	10,1	9,2	8,3	7,4	6,5			
24,24	6	11,1	10,2	9,3	8,4	7,5	6,6		
26,26	6	12,1	11,2	10,3	9,4	8,5	7,6		
28,28	7	13,1	12,2	11,3	10,4	9,5	8,6	7,7	
30,30	7	14,1	13,2	12,3	11,4	10,5	9,6	8,7	
32,32	8	15,1	14,2	13,3	12,4	11,5	10,6	9,7	8,8
34,34	8	16,1	15,2	14,3	13,4	12,5	11,6	10,7	9,8

The number of rhombs possible is clearly equal to the value of q in the "fattest" rhomb, or is equal to $\frac{m}{4}$ if m is exactly divisible by 4 or to the integral part of m/4.

In general, the number of rhombs possible is equal to the integral part of $m/4$.

Fig. 18.

2:1 pair $= 2(p+q)+2q,\ \frac{2(p+q)+2q}{2}$. 1:2 pair $= 2(p+q)-q,\ 2[2(p+q)-q]$.
$= 2p+4q \quad = p+2q \qquad\qquad = 2p+q \quad = 4p+8q$

Fig. 19

Thu 6 Nov 1975

(p,q) rhomb size	No. of Solns.	Numerical Values, in The form (M, N).* * see notes on p.22 when p = q.													M =	N
1,1	2 (3)	6,3	4,4		3,6										2N/N-2	2M/M
2,1	4	12,3	8,4		6,6			5,10							4N/N-2	2M/M
3,1	6	18,3	12,4	10,5	9,6		8,8			7,14					6N/N-2	2M/M
4,1	5	24,3	16,4		12,6			10,10			9,18				8N/N-2	2M/M
5,1	6	30,3	20,4		15,6	14,7			12,12			11,22			10N/N-2	2M/M
6,1	8	36,3	24,4	20,5	18,6		16,8	15,10		14,14				13,26	12N/N-2	2M/M
2,2	3 (5)	20,5	12,6	8,8	6,12	5,20									4N/N-4	4M/M
3,2	8	30,5	18,6	14,7	12,8	10,10	9,12	8,16	7,28						6N/N-4	4M/M-
4,2	6	40,5	24,6	16,8	12,12	10,20	9,36								8N/N-4	4M/M
5,2	8	50,5	30,6	20,8	18,9	15,12	14,14	12,24	11,44						10N/N-4	4M/M-
6,2	10	60,5	36,6	28,7	24,8	20,10	18,12	16,16	15,20	14,28				13,52	12N/N-4	4M/M-
7,2	8	70,5	42,6	28,8	22,11	21,12	18,18	16,32						15,60	14N/N-4	4M/M-
3,3	5 (9)	42,7	24,8	18,9	15,10	12,12	10,15	9,18	8,24	7,42					6N/N-6	6M/M
4,3	10	56,7	32,8	24,9	20,10	16,12	14,14	12,18	11,22	10,30	9,54				8N/N-6	6M/M
5,3	12	70,7	40,8	30,9	25,10	22,11	20,12	16,16	15,18	14,21	13,26	12,36	11,66		10N/N-6	6M/M
6,3	12	84,7	48,8	36,9	30,10	24,12	21,14	20,15	18,18	16,24	15,30	14,42		13,78	12N/N-6	6M/M-
7,3		98,7	56,8	42,9	35,10	28,12	26,13		21,18	20,20					14N/N-6	6M/M-
8,3		112,7													16N/N-6	6M/M-
4,4	4 (7)	72,9	40,10	24,12	16,16	12,24	10,40	9,72							8N/N-8	8M/M-
5,4	10	90,9	50,10	30,12	26,13	20,16	18,18	15,24	14,28	12,48	11,88				10N/N-8	8M/M-
6,4	12	108,9	60,10	44,11	36,12	28,14	24,16	20,20	18,24	16,32	15,40	14,56		13,104	12N/N-8	8M/M-
7,4															14N/N-8	8M/M-
8,4															16N/N-8	8M/M-
9,4															18N/N-8	8M/M

Integral Numerical Values for The equations $M = 2Np/N-2q$; $N = 2Mq/M-2$
When $M = N$ figures are given in red. When They are in The ratio 2:1 or 1:2 figures are given in green.

From inspection, the lowest pair of values for a given rhomb p, q is

$2p(2q+1),\ 2q+1$; and the highest pair of values is $2p+1,\ 2q(2p+1)$.
When m and n are the same, we have $\boxed{m = n = 2(p+q)}$

15 Wed 7 June 1978.

Designation symbols for the general rhombus & axially centred rhombic patterns.

The general rhombus may be designated R(p×q)m, n.
p, q being the numbers of divisions in the ½-angles of the rhomb, and m, n being the numbers of primary rays or points in the stars or rosettes centred on the vertices of the rhomb. The prefix E is used when we are referring to a euclidean rhombus, similarly S is used for spherical and H for hyperbolic.
If it is necessary to deal with parallelograms as well as rhombuses, the letter P is used instead of R. Having chosen values for m, n, p and q, the type of pattern drawn within a particular rhombus follows the rhomb designation, as follows

ER(p×q)m, n/T ; recognised types are given Roman numerals with subdivisions to represent various derivative patterns. When the rhombus forms a pattern the symbol for the rhombic tessellation concerned, if it has one, precedes the whole rhomb symbol. Thus,

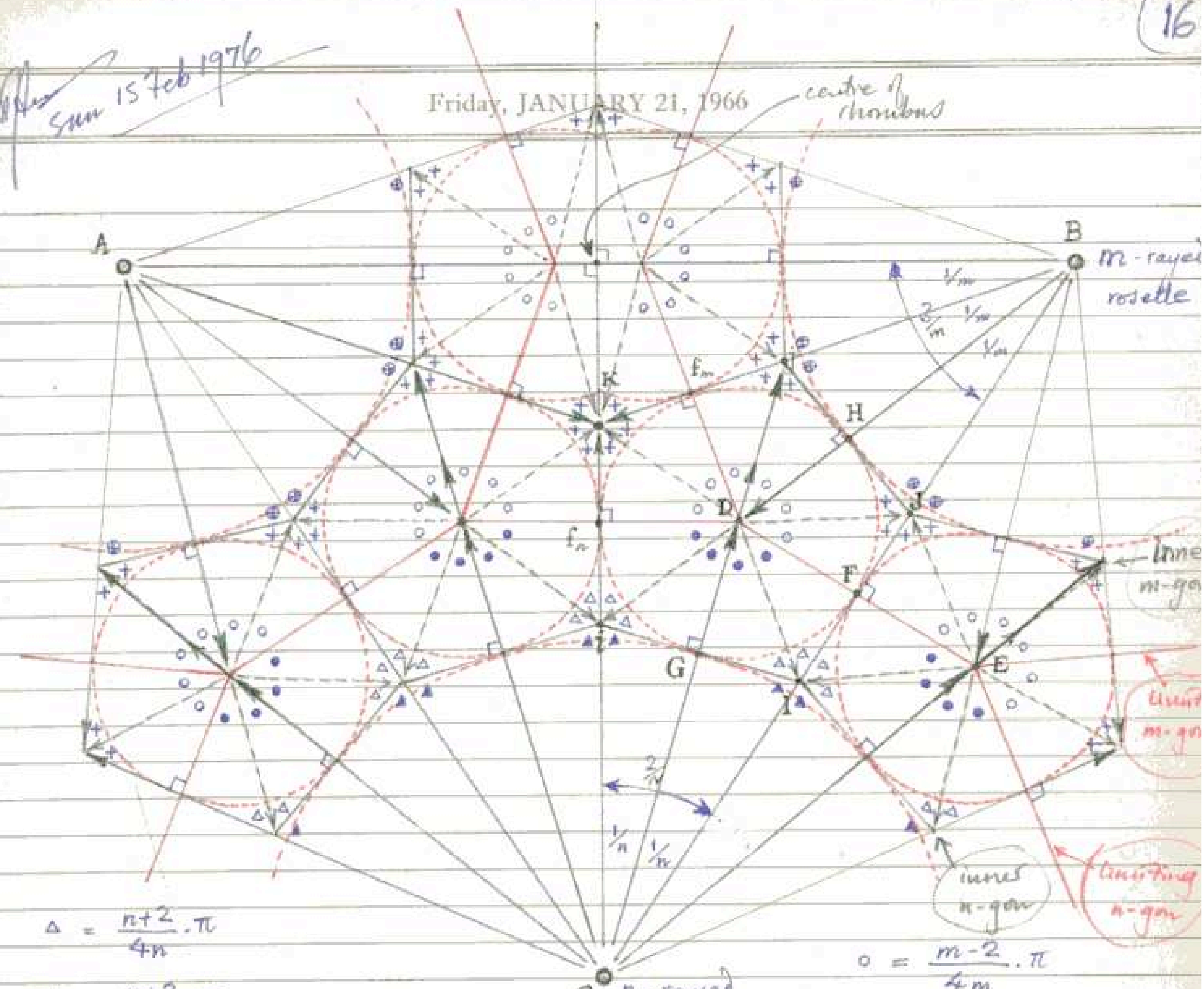
Rp1. ER(3×2)10,10/I ; or the ER may here be omitted, since Rp1 refers to a rhombic tessellation in the euclidean plane. If the pattern contains more than one size of rhomb, the designation is modified:-

Rs2(p×q, p'×q')m, n/T,
or Rs3(p×q, p'×q', p"×q")m, n/T and so on.

The series of pattern types T, strictly applies in the case of (3×2) rhombs, but may be partly adapted to other sizes of rhombus.

Note that although the symbol ER(p×q)m, n defines the shape of a particular rhombus precisely, the symbol EP(p×q)m, n may refer to an infinite number of parallelograms, sharing only the sizes of their internal angles, but differing in the ratio of the lengths of their sides.

Sun 15 Feb 1976



$\triangle = \frac{n+2}{4n} \cdot \pi$

$+ = \frac{m+2}{4m} \cdot \pi$

$\blacktriangle = \frac{n-2}{2n} \cdot \pi$ $\oplus = \frac{m-2}{2m} \cdot \pi$

$\circ = \frac{m-2}{4m} \cdot \pi$

$\bullet = \frac{n-2}{4n} \cdot \pi$

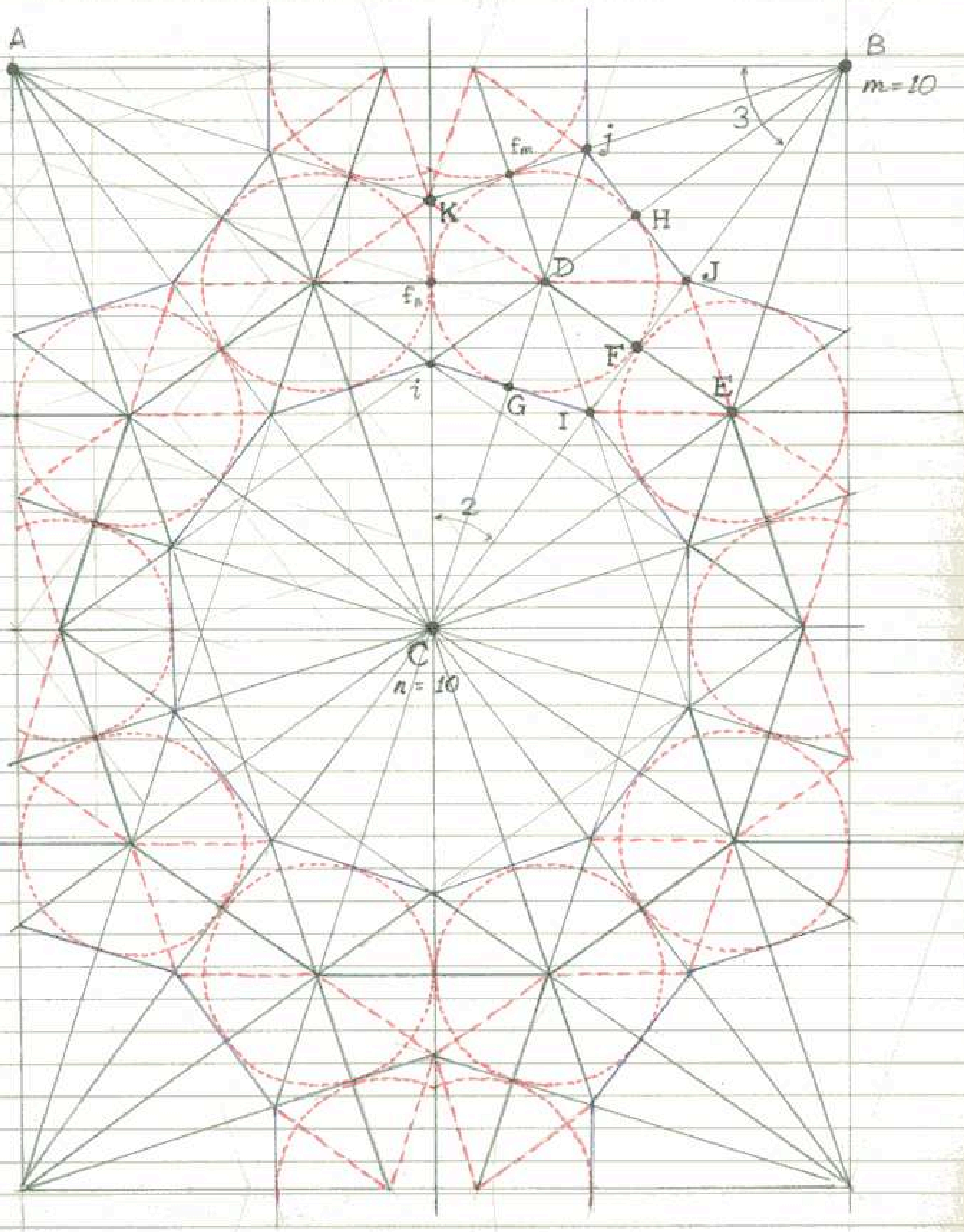
The basic constructional net for patterns on 3x2 rhombs, showing general expressions for angle sizes in terms of m and n. (angles are expressed as fractions of π radians, or 180°)

N.B. CDj are collinear in all 3x2 rhombs; BDi are collinear only when m=n=10

NOTE THAT this construction can be drawn in any ~~size~~ shape of (3x2) rhomb, whether m, n are integral values or not. But of course if neither m, n are integral, as here, then centres A, B and C will not close up to form stars or rosettes.

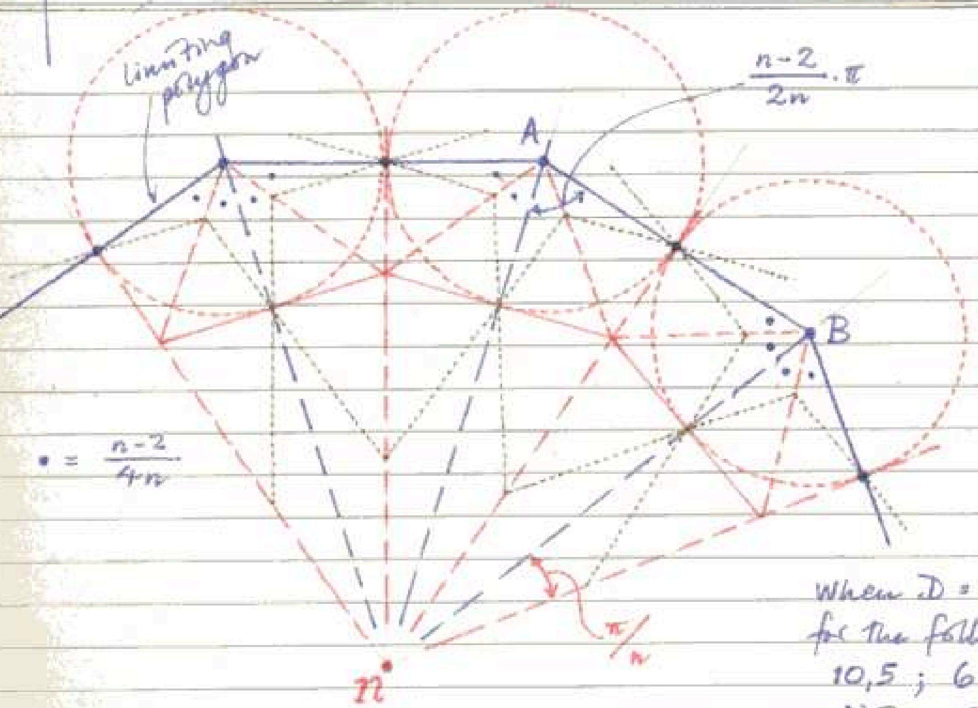
Sun 15 Feb 1976

Monday, JANUARY 24, 1966



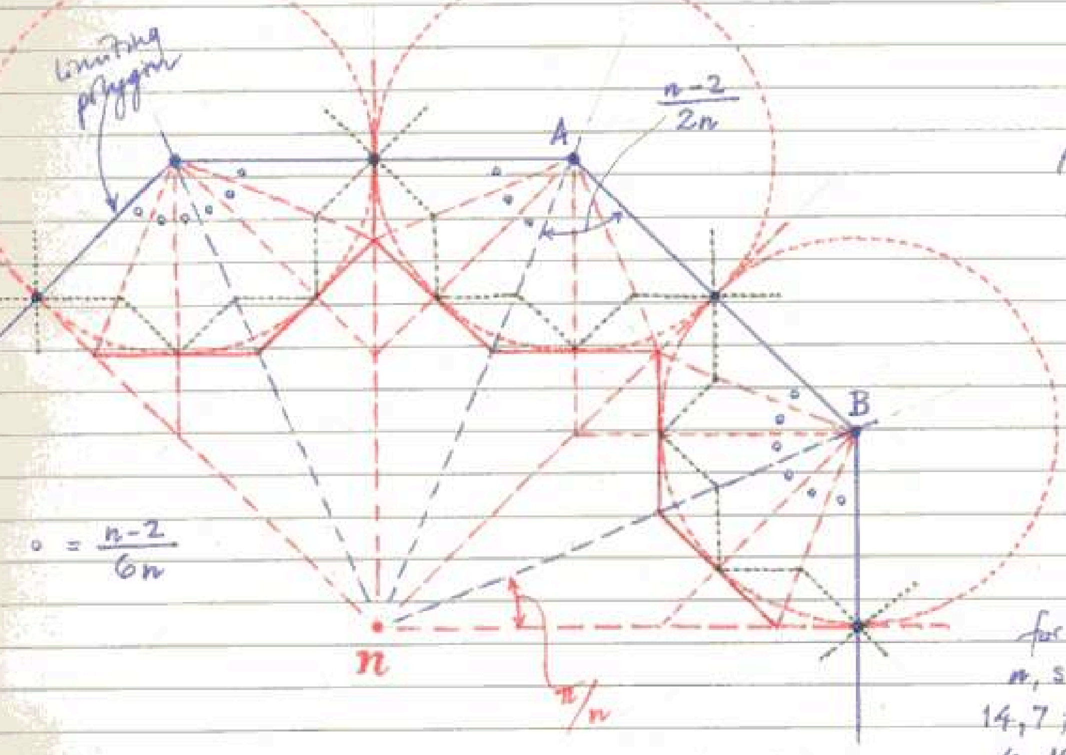
Tue 2 Mar 1976

Tuesday, JANUARY 25, 1966



Here D = 2 (The number of equal divisions of angle nAB).
Each division is equal to $\frac{n-2}{4n}$; or, in general, to $n-2/2Dn$

When D = 2, regular stars may be formed for the following pairs of values n, s: 10,5 ; 6,6 ; 4,8 ; 3,12 (and, in addition, the points at infinity ∞,4 ; 2,∞). — see the curves opposite.



Here, D = 3. Each division is equal to $n-2/6n$.

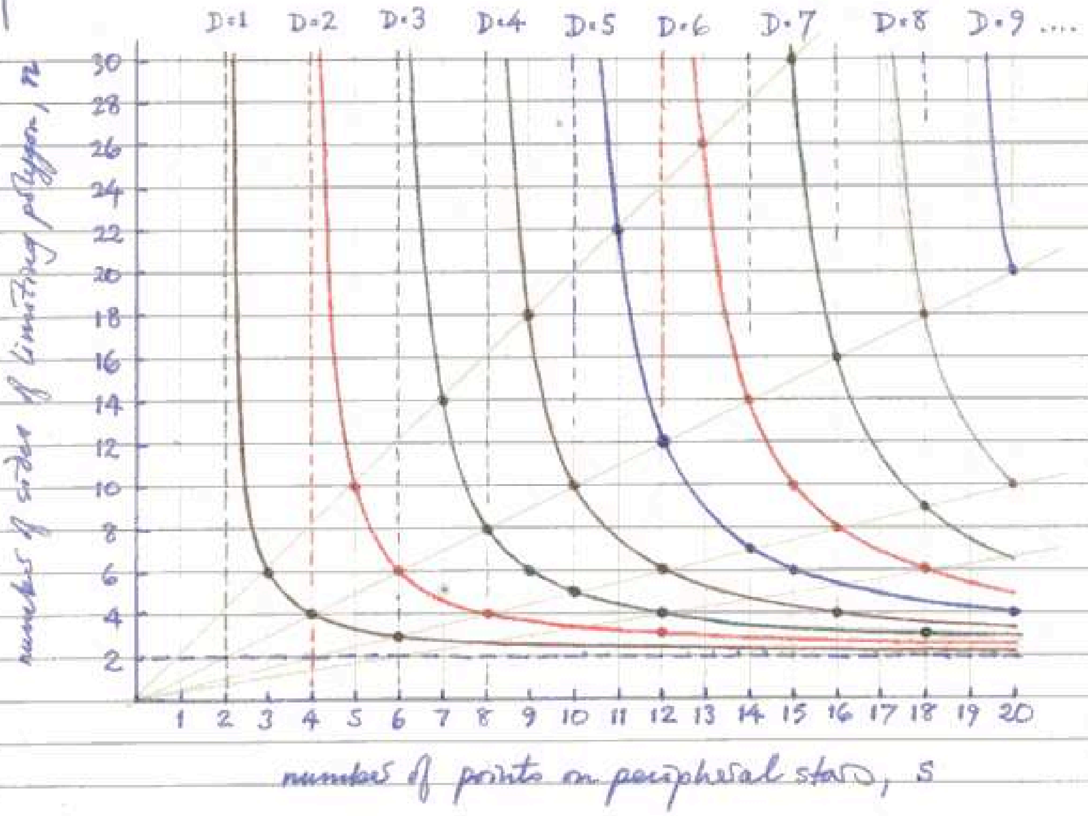
When D = 3, the peripheral stars become regular for the following pairs of values n, s:
14,7 ; <u>8,8</u>* ; 6,9 ; 5,10 ; and 4,12.

See also pp. 181–190

* Illustrated here

These are effectively curves for (p, 1) rhombs i.e. D = p and q = 1, and s = m. Derived from formulae on p. 11 | 20

Tue 2 Mar 1976



$$s = \frac{2Dn}{n-2} \quad (1)$$

$$n = \frac{2s}{s-2D} \quad (2)$$

N.B. approximate su... are possibilities have ... by examining different curves, e.g. the point 5, 17 for D = 5.

Asymptotes for each curve are shown as d... vertical lines. n = 2 is the other asymptote for a... curves, as seen from equation (1) above.

This diagram and those opposite generalize the concept of the peripheral star construction. Originally regarded as an incidental accompaniment to Type I rosette formation, especially in central patterns in the 3×2 rhomb series, the peripheral star is here regarded as a main pattern forming element, and the particular method of constructing 5-pointed peripheral stars in 3×2 patterns is widened and generalized to include "peripheral stars" of any number of points. As in the case of pairs of values possible in p×q rhombs, we find that the number of sides of the limiting polygon, n, is related by algebraic expressions to the number of points, s, in the peripheral star, and to the number of divisions D, in the angle n-2/2n of the n-gon. These pairs of values are shown in the series of curves drawn above, for varying values of D.

If D is the number of divisions of angle n-2/2n*, then each division = $\frac{n-2}{2Dn}$. If the peripheral star has regular and equal divisions, then each of these same divisions is equal to 1/s (as a fraction of π or 180°), i.e. $\frac{n-2}{2Dn} = \frac{1}{s}$ or $s = \frac{2Dn}{n-2}$

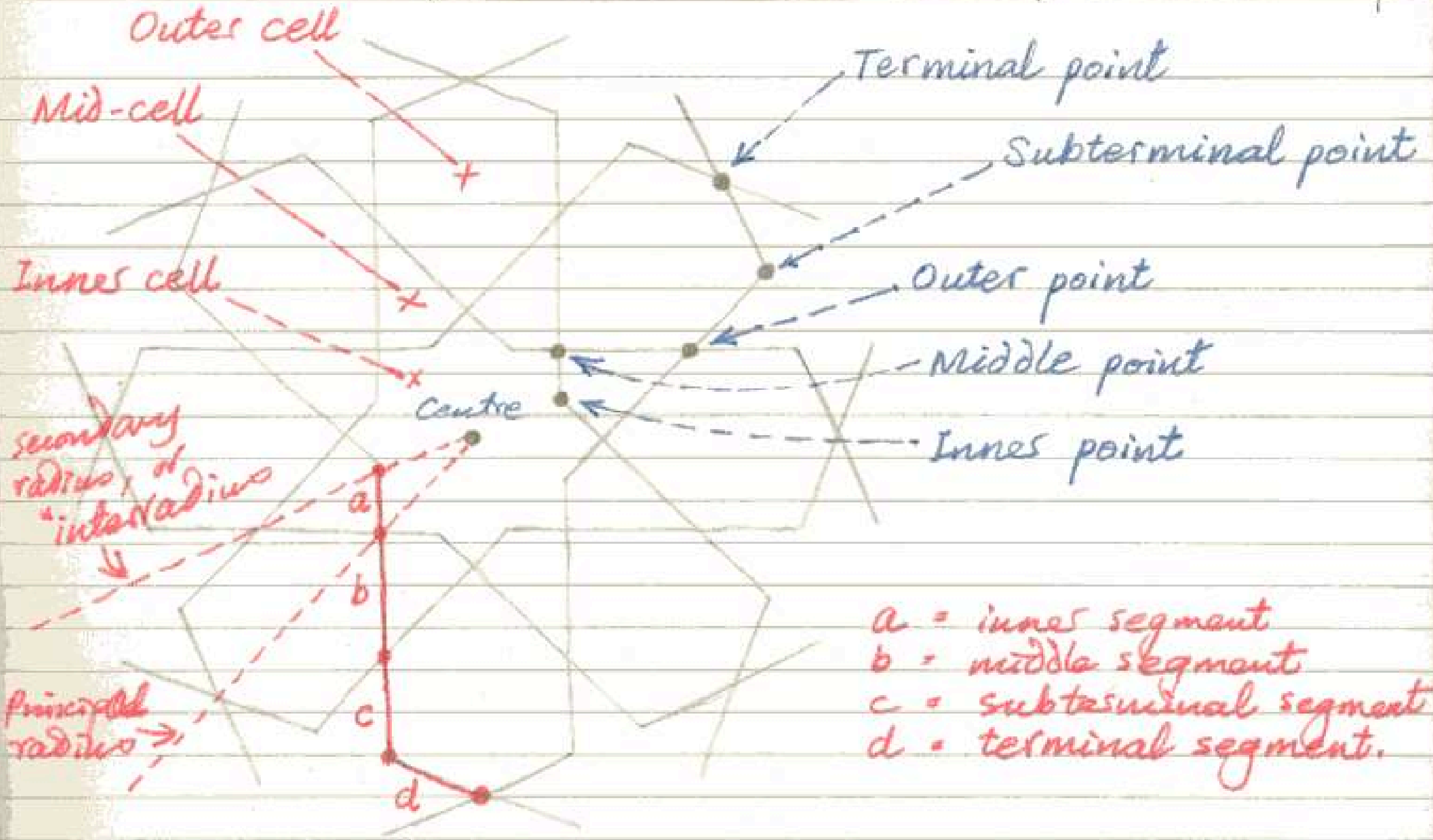
Rearranging, $n = \frac{2s}{s-2D}$. These are the expressions given above relating the 3 quantities n, s and D.

* = angle nAB, opposite.

Tue 2 Mar 1976

21 | TERMINOLOGY OF PARTS OF GEOMETRICAL ROSETTE

Revised May 1976, March 1977 Thursday, JANUARY 27, 1966 Sat 3 Dec 1977



a+b+c together constitute the lateral segment (c might therefore be termed the outer segment of the lateral segment; or perhaps a, b, c should be termed inner, middle and outer divisions of the lateral segment — I don't know!)

~~Thu 1 June 1978~~

 $p \to m$, $q \to n$ $\quad m \gtreqless n$, $p \geq q$

Rule for labelling $\underline{m}$, $\underline{n}$ in the general $(p \times q)$ Rhombus.

If $p > q$ there are p divisions at $\underline{m}$ and q divisions at $\underline{n}$. The values for p, q are chosen so that $p \geq q$.

If $p = q$ then p is still associated with centre $\underline{m}$, such that $\underline{m} > \underline{n}$.

If $p = q$ and $\underline{m} = \underline{n}$, then the rhombus is a square, and it is immaterial which centre is $\underline{m}$ and which $\underline{n}$.

$p > q$, $m \gtreqless n$
$p = q$, $m \geq n$

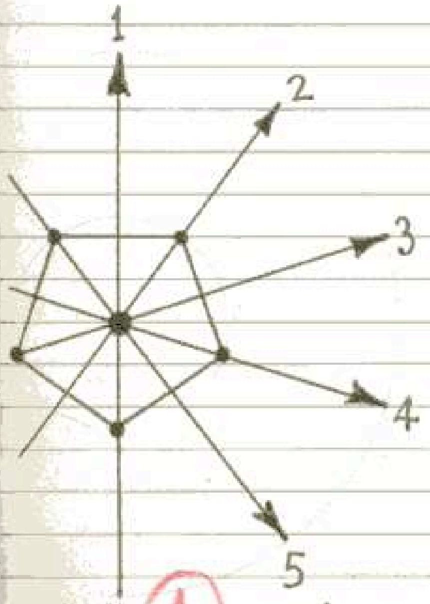
Note that the primary choice is that $p > q$. It may then happen that $\underline{m} < \underline{n}$. In the lists on p. 14 m, n pairs have been fixed in order by the first chosen values. For example, according to the rules we have just given, the m, n pairs listed for (2×2) rhombus should be 20, 5 12, 6 8, 8 – No! in fact one notices here that the pairs of values reverse after the $m = n$ pair, and can be ignored; a point I had momentarily forgotten.

23

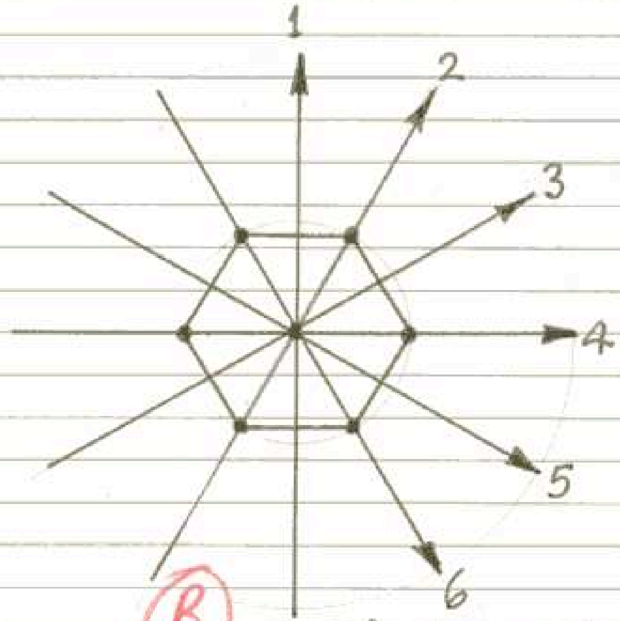
STAR-CENTRE

Fri 2 June 1978

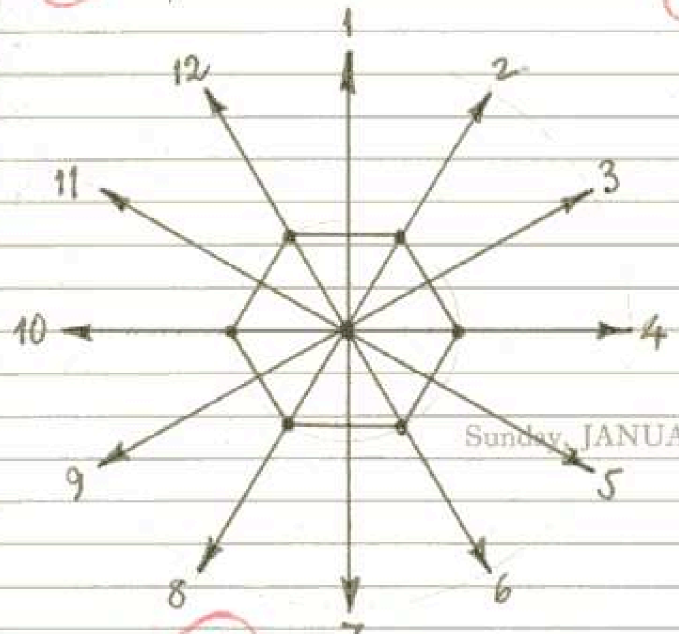
Saturday, JANUARY 29, 1966



A 5-fold star-centre



B 6-fold star-centre with n lines through its centre.



← N.B. as drawn here, even-numbered radii are principal radii, while odd-numbered radii are interradii or secondary radii.

Sunday, JANUARY 30, 1966

C 6-fold star-centre with 2n radii.

In a geometrical sense the "star-centre" might be termed simply a star, and a distinction made between the latter and a star-motif, an ornamental device with n-fold rotational symmetry constructed on the basis of the star.

Thu 7 June 1984

Fri 2 June 1978 Monday, JANUARY 31, 1966

STAR-CENTRES (See also p. 181)

A star-centre is a configuration consisting of n regularly spaced straight lines through a central point. Since the angle between adjacent lines is constant, a regular star-centre underlies any regular polygon or star-motif (figs $\boxed{A, B}$) opposite.

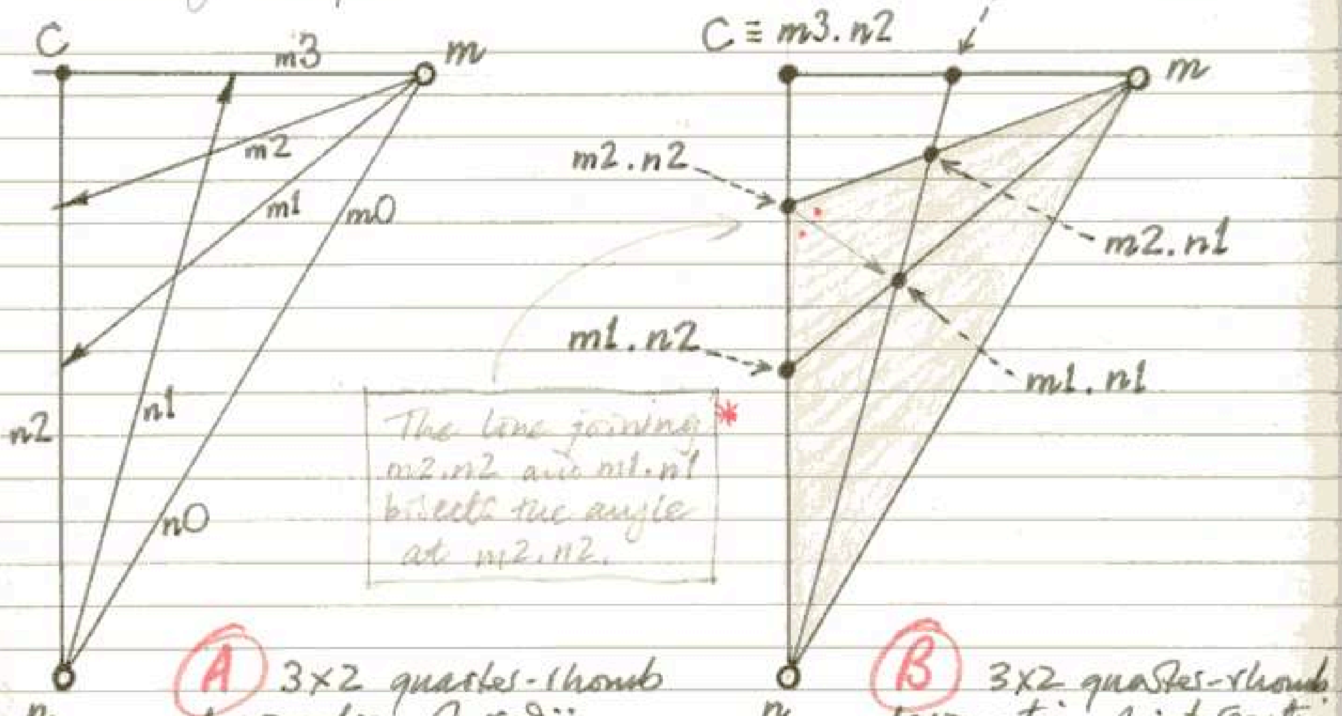
C Alternatively, it is also convenient to regard a star-centre as comprising $2n$ radii originating from a central point. The angle between adjacent radii is constant for a given star-centre. n of the radii will pass through the outer points of a star or rosette, and may be termed the principal radii; while the remaining n radii alternate with the principal radii and pass through the inner points of a primary star. These may be termed secondary radii, or inter-radii.

A star-centre with n straight lines or $2n$ radii will be termed an n-fold star-centre (it being understood that we are only concerned with regular star-centres* in which the angle between successive radii is constant for a given star-centre). The angle between the radii of any star-centre is equal to π/n or simply $1/n$.

N.B. The different categories of Links between a pair of adjacent star-centres, or star motifs, are defined on p. 157.

* The insistence on regular star centres is indicative of a purely theoretical interest in geometrical star patterns of a type often only approximated by authentic Islamic ornament. It seems likely that the original artisans and pattern designers on the whole made no mental distinction between geometrically exact patterns and those, often involving less than regular star centres, in which some small error was present, to be masked by the skill of the pattern makers. Any account which pretends to deal with authentic Islamic star patterns must consider many patterns which in an absolute mathematical sense are impossible. Thu 7 June 1984

Fri 2 June 1978 Tuesday, FEBRUARY 1, 1966



(A) 3×2 quarter-rhomb designation of radii

(B) 3×2 quarter-rhomb designation of intersections of radii

m0, n0 : aligning radii
m1, n1 : 1st collateral radii
m2, n2 : 2nd collateral radii
etc.

m1.n1 = first collateral intersection
m2.n2 = second collateral intersection

The shaded triangle, formed on points m, n and m2.n2, may be termed the <u>second collateral triangle</u>

Although in the p×q rhomb the named intersections extend no further than $m_q.n_q$, the intersections in a general pair of aligned star centres are more numerous, extending to "points at infinity".

Fri 9 June 78

* Within triangle (m, n, m2.n2) the bisectors of the three angles meet at the point m1.n1, which is the incentre of that triangle.

Fri 2 June 1978 Wednesday, FEBRUARY 2, 1966

Designation of radii and intersections in the (p×q) rhombus

mnC represents one quarter of a rhombus, point C being the centre of the rhombus, and the angle at C a right angle. In the general rhombus, the rosette or star at m (or the star-centre at m) will contribute p equal subdivisions of angle Cmn, each division of π/m. Similarly, angle mnC will have q equal divisions of π/n each. In this case the rhombus as a whole is referred to as a (p×q) rhombus. The m-star is chosen such that $p > q$; if, however, $p = q$ then the m-star is chosen such that $m > n$.* If $p = q$ and $m = n$ then the rhombus is a square and it becomes immaterial which of the equal stars is labelled m and which n.*

(A) Radii from the centres of the m- and n-stars (i.e. points m and n resp.) are labelled m0, m1, m2 ..... m_p, and n0, n1, n2 ...... n_q, starting with those radii which coincide with the edge of the rhombus, that is, with side mn in the right triangle Cmn. Radii m0 and n0 are therefore normally collinear. Furthermore, in a (3×2) rhombus radii m3 will form the m-axis of the rhombus, and radii n2 the n-axis. In general, in a (p×q) rhombus m_p is the m-axis, n_q the n-axis.

The collinear radii m0 and n0 form the aligning radii. m1, n1 are the first collateral radii, m2, n2 the second collateral radii, etc.

(B) The point of intersection between m1 and n1 may be designated m1.n1; that between m2 and n1 as m2.n1, and so on. Note that the intersection $m_p.n_q$ is always the centre point of the (p×q) rhombus.

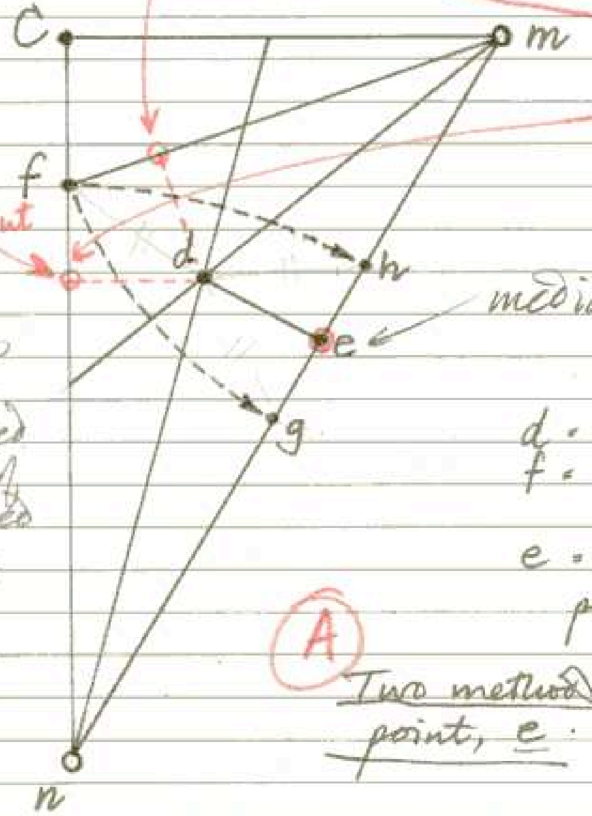
Two intersections of major importance, especially in the construction of (3×2) rhomb patterns, are m1.n1, which may be termed the first collateral intersection, and m2.n2, which may be termed the second collateral intersection. Intersection m2.n1 is of importance in patterns of types IV and VII of the (3×2) rhomb series.

* if $p = q$, $m \neq n$ the values "flip over" on the curves $q/n + p/m + 1/2 = 1$
The rule given in effect takes account only of the right half of the curve.

27

Fri 2 June 1978 Thursday, FEBRUARY 3, 1966

THE MEDIAN POINT



Note that perpendicular from d to radii m2, n2 are equal in length to de. d is thus the incentre of the triangle formed by, m, n and intersection m2.n2.

In 3x2 rhombs the peripheral step is constructed in the incircle of the triangle formed by points m, n and m2.n2.

d = 1st collateral intersection
f = 2nd collateral intersection.

e = median point; de is perpendicular to mn.

A

Two methods of obtaining the median point, e.

Proof: It is obvious that $df = dh$ (symmetrical with resp. to point n) similarly, with resp to m, $df = dg$.
Therefore $dg = dh$ and triangle dgh is isosceles.
Hence $eg = eh$ Q.E.D.

Note:— by convention one might define radius mO as extending no further than the median point, and similarly with radius nO. By a similar convention the median point e might be regarded as the intersection mO.nO, otherwise indeterminate.

Fri 2 June 1978 Friday, FEBRUARY 4, 1966

The Median Point*

The median point is of great importance in the construction of star patterns in ($p \times q$) rhombic alignments. When $m=n$ the median point coincides with the mid point of the line mn, but when $m \neq n$ the median point lies nearer to the centre of the star with the lesser number of points.

The median point marks the ~~shared~~ coincident outer points of a pair of aligned stars. Its position is obtained as follows.

a) From the 1st collateral intersection m1.n1 (d in fig. A opposite) drop a perpendicular to the line mn, intersecting it at a point e. Point e is the median point.

b) Using the 2nd collateral intersection m2.n2 (f in fig. A, opposite) an arc centred on m with radius mf intersects mn at g. Similarly, an arc centred on n with radius nf intersects mn at h. Point e is then the midpoint of the segment gh. (Proof is given opposite)

Method a) of obtaining the median point was known to the original designers and artists executing authentic Isl. patterns, and has been used by recent authors (Bourgoin, 1879; Hankin, 1905, 1925; Critchlow, 1976). Method b) does not seem to have been previously ~~discovered~~ known (it was personally discovered in ~~1965~~ December, 1964 before knowledge of Hankin's work - A.J.L.). Method b) is more accurate than method a) however, and is to be recommended in practical drawing of these patterns.

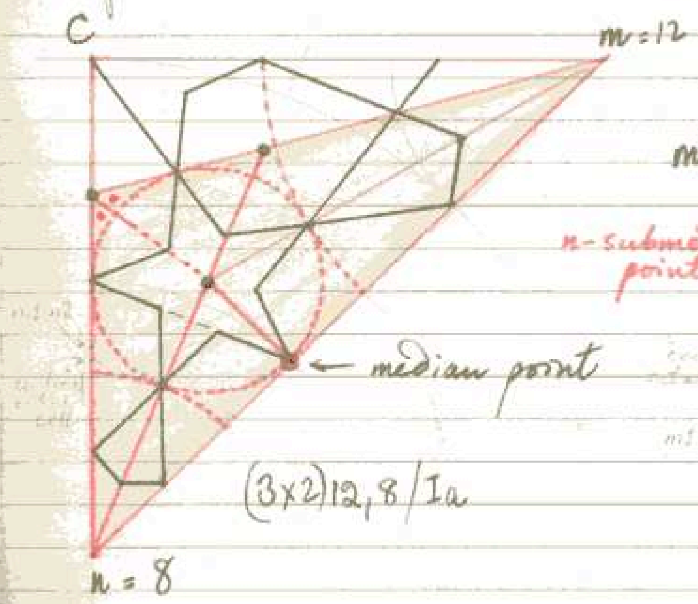
When m and n are integers de represents half the shared edge of two regular polygons centred on m, n and of m and n sides, respectively. Hence Hankin's "polygons in contact" (PIC) method of construction. However methods a) and b) give the same result whether or not m and n represent integral values.

* The term "median" for this point is not a particularly appropriate one, and must be regarded as provisional only.

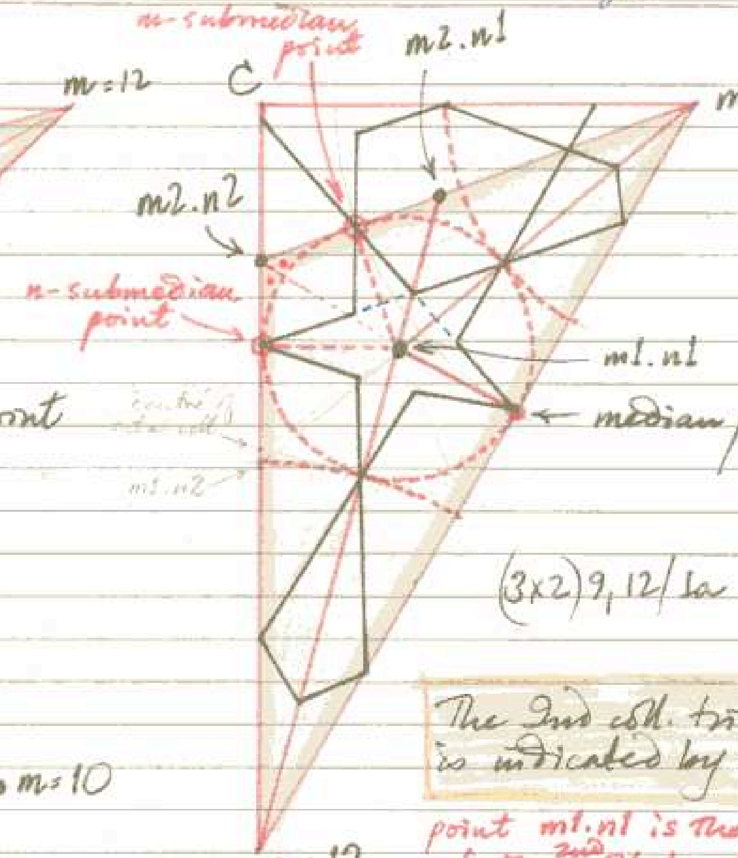
Fri 2 June 1978

(3x2) Type Ia general constr.

see p. 83 for revised type labels.



(3x2)12,8/Ia

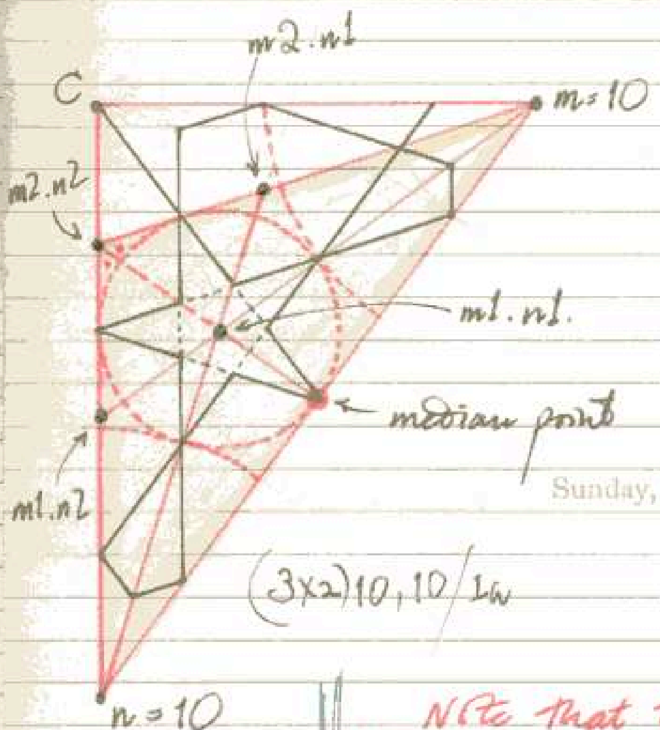


(3x2)9,12/Ia

The 2nd coll. triangle is indicated by shading.

point m1.n1 is the incentre of the 2nd collateral triangle, and the meeting point of the bisectors of the three angles.

In higher rhombs one might refer to the 1st, 2nd etc submedian points. Mon 5 June 1978



(3x2)10,10/Ia

Note that the pattern within the 2nd collateral triangle is topologically equivalent to patterns of the same type in (2x2) rhombs. The angle at m2.n2 becomes a right angle in this deformation.

Mon 5 June 1978 [30]

Monday, FEBRUARY 7, 1966

"Second collateral triangle"

In type I constructions ~~the~~ in the (3×2) rhomb series the ~~2×2~~ triangle (formed on points m, n and m2.n2) is of importance, together with its incircle and incentre, point m1.n1.

The incircle of the second collateral triangle determines the radius of the outer mid-circle of both rosettes.

When correctly constructed, the intersections m2.n1 and m2.n2 coincide with the "centres" of the m-cell and interstitial cell, respectively, in which they lie. In the case of m=n=10 point m1.n2 also coincides with centre of the outer n-cell in which it lies, but this is not so when m≠n. The "centre" of any cell is the point at which the bisectors of its angles meet. Thus the "centre" of the peripheral star in (3×2) rhomb pattern is the first collateral intersection m1.n1.

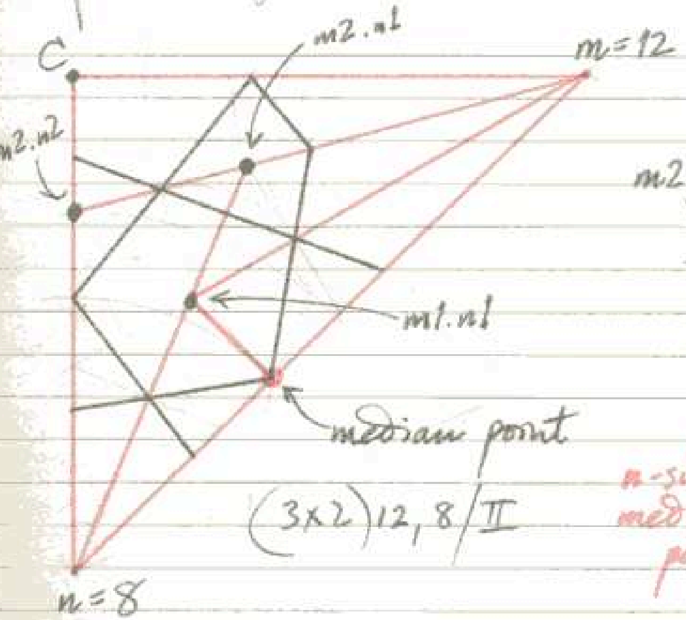
Since the centres of the more important pattern cells coincide with named major intersections or constructed points, it is sometimes convenient to name a cell after the designation given to its centre point. Thus, in type I patterns, opposite, the peripheral star may be referred to as the m1.n1 cell, the interstitial cell as the m2.n2 cell, or they may be referred to also as the 1st and 2nd collateral cells, respectively. The outer cells of the m-rosette may be called the m2.n1 cell and the (m2.n1)′ cell, the symbol (m2.n1)′ indicating that the repetition of point m2.n1 by mirroring across radius m1 is referred to. Similarly, the outer cells of the n-rosette may be termed the m1.n2 and (m1.n2)′ cells, with the reminder that only when m=n=10 does the centre of the outer n-cell coincide with m1.n2. When m=9 n=12 the true centre lies *above* m1.n2; when m=12 n=8 the true centre lies *below* m1.n2. In more general terms, when m=n=10, then m1.n2 coincides with the centre of the n-cell in which it lies; when m<n, then m1.n2 lies nearer to the n-centre; when m>n m1.n2 lies further from the n-centre.

Sun 4 June 1978

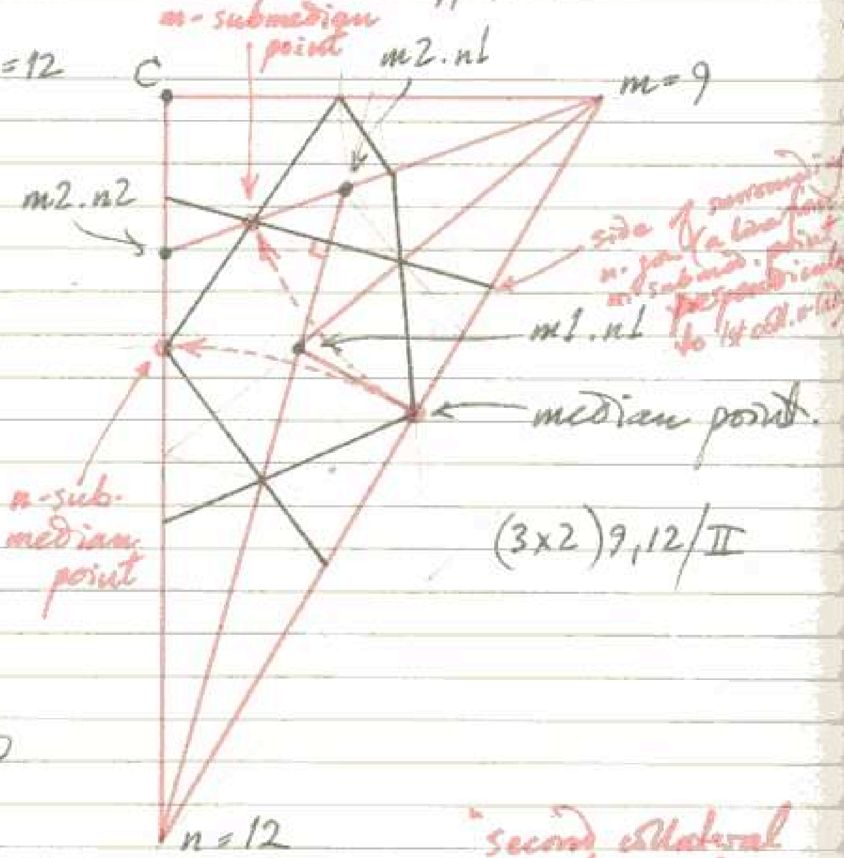
(3X2) Type II general contd.

Tuesday, FEBRUARY 8, 1966

IX, X are direct derivatives of II see pp 41-42.

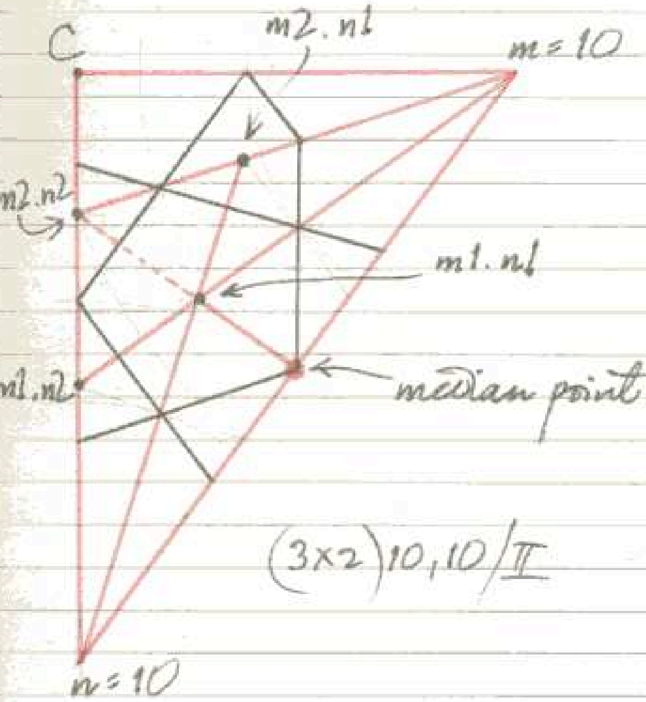


(3x2)12,8/II



(3x2)9,12/II

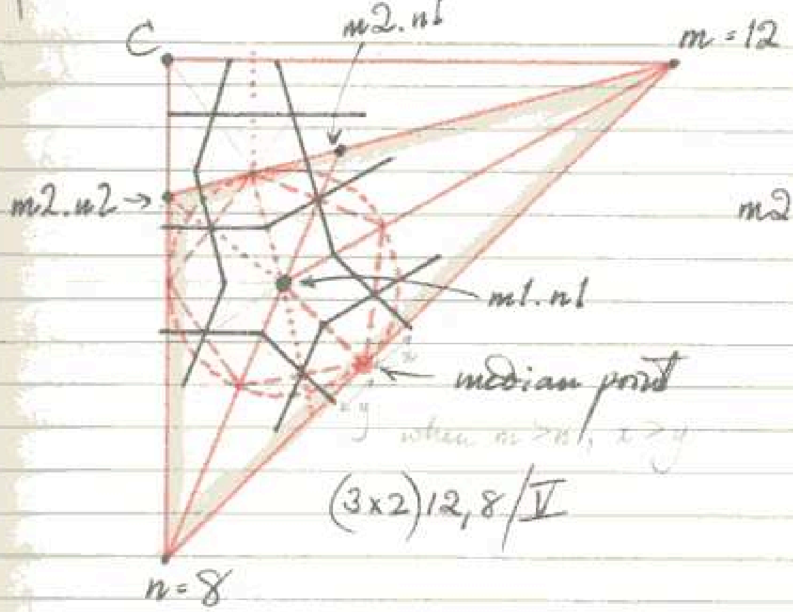
"second collateral triangle"



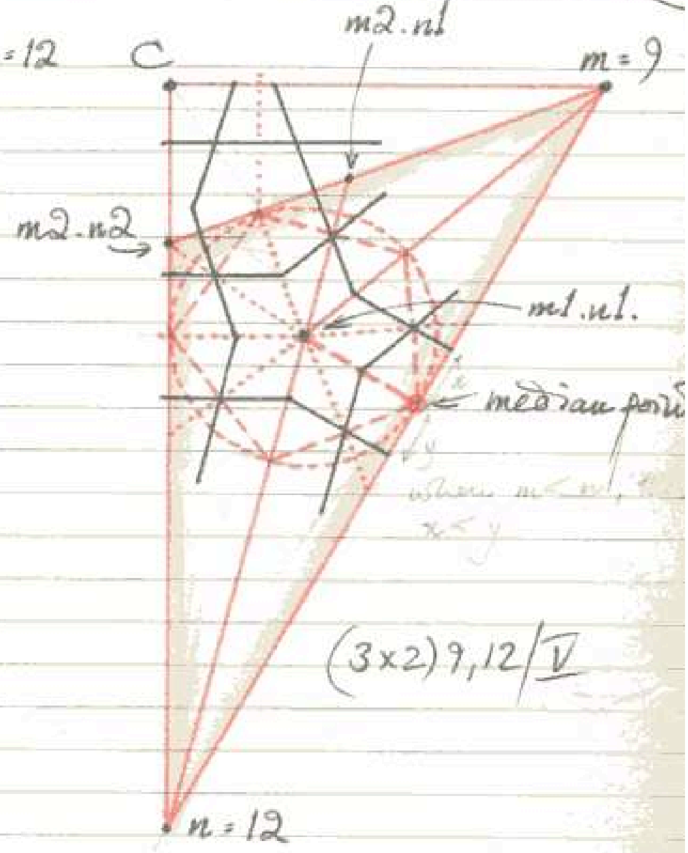
(3x2)10,10/II

In (3x2)/II patterns the peripheral elements are not formed on the incircle of the 2x2 triangle. Indeed except in the case of the 10,10 rhomb, they cannot be so formed. Instead, a straight line through the submedian points is continued in both directions and thereby determines the mid- and in-circles of the m- and n-star, and hence the slope of the sides of the outer cells of these stars.

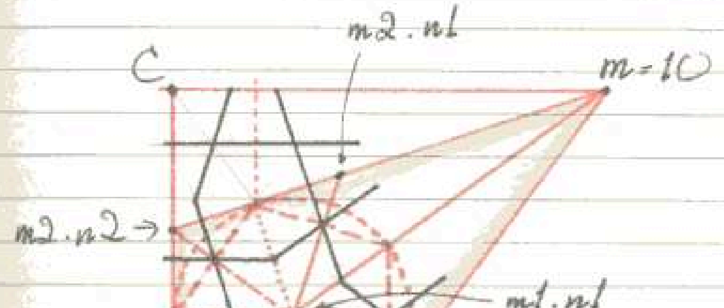
When m=n=10 the incircle of the 2nd coll. triangle (or peripheral circle) is tangent to the midcircles of both rosettes. In the other cases, it is always tangent to the midcircle of the m-star, but when m>n the two circles overlap; when m<n they do not meet.



(3x2)12,8/V



(3x2)9,12/V



(3x2)10,10/V

In (3x2)/V patterns the peripheral stars are formed on the midpoints of the edges of a pentagon inscribed in the peripheral circle. It is possible, however, to form the peripheral stars on the mid edges of the pentagons of the corresponding type II pattern. When the values of m, n are not too dissimilar there is little difference in the two constructions.

Note: The cell centred on the m-submedian point is exactly similar to the cell centred on intersection m2.n1 in type IV patterns, for a given pair of values m, n, and can be made to form the outer cell of an m-fold type Ia rosette — see p. 40.

53) Mon 5 June 1978 (3x2) IV

Type IV patterns differs from other common (3x2) rhomb types in that the first collateral intersection and median point are not used in the construction. Instead, the line segment between m2.n1 and m2.n2 becomes the side of a rhombus, completed by mirroring across radius n1. Pattern lines pass through the midpoints of the sides of this rhombus, and a continuation of the straight line through points x, y determines the radius of the mid-circle and incircle of the m-star. A complete n-gon is drawn through point x, its side perpendicular to radius n0.

The completion of the pattern within the n-gon is arbitrary, but usually ~~those~~ the pair of pattern lines running parallel to radius n1 bend inwards, crossing that radius, and forming the outer points of a simple star where they do so, as indicated in fig. C.

Type IV patterns are related to type Ib, 1st method, the cell cell centred on point m2.n1 being identical in each case. In fact the pattern lines of type IV above the major axis of the constructional rhombus coincide with pattern lines of type Ib (1st method).

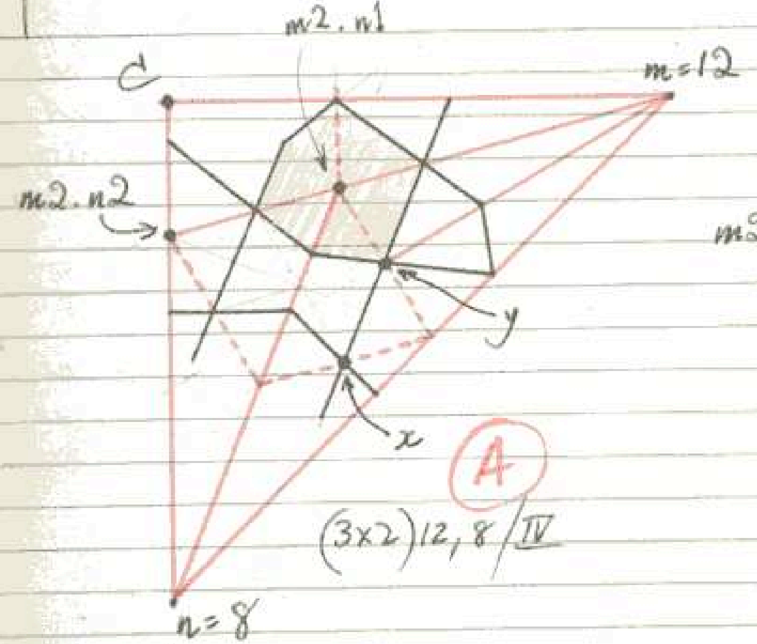
A type IV pattern is one construction where the two centres m and n need to be distinguished, since a reversal of the relationship between the pattern lines and the star centres results in an inexact pattern, in that the n-stars (as they will now be) cannot be regularly formed. In this second construction (termed IVb) the first side of the constructional rhombus lies on radius n2, not m2 as in IVa. (In these remarks it is assumed that $m \neq n$; when $m = n = 10$ ~~it is of course immaterial which pattern is~~ the two varieties are indistinguishable).

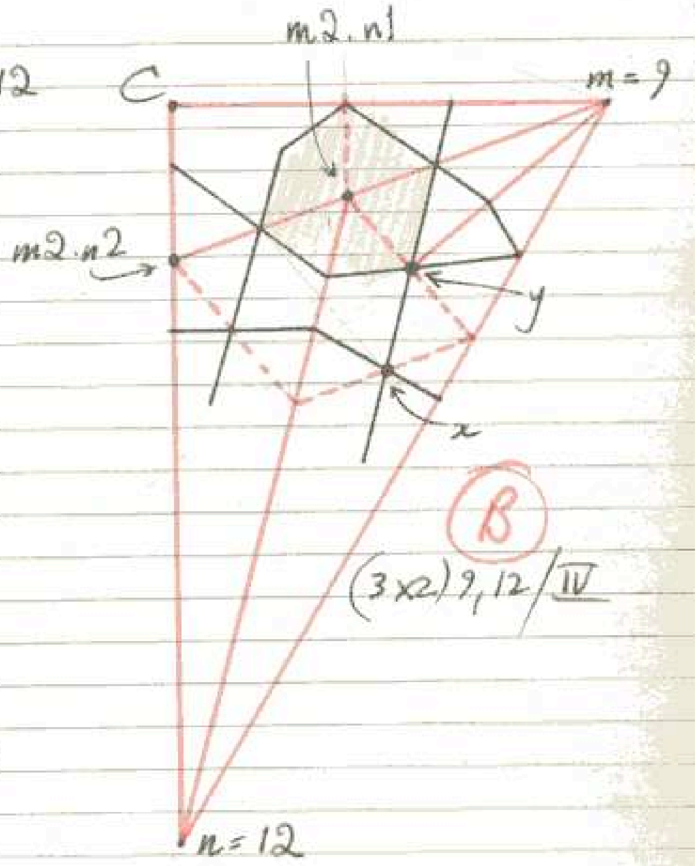
Mon 5 June 1978

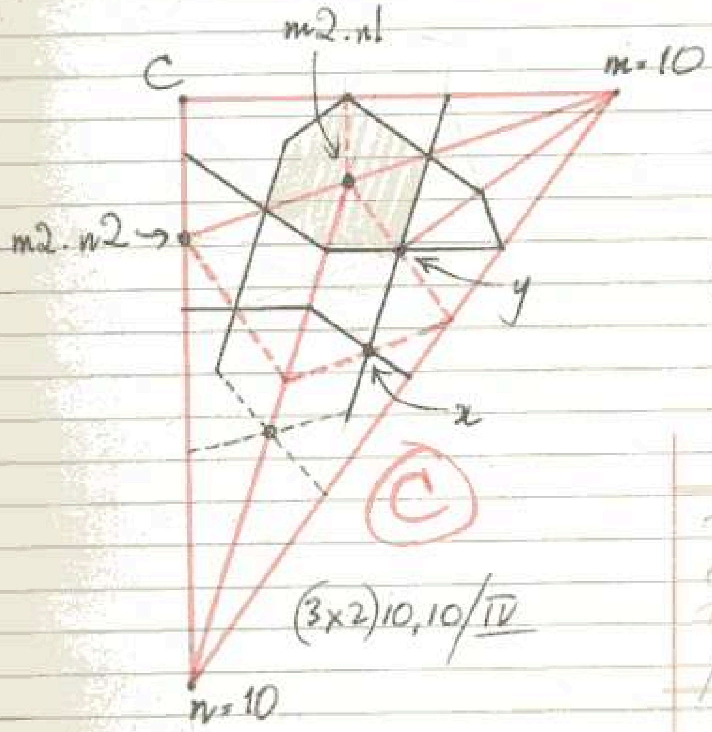
Friday, FEBRUARY 11, 1966

(3x2) IV general const

see p. 83 for revised type labels.







(3x2) IV patterns can also be constructed using small tangent circles on the vertices of the constructional rhomb, but these are unnecessary.

The cell centred on m2.n1 is exactly similar to the cell on the m1 submedian point in type 3 patterns.

35)

(3x2) III

Mon 5 June 1978

may be [illegible]

Type III patterns are derived from type II by enlarging the outer cells and peripheral cells about their centres until they overlap adjacent cells in small rhombuses, or rhomboidal shapes. Apart from m = n = 10 the peripheral pentagons of type II patterns are not regular, but nothing can be done about such a defect since the pattern lines of type II are rigidly determined. However, in type III patterns it becomes possible to make the peripheral pentagons slightly more regular by individual adjustment of the sides of various overlapping outer and peripheral cells, since pattern lines no longer have to follow such straight courses over long distances. Only in (3x2)10,10 can the pentagons be perfectly regular, but the degree of regularity achieved is sometimes sufficient to deceive all but the most discerning eye (fig. B, opposite).

The m and n stars are usually felt to be unsatisfactory in the condition illustrated in fig. B, and usually a starred type Ia rosette is inscribed directly on to the inner points of the peripheral pentagons, as shown at fig. C.

The usual type III pattern is produced as stated, by over-lapping the outer and interstitial cells of a type II pattern. Theoretically another kind of pattern is possible by overlapping of the peripheral elements, but this does not seem to occur as an authentic pattern, and certainly gives much less satisfactory results.

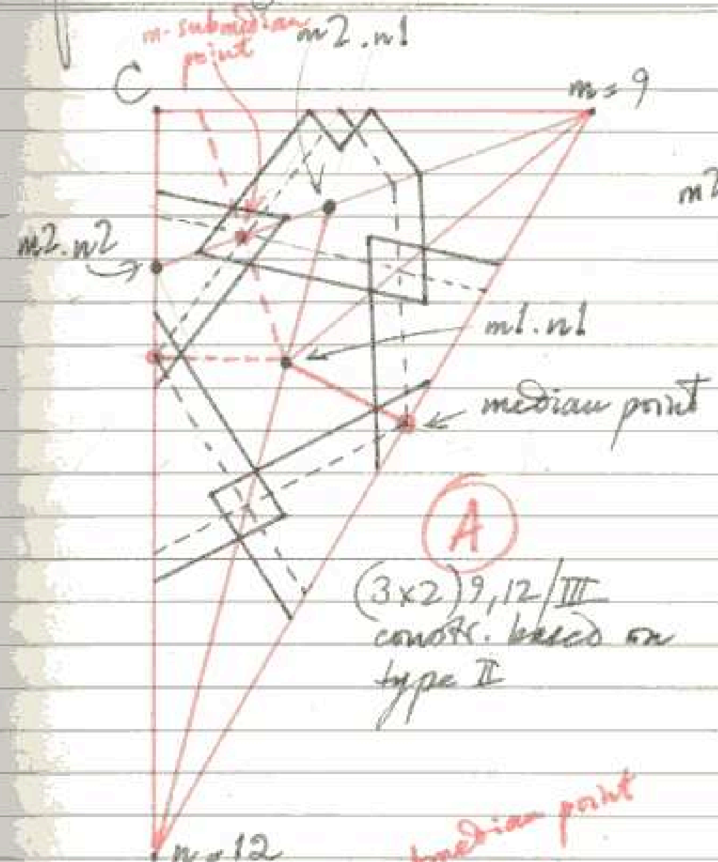
Note: in fig. B we cannot alter the positions of points A, B without reducing the regularity of the 7 and 12-fold stars, but point D can be placed midway between A and B on the circumference of the circumscribing circle; similarly, points E, F can be arranged on the same circumference to make distances AE, EF and FB all equal. This would mean, however, that the small rhombuses in the median and submedian points become kites, although hardly noticeably so.

Mon 5 June 1978

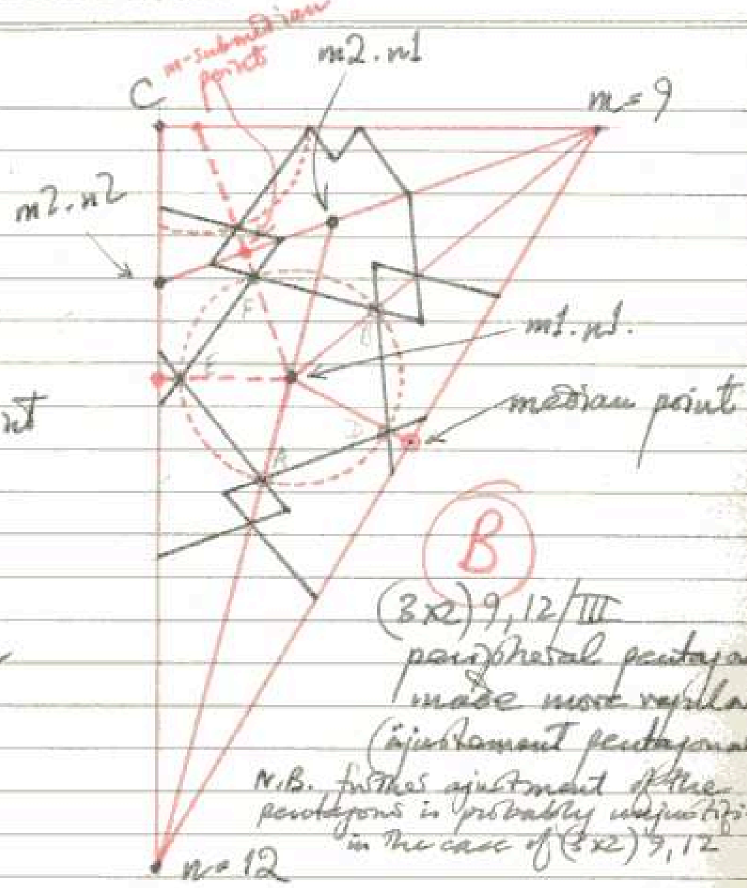
see p. 83 for revised type labels

(3x2)III

Monday, FEBRUARY 14, 1966

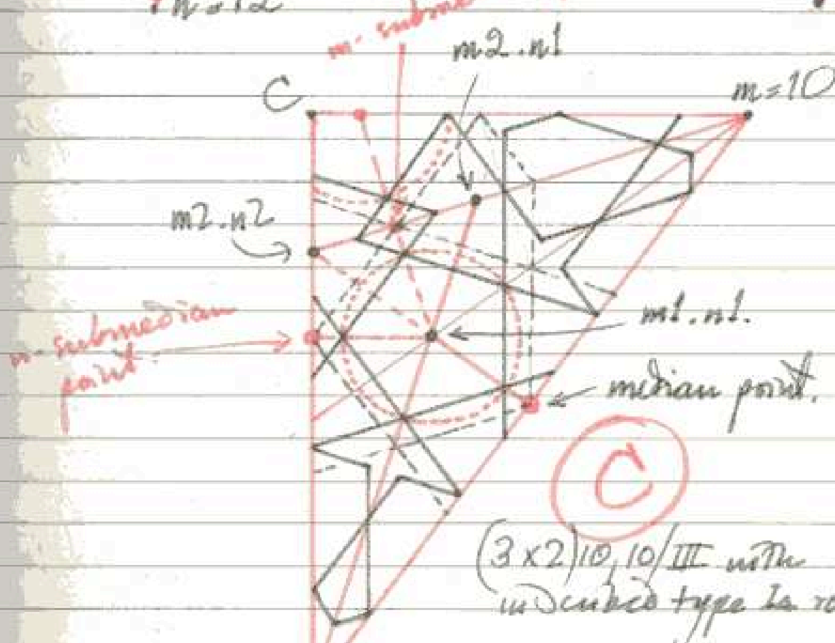


(3x2)9,12/III
constr. based on
type II



(3x2)9,12/III
peripheral pentagons & made more regular
(ajustament pentagonal)

N.B. further ajustment of the pentagons is probably unjustified in the case of (3x2)9,12



(3x2)10,10/III with inscribed type 1a rosettes.

Note: The pentagon should line up with centres m,n from its centre m1.n1, so that the angle between the first coll. radii is usually taken as fixed. The closer this angle is to $4\pi/5$ the more regular is the pentagon likely to be. The remaining vertices of the pentagons need not align with the sides of the limiting polygons, however, but can be obtained by bisecting the 1st coll. angle, etc.

As with so many of these patterns, the construction used depends on exactly which features we wish to regularise the most, and which can be ignored.

37

Mon 12 June 1978

Type VIII This is known so far only from (3×2)10,10/VIII It can be easily adapted to other m, n values but no authentic examples seem to exist. The peripheral pentagrams obviously become more regular if constructed on peripheral circles, using as points of the pentagram the vertices of a circumscribed pentagon (fig. B).

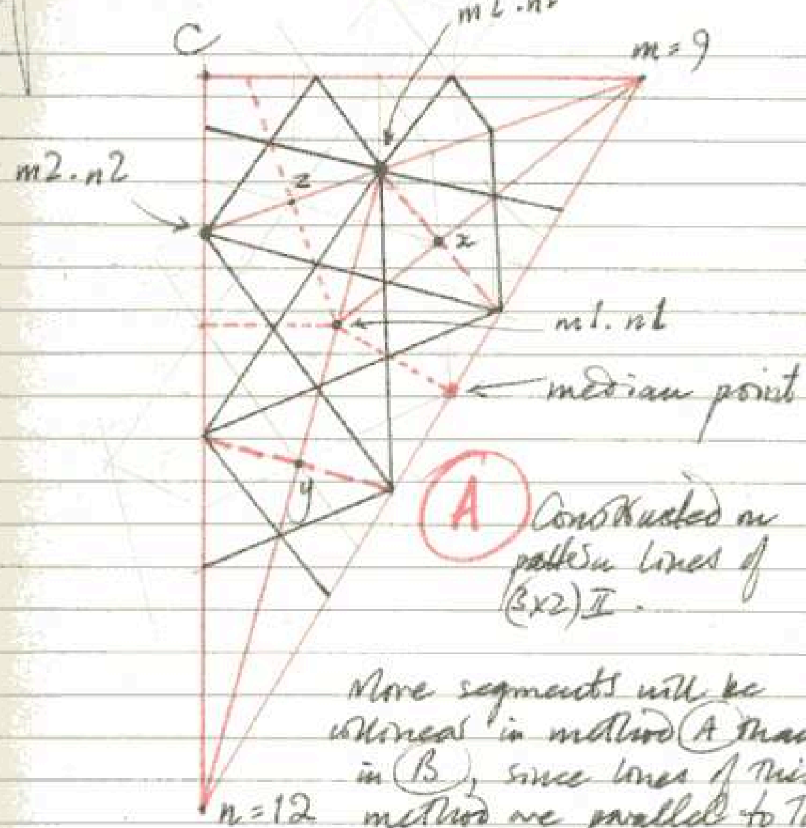
A pattern "type" is a definitive treatment of a (3×2) rhomb skeleton where the same general construction has been used for different pairs of m, n values, on authentic examples. The original artists did not necessarily distinguish clearly between the two centres m and n, where this distinction needs to be made, for example in type IV patterns. But the occurrence of different patterns of the same general type shows that the analogy was made, if not always understood. Indeed, it is likely that the original designers did not understand the common basis for the pattern types in the (3×2) rhomb series, and that patterns appropriate for (3×2) rhombs were tried with other sizes of rhomb. These usually resulted in unsatisfactory arrangements and would therefore not survive as finished patterns. One such "failure" which has survived appears to be exemplified by a pattern of 14-pointed stars on the main entrance to the masjid-i jami Fatchyah Sikri (see Hankin 1925 fig.). This seems to be an attempt to adapt a pattern of the (3×2)/Ia type to the (4×3)14,14 rhomb where interstitial cells congruent to the outer cells of the m-star occur. However, in the (4×3) rhomb it happens that not both of the stars can then be completed simultaneously.

Strictly speaking, the designation (3×2) refers to each of the four right triangles delineated by the two axes, or diagonals, of the rhombus, but it is convenient to extend the designation to the rhomb itself.

see p. 83 for revised type labels.

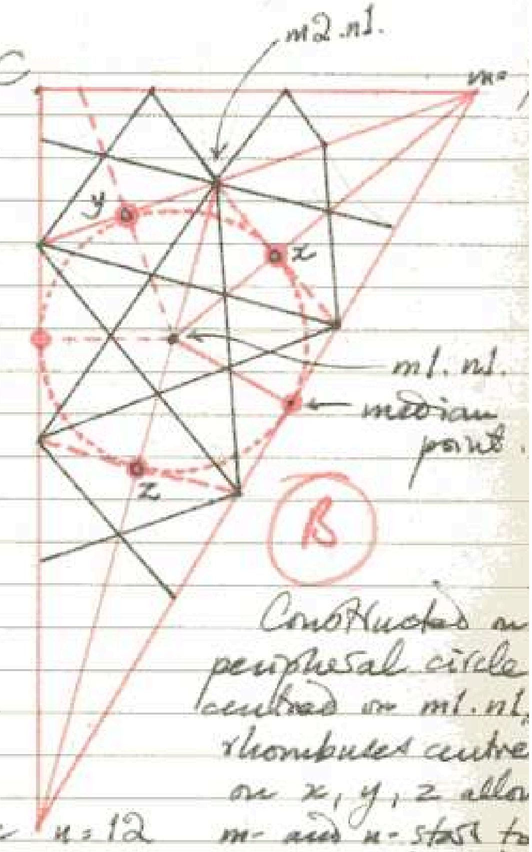
Now 8 June 1978

(3x2) VIII



A Constructed on pattern lines of (3x2) II.

More segments will be collinear in method (A) than in (B), since lines of this method are parallel to those of type II.

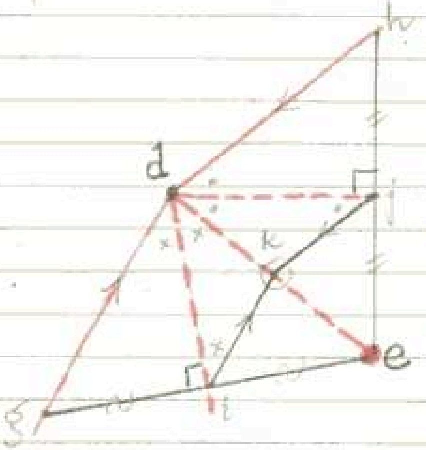


B

Constructed on peripheral circle centred on m1.n1, rhombuses centre on x, y, z allo m- and n- stars to be completed and the interstitial patt

Thursday, FEBRUARY 17, 1966

Tue 6 June 1978



N.B. The pattern lines shown will always meet on line de. at point k.

deg and deh are Isosceles Triangles i, and j are the midpoints of their respective bases.

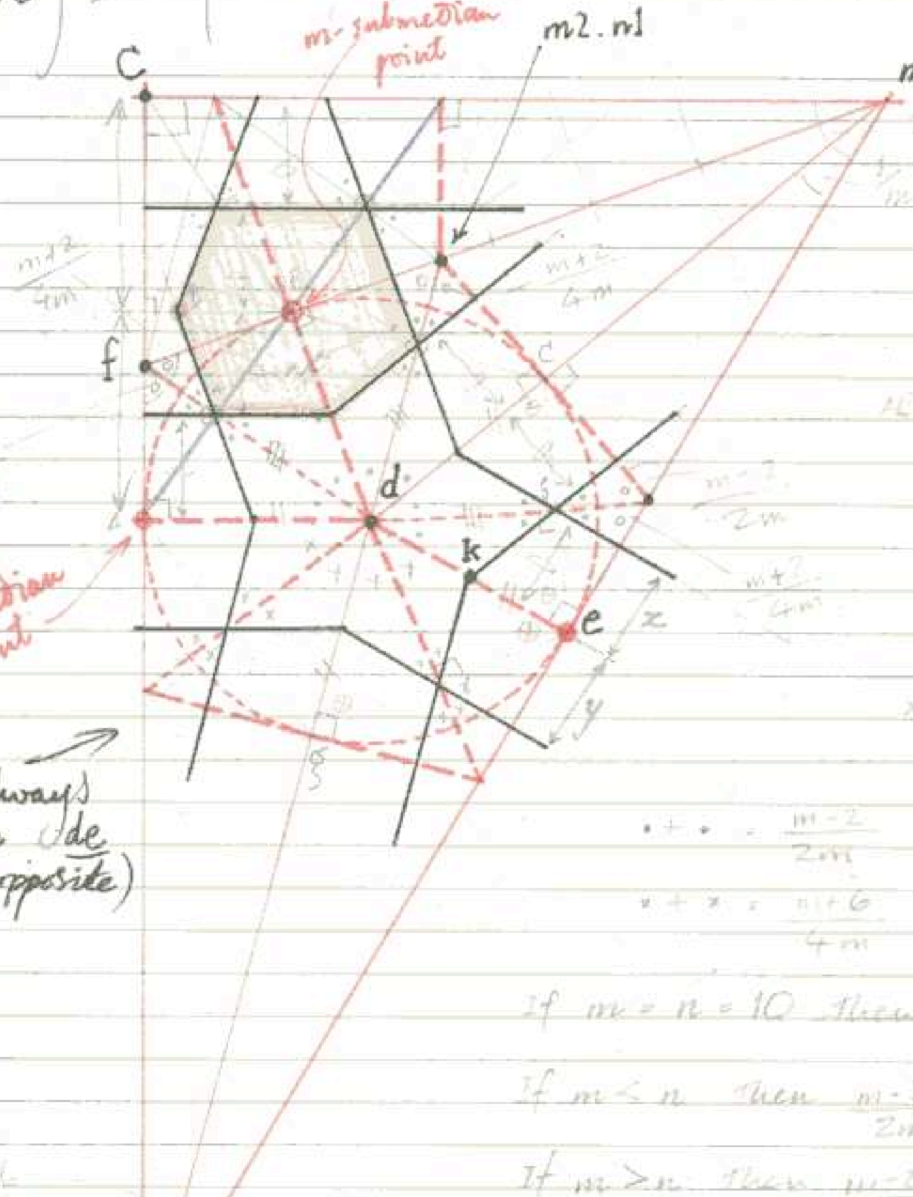
ik is parallel to dg, and jk to dh
Here ik bisects line de, in triangle deg
and jk bisects line de in triangle deh.
since de is common to both triangles,
Therefore k always lies on de, whatever the
value for m and n.

Tue 6 June 1978

Friday, FEBRUARY 18, 1966

(3×2) IV patterns

see p. 83 for revised type labels



Point k always lies on line de (see proof opposite)

$o = \frac{m+2}{4m}$

$\bullet = \frac{m-2}{4m}$

$x = \frac{m+6}{8m}$

$\bullet + \bullet = \frac{m-2}{2m}$

$x + x = \frac{m+6}{4m}$

If $m = n = 10$ then $\frac{m-2}{2m} = \frac{m+6}{4m}$

If $m < n$ then $\frac{m-2}{2m} < \frac{m+6}{4m}$

If $m > n$ then $\frac{m-2}{2m} > \frac{m+6}{4m}$

Similarly, if $m = n = 10$, then $x = y$;
if $m < n$ then $x < y$;
and if $m > n$ then $x > y$.

The m-submedian cell (shaded light orange) is symmetrical about the blue line, and forms the outer cell of an m-fold type Ib rosette.
This cell is exactly similar to the cell centred on m2.n1 in (3×2) type IV patterns.

A1

Wed 7 June 1978 Saturday, FEBRUARY 19, 1966

Type IX and X patterns are both derived by elaboration from type II. The lines of type II are used, although certain small segments are omitted, and additional pattern lines are drawn inside the peripheral pentagons and, in the case of type X inside the outer and interstitial cells also. The patterns are not of course interlacing patterns, and both have an inherent "handedness", which may be defined arbitrarily as left-handed or right-handed. Type X is authentic, from Persia, but type IX is so far known to me only from a Seljuk border of pentagons in carved stone, in Turkey. Both patterns may be adapted to (3x2) rhombs with differing m, n values, and in fact 12,8/X occurs as an authentic Persian pattern.

In view of their close derivation from type II patterns a revision of the type numbers seems called for, which will express this fact better than the present system of numbering does.

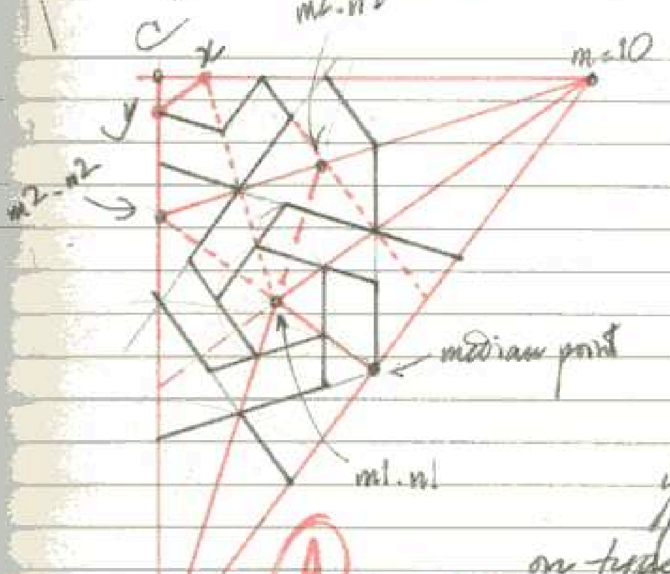
Patterns in types IX and X should be drawn with the peripheral designs of the same handedness throughout any single pattern, but it is permitted to have left- and right-handed patterns on two disconnected panels symmetrically placed each side of some central axis.

Sunday, FEBRUARY 20, 1966

see p. 83 for revised type labels

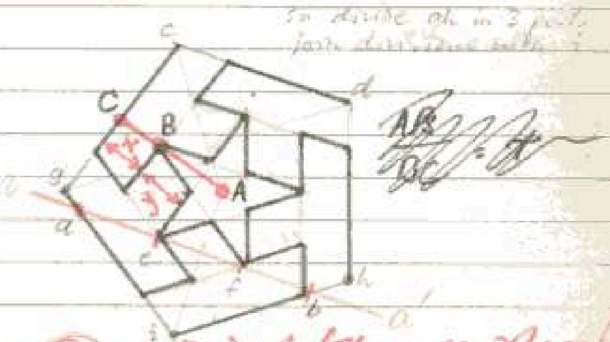
Wed 7 June 1978

(3x2) IX, X

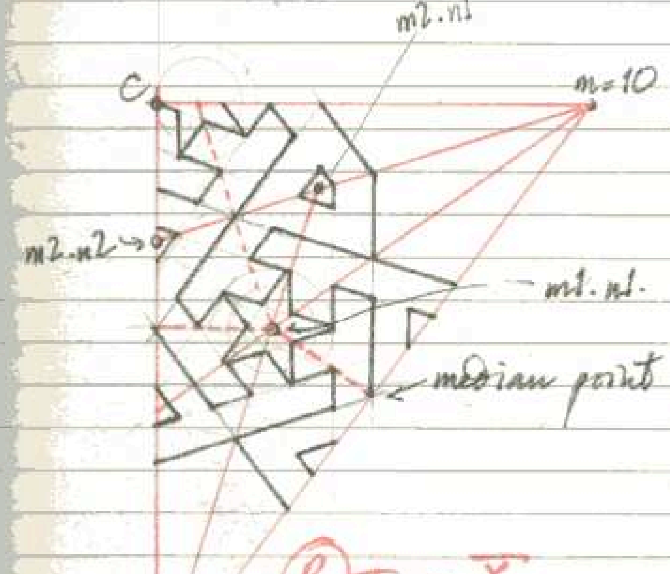


(A) Type IX

The determining factor in type IX patterns is that two small pentagons centred on point x should share an edge (centred on the middle point C of the rhombus) across the n-axis of the rhomb. The same sized pentagon is then drawn on the first collateral intersection, m1.n1. Pattern lines of type IX are largely superimposed on type II – shown in blue lines.



(C) method of filling peripheral pentagons of type X.



(B) Type X

The determining factor in type X patterns is the method of filling the peripheral pentagons of a type II, from which type X derives. This must be constructed so that x = y. Unfortunately it will then be found that two such central pentagrams will not meet exactly at the centre of the rhombus but their points instead overlap. This small unavoidable blemish must be disguised as much as possible when drawing the pattern. — see also p. 245 →

43)

Thur 8 June 1978

Tuesday, FEBRUARY 22, 1966

Types VI and VII patterns, of which the bare outline of the essential construction is given opposite, are characteristically realized as wooden lattices, mainly in Central Asia. The only named intersection used is m1.n1, and the construction is easily adapted to other rhomb sizes.*
In type VIa the first collateral intersection determines the outer point of an m-pointed star centred on m, and the midpoint of the edge of an n-gon centred on n. The vertex x of the n-gon, on the aligning radius nO, determines the inner point of the m-pointed star. In type VIb point m1.n1 determines the outer point of an n-pointed star centred on n, and the midpoint of the edge of an m-gon centred on m.
Type VII is similar to VI except that instead of an m-gon or n-gon as the case may be, we have a 2m-gon or a 2n-gon, a vertex of which is determined by point m1.n1.
In type VI points x, y are determined by a line through point m1.n1 perpendicular to radii n1, m1 respectively.
Similarly in type VII points x, y are determined by circles centred on n, m respectively, through point m1.n1, striking the aligning radius.

Note that in Rp1(3x2)10,10/VI and VII both versions, a and b, are necessarily used in the same pattern, and the rhombic tessellation uses rhombs with two kinds of filling and is therefore properly Rp1′. In other cases, when m ≠ n, the pattern as a whole can be labelled a or b.

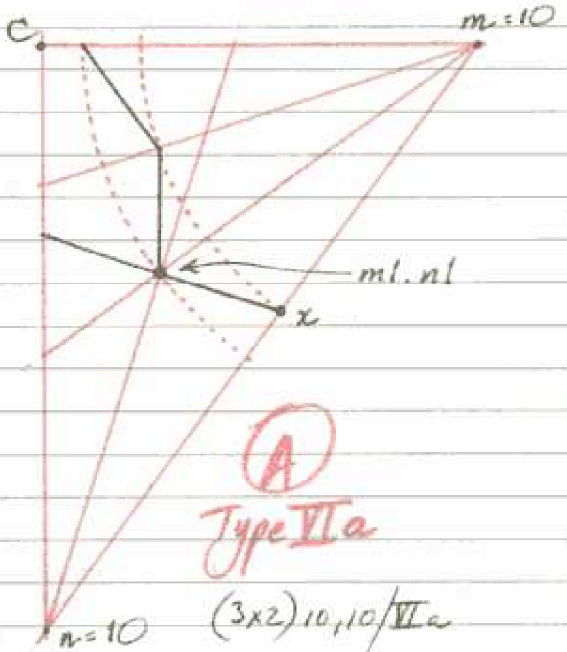
* Size here refers to the specific (pxq) values of any particular rhomb. Thus all (3x2) rhombs are the same "size", but not the same "shape".

Wed 7 June 1978

see p. 83 for revised type labels

3x2 rhombs

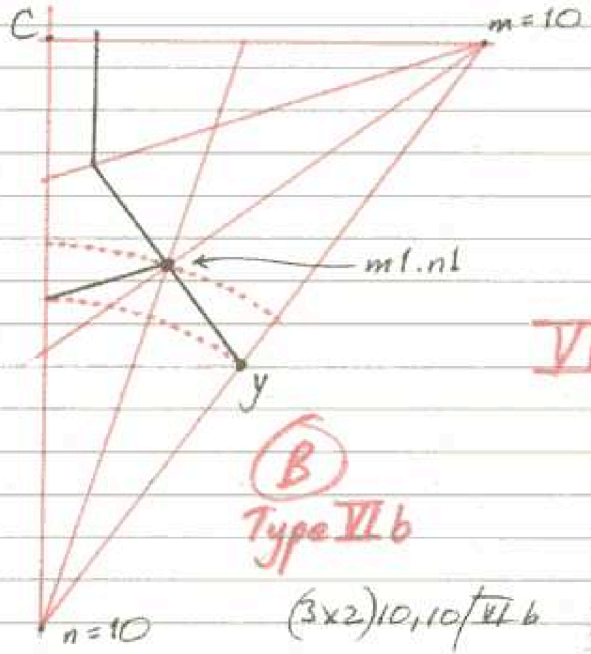
Types VI, VII



A

Type VIa

(3x2)10,10/VIa

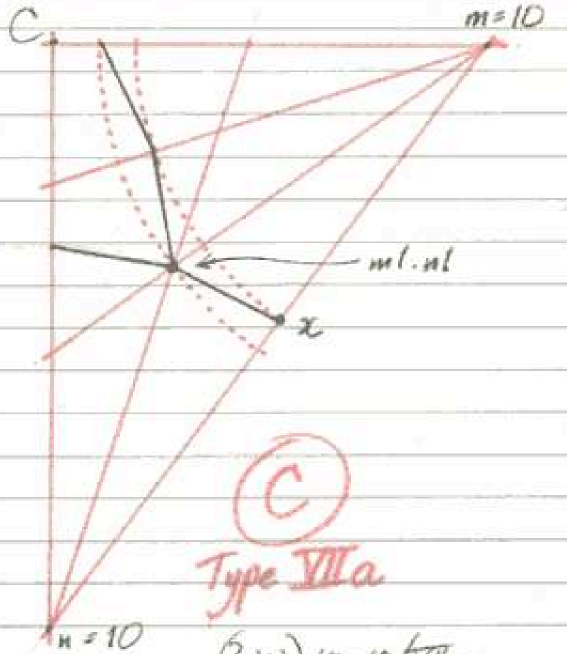


B

Type VIb

(3x2)10,10/VIb

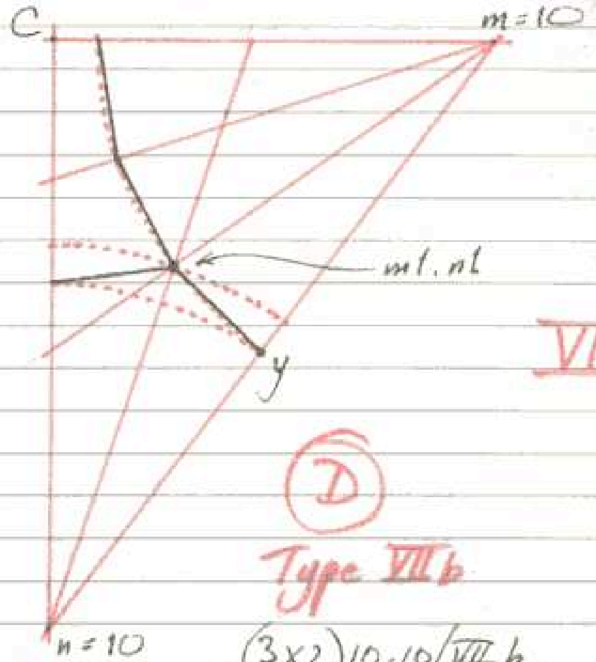
VI



C

Type VIIa

(3x2)10,10/VIIa



D

Type VIIb

(3x2)10,10/VIIb.

VII

Thu 13 June 1978

Rp1(3x2)10,10/Ia This is one of the most widespread of all patterns and was one of the earliest geometrical rosette patterns to appear. One of the earliest examples occurs in the North Dome Chamber of the Masjid-i Jami, Isfahan dated 1088. It also occurs on woven minbars in the Aqsa mosque Jerusalem (1168) and mosque of Alā ad-Din, Konya (1155). It is certainly one of the easiest patterns to construct, due largely to the fact that it contains rosettes of one kind only, and also to the fact that line segments align with many others across the entire pattern. It is perhaps best to begin the pattern by drawing starred rosettes, tangent across the minor axis of the rhombus, but overlapping on its edges. The peripheral stars are the outer borders of regular pentagrams, but those in the rhomb centre are incomplete, having two of their points truncated. The interstitial cells are congruent to the outer cells of the rosettes, and their inner points meet exactly at the centre of the rhombus

Rp1 (3x2)10,10/Ib This version seems to be later in appearance and is much less widespread throughout Islam. The re-entrant angles of the peripheral stars are much flatter than in type Ia due to the fact that the further sides of adjacent points are parallel. The interstitial cells again are congruent to the outer cells of the rosette, but they no longer meet at the rhomb centre. The midcells of the rosettes of type Ib are the same shape as the outer cells of the stars of type II. It is possible to continue the inner points of the interstitial cells until they meet to form a rhombus at the centre of the rhombus.

Types Ia and Ib are special cases in an infinite sequence in which the slope of the pattern lines is continuously variable through certain fixed points, termed 'nodal points'. (x, y) When $m=n=10$ the series is continuous with type II, which uses the same nodal points. However, when $m \neq n$

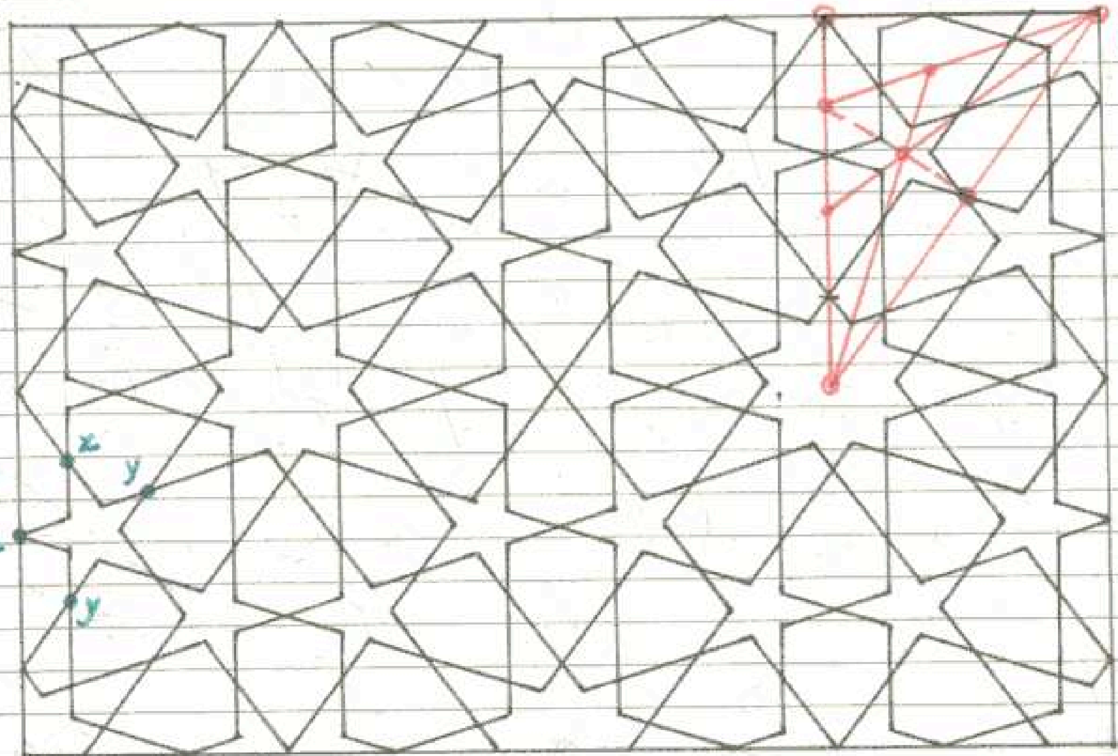
Tue 13 June 1978

Type numbers in blue circles are the suggested definitive designations (Oct 1979)

(3x2)10,10/I

II

"PIC"



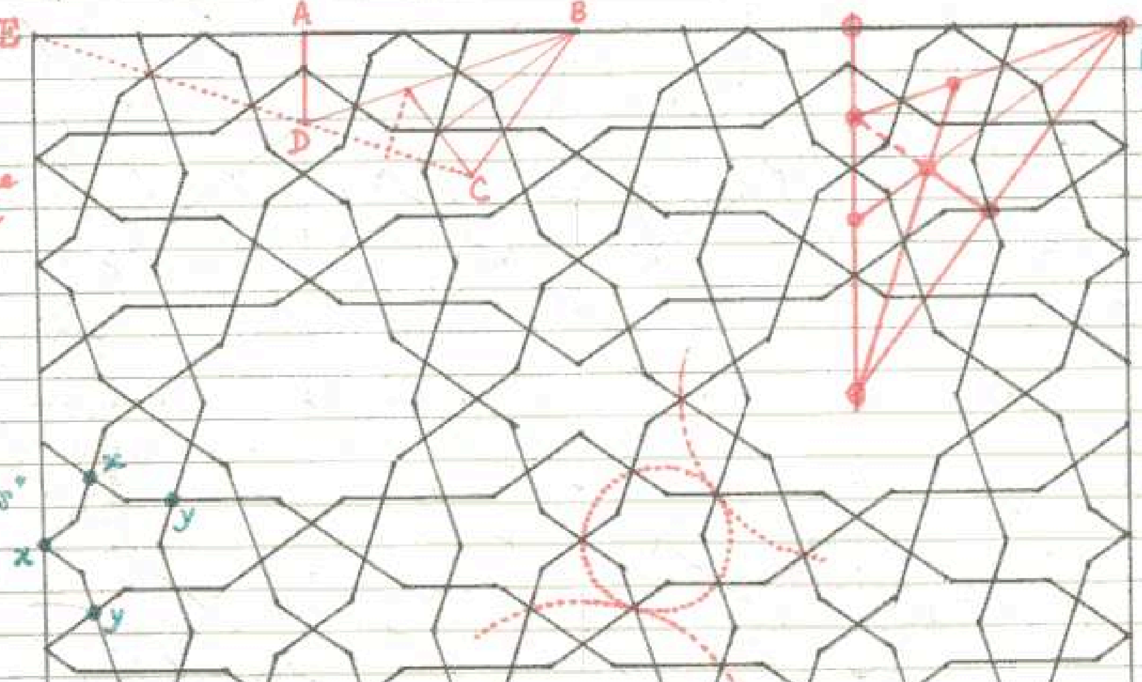
Rp1(3x2)10,10/Ia

Ia

Radius of inner star is half the distance from intersection m1.n2 to point n.

III

"P1"



Note that CDE is a straight line only when m = n = 10

Ib

Rp1(3x2)10,10/Ib

Note that pattern lines within quadrilateral ABCD are identical to those in type IV. (see p. 50)

Saturday, FEBRUARY 26, 1966

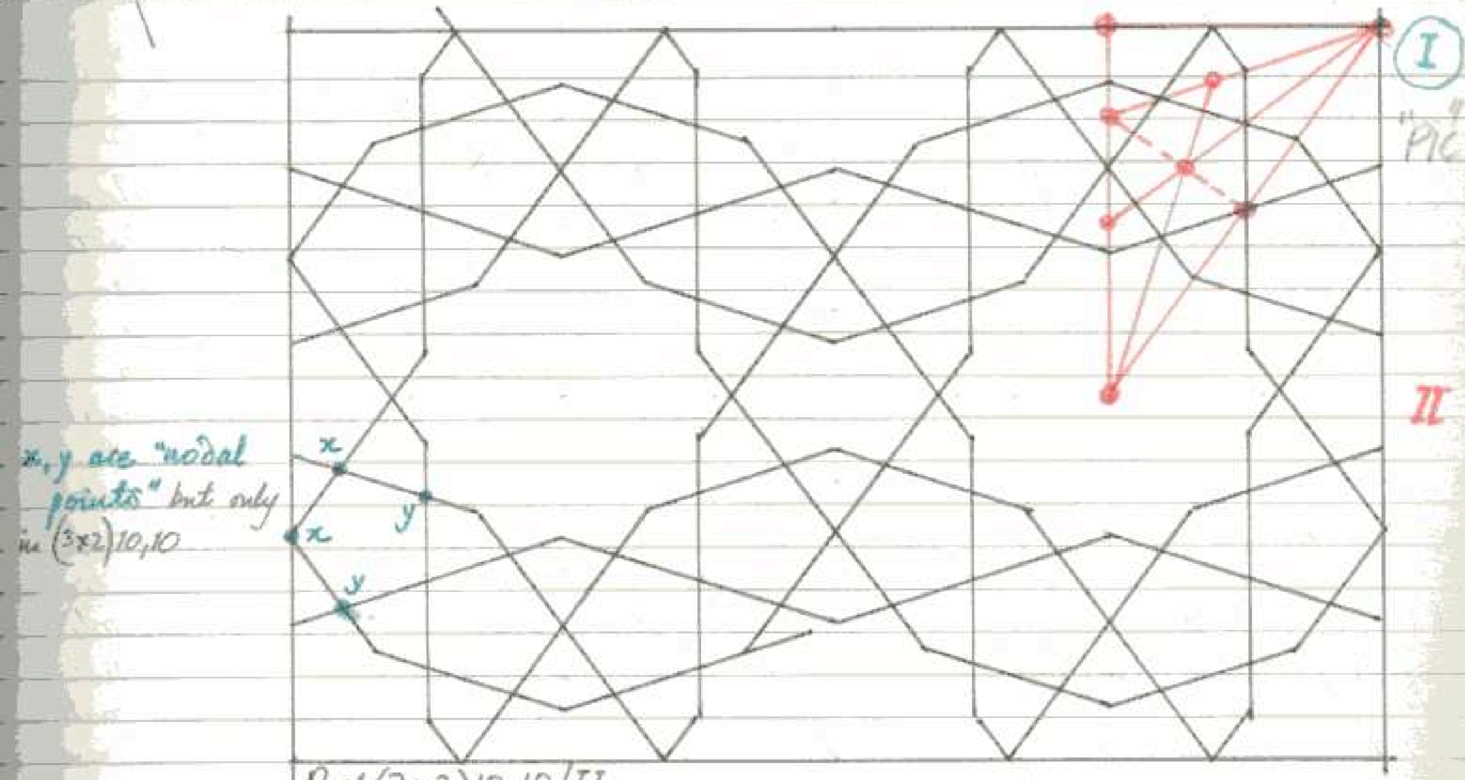
This is not always so

Type II patterns are distinctive in that apart from the case $m=n=10$ the cell centred on $m1.n1$ is not formed entirely from the incircle of the 2nd collateral triangle, and the centre of the outer n-cell on radius $n2$ does not correspond to that in type I patterns.

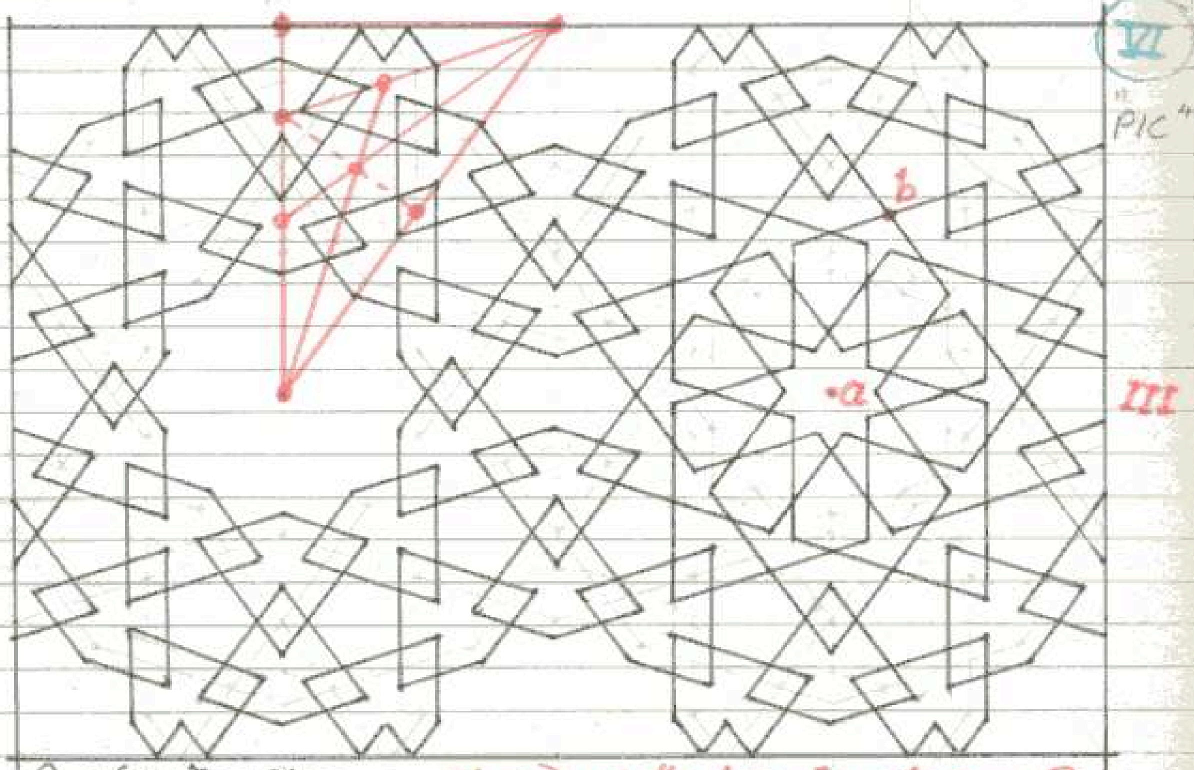
Sunday, FEBRUARY 27, 1966

Tue 13 June 1978

(3×2)10,10/II, III



Rp1(3×2)10,10/II



Rp1(3×2)10,10/III.

Starred rosette type Ia at a, with radius ab.

49

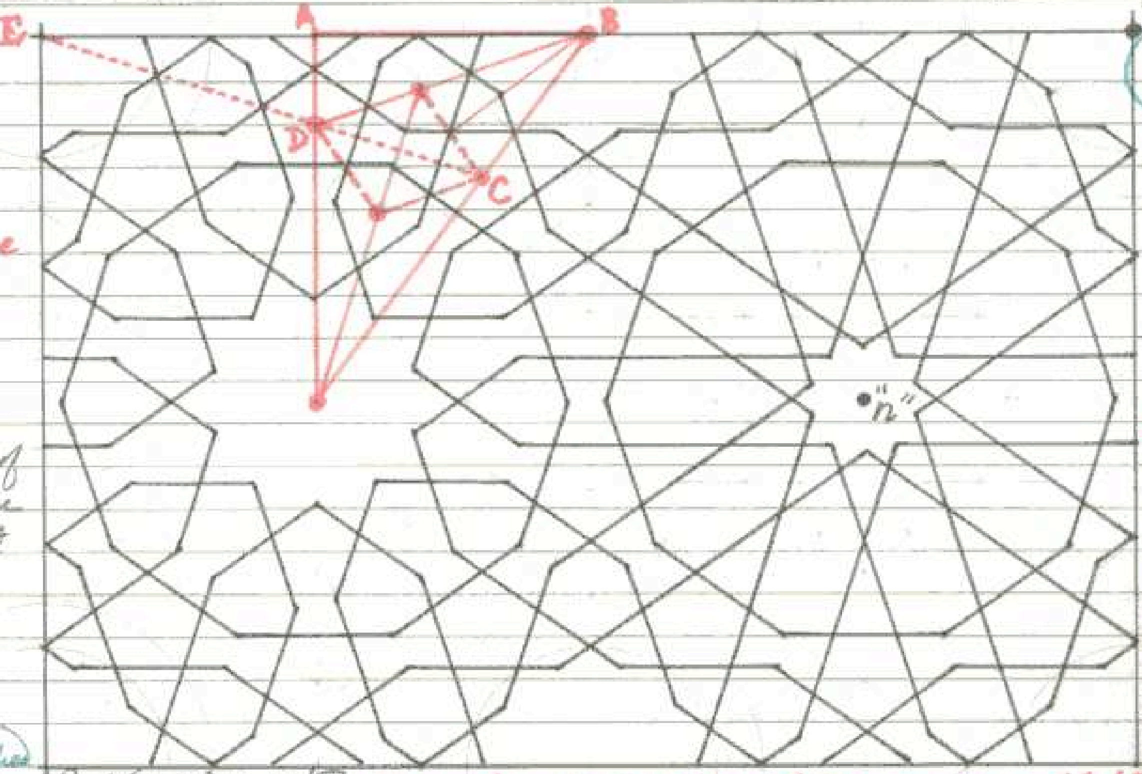
Apr Fri 16 June 1978

Rp1(3x2)10,10/IV Of the pattern types so far recognised in the 3x2 rhomb series, types IV, VI and VII are distinctive in that they possess motifs of two different kinds on alternate star centres. The rhombs of the Rp1 pattern outlined by the star centres occur as two varieties according to their content and whether one or the other variety of motif is centred on the m or n vertex. The rhombic tessellation is then equivalent to a dichromatic colouring of the original Rp1 pattern, and might be distinguished as Rp1′.

In an Rp1(3x2)10,10 pattern there is only one kind of vertex and the star-centre forming the m-vertex in one rhomb also forms the n-vertex in the next rhomb, and vice versa. In fact each vertex is common to four rhombs, to two of which it contributes an m-centre, and two an n-centre. In other patterns in the (3x2) series, where $m \neq n$ the two centres are consistently distinct throughout the pattern, i.e. there are are two kinds of vertex throughout the pattern. In such patterns it is found that type IV patterns can only be constructed exactly if one type of motif is constructed on the m-vertex, and the other kind on the n-vertex; these two kinds are appropriately labelled in the upper right corner of the diagram on p.50, opposite. In Rp1(3x2)10,10 the pattern works both ways, since every vertex is simultaneously an "m" and an "n" vertex, but in other cases an exact construction is possible only if the "m"-motif is centred on the m-vertex and the "n"-motif on the n-vertex. This version of the pattern may be termed type IVa. It is possible to construct a pattern with the motifs reversed, and in fact Sp1(3x2)12,8/IVb exists as an authentic pattern in Persia, but both centres cannot be regularly formed in this type IVb. In the case of Rp1(3x2)10,10/IV of course one cannot distinguish type IVa from IVb, but such a panel as that illustrated opposite can obviously be drawn either with the "n"-motif prominent, as here, or with the "m"-motif prominent, and these might be distinguished in a loose sense as IVn and IVm respectively. We may further note that the m-motif of a type IV pattern resembles the rosettes of a type Ib pattern, and the n-motif, those of type V.

$(3\times2)_{10,10}$/IV, V

Ray 14 June 1978



VII

NOT PIC

IV

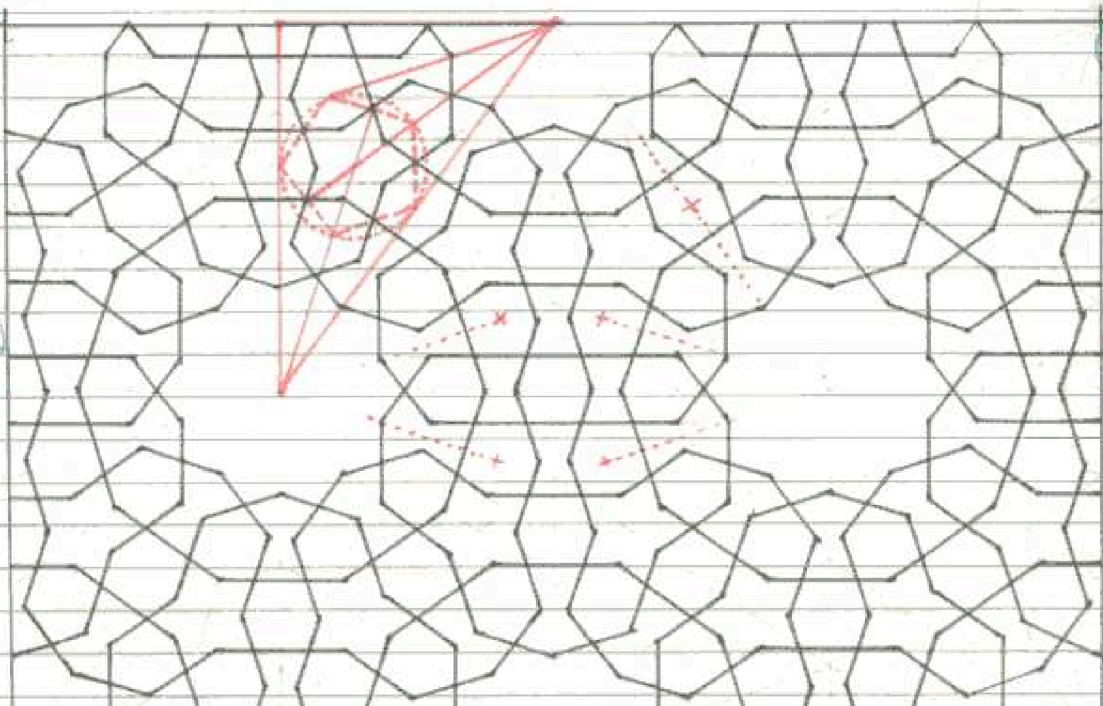
N.B. That CDE is a straight line only when m = n = 10

N.B.
The m-rosette resembles that of type Ib and the n-rosette that of type V.

Rp1 $(3\times2)_{10,10}$/IV For pattern lines within ABCD, compare p. 46/Ib.

N.B. as a published plate, illustrate the two ways of drawing such a panel as the type IV pattern: with the "n"-motif central, as here, or with the "m"-motif central. Where appropriate, these might be distinguished as the n-version and the m-version, respectively.

Ray 16 June 1978



VIII

PIC

V

Rp1 $(3\times2)_{10,10}$/V

N.B. This is geometrically related to XII on p. 54.

~~Mon~~ Tue 20 June 1978

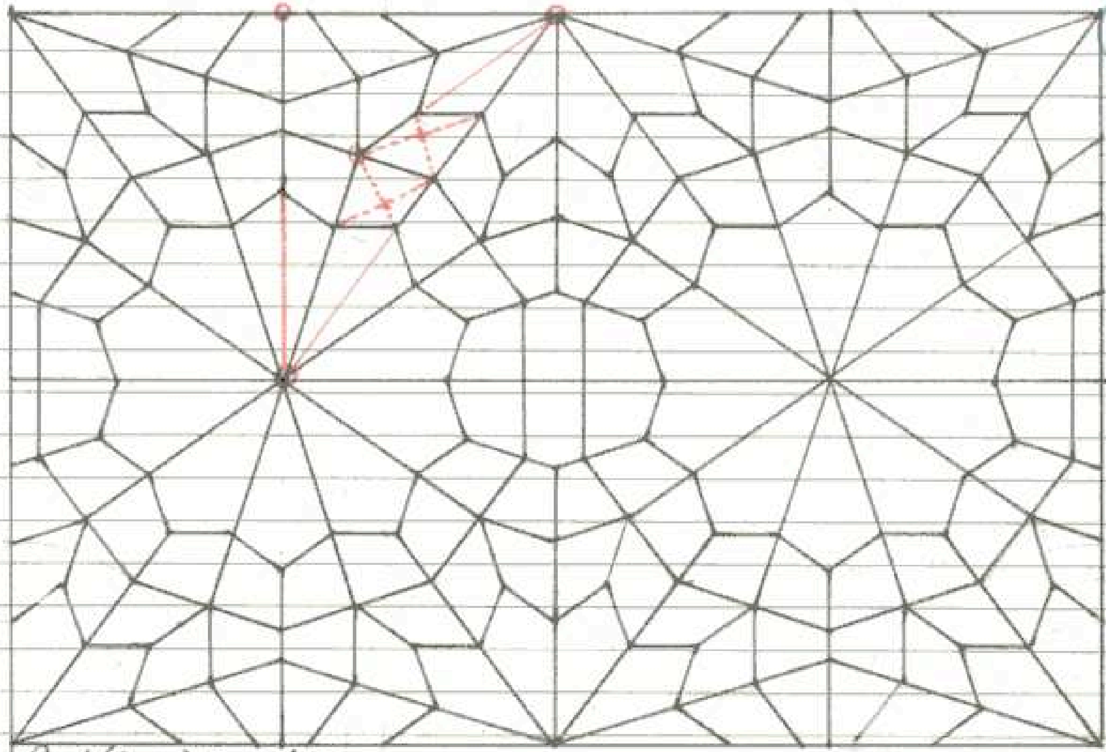
Types VI, VII Patterns of these types are typically constructed as wooden lattice (see L. Rempel, 1957 and R. Orazi, 1976). The rigidly determined parts of these patterns are of limited extent and are shown on p. 44. The precise location of the remaining pattern lines, of which arbitrary examples are shown opposite, is variable, although certain rationalisations are normally carried out, as noted by Orazi. The small areas round the edges of the star-like or polygonal motifs are typically, though not invariably, drawn as kites, each with its own local axis of symmetry. This procedure completely determines the size and shape of the inner star of each motif, however. The appearance of these patterns can be greatly varied by placing the points of the inner stars on either odd or even radii (cf. diagram A on p. 25), and since this can be done independently with either the m-centre or the n-centre there are theoretically four distinct combinations. For example, in the type VI example shown opposite the central motifs have the points of their inner stars on even radii, whereas the corner motifs have the same points on odd radii. With Rp1(3×2)10,10 it is not possible to differentiate the m- and n-centres, but the two types of motif are still of course quite distinct. In fact in type VI and VII patterns the construction is possible after interchanging the two kinds of motif between the m- and n-centres, so that in cases where the m- and n-vertices can be distinguished there are theoretically eight different varieties possible, along the lines just mentioned.

Wed 4 Jan 1978

Friday, MARCH 4, 1966

(3×2)10,10/VI, VII

IX

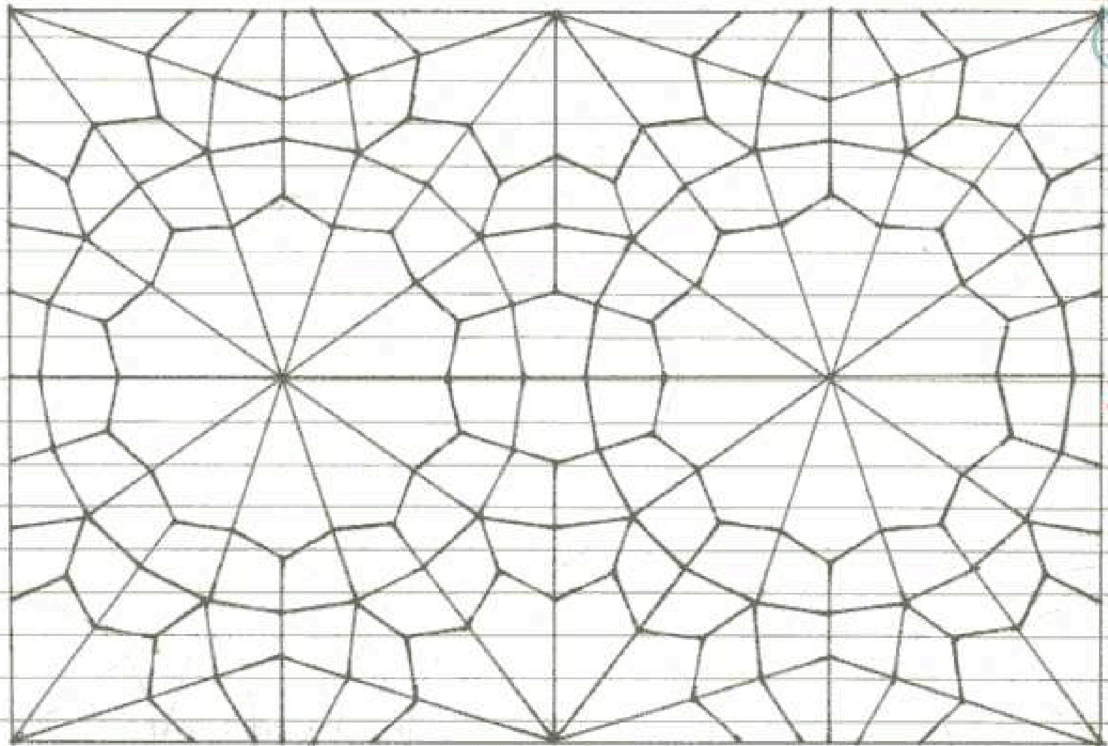


VI

Non-pic, but using intersection ml. nl

Rp1(3×2)10,10/VI

X



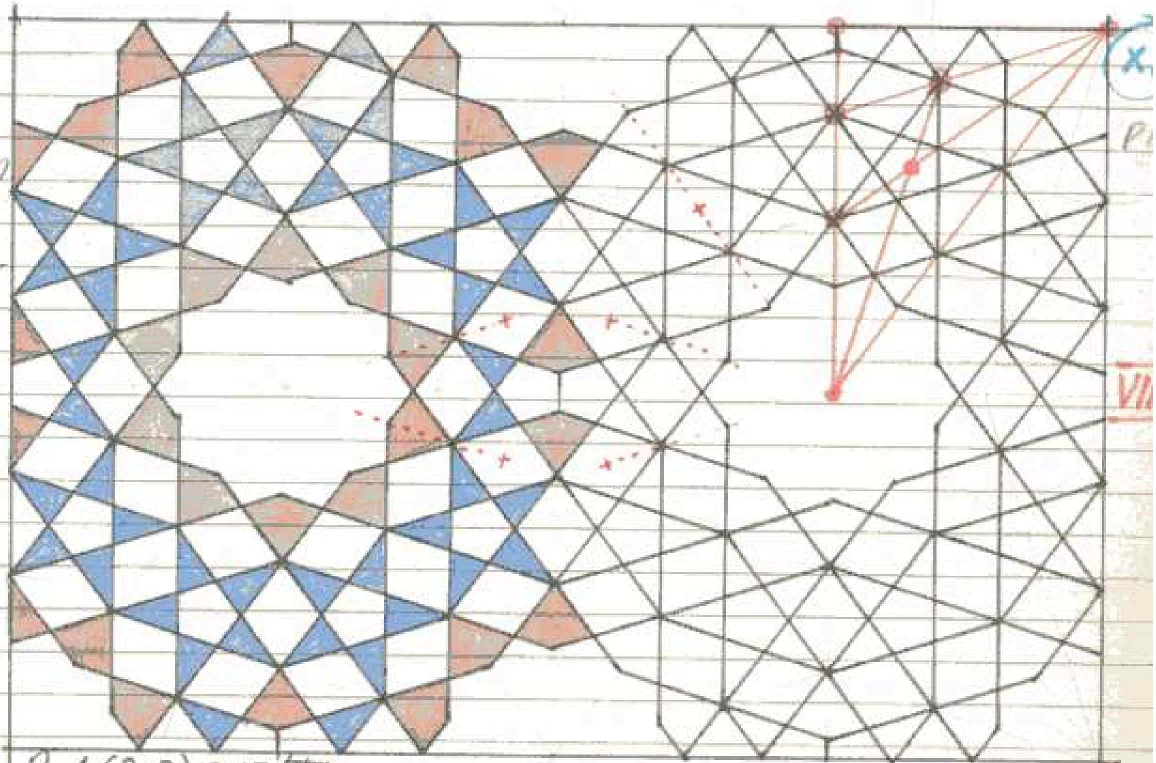
VII

Rp1(3×2)10,10/VII

[illegible] June 1978

Monday, MARCH 7, 1966 (3×2)10,10/VIII

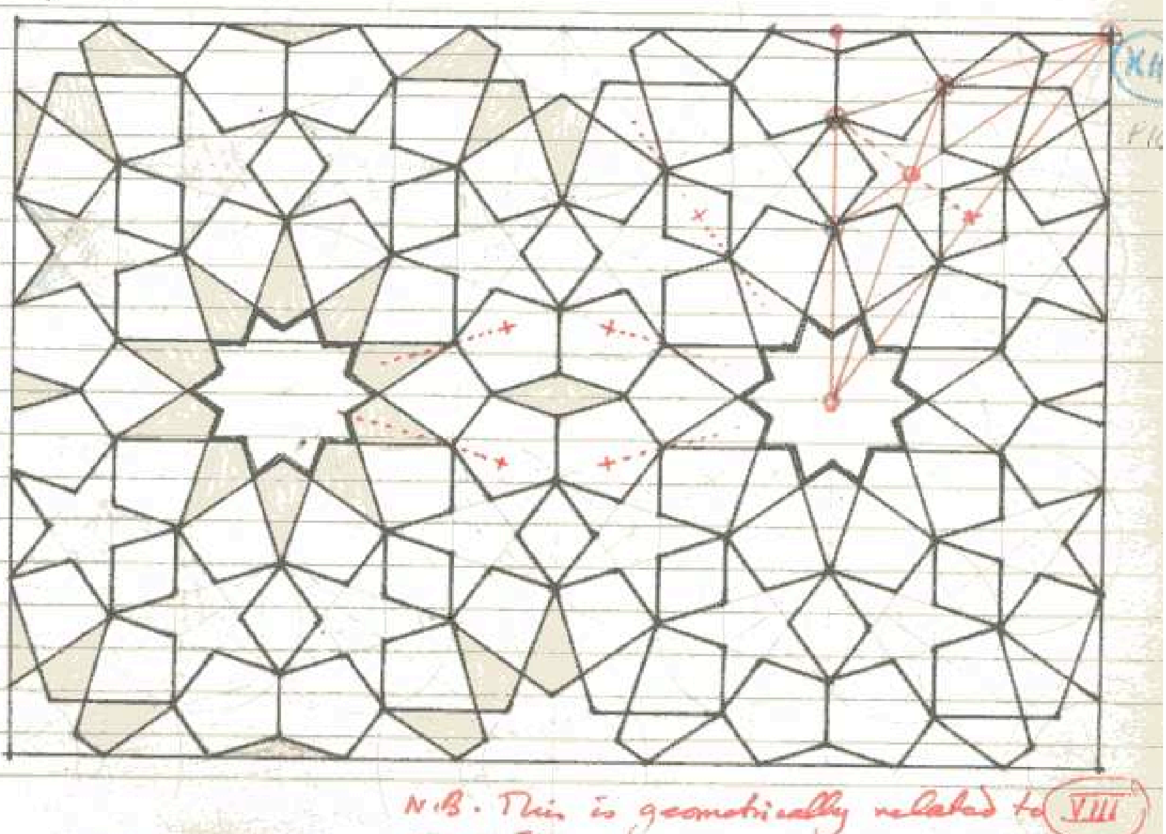
The small pentagons are the same size as those of type IX.



Rp1(3×2)10,10/VIII

A closely related pattern to the above. Derived from a different arrangement in Persia, but it may well occur in the form shown here.

[illegible] 13 Sept 1978

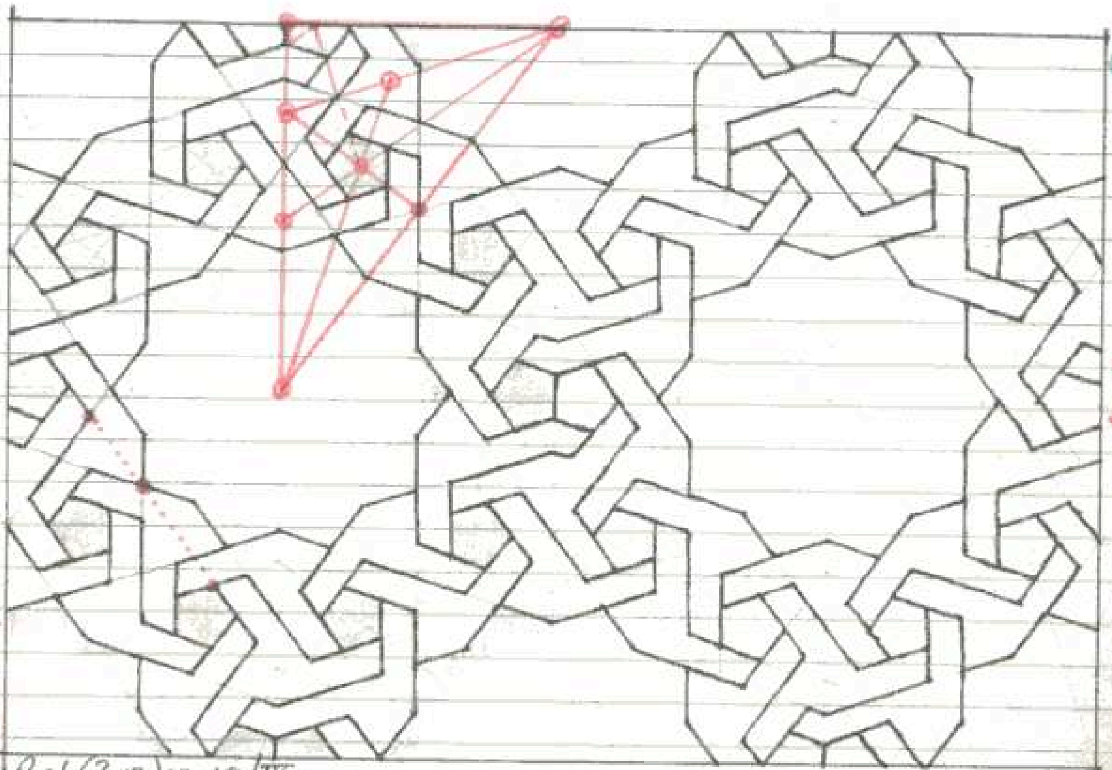


N.B. This is geometrically related to VIII on p. 50.

Wednesday, MARCH 9, 1966

$(3\times2)10,10/\overline{IX}, \overline{X}$

The small pentagons are the same size as those of Type $\overline{VIII}$



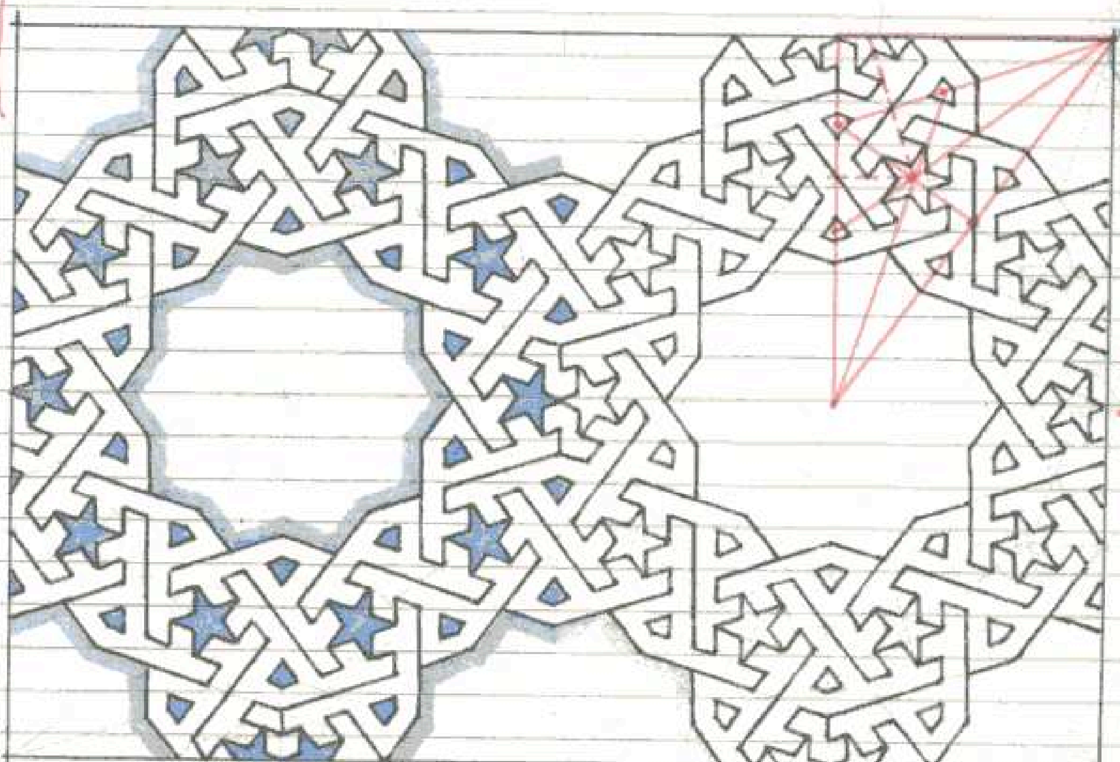
Rpl $(3\times2)10,10/\overline{IX}$.

IX

These should not be regarded as distinct "types", but merely as elaborations of Type I — see pp 83-84 for a brief classification of basic pattern groups in (3×2) Rockets.

Sun 27 May 1984

See also p.p. 22 and 225



Rpl $(3\times2)10,10/\overline{X}$

X

After Fri 22 September 1978

PATTERN VARIATION

Rosettes or stars in a pattern may touch their nearest neighbours at a shared point (x in fig. A, opposite) or they may be separated by a greater or lesser distance (fig. B). This intervening space must be filled with pattern lines of some kind or other, the style of which should ideally match the treatment of the stars themselves. In the figures shown opposite the stars are drawn in black, while the intervening pattern lines which lie outside the stars are shown in red. These red lines may be referred to as __interstitial pattern__ lines. In fig. A the interstitial pattern is of very limited extent, and consists of __interstitial cells__* which are congruent to the outer cells of the stars themselves. – see note on p. 58, opposite

In the decagonal series of patterns, examples of which are shown here, an infinite number of arrangements is possible, using very few basic cell shapes. Apart from the cells in the stars, fig. B uses only four different shapes of cell, and these four shapes are in fact by far the most common throughout all authentic Islamic patterns of this class. Three of these shapes are already present in the parent pattern of the series (fig. A, Rp1(3x2)10,10(a)) and the remaining vase-shaped cell is easily derived from arrangements of the other cells.

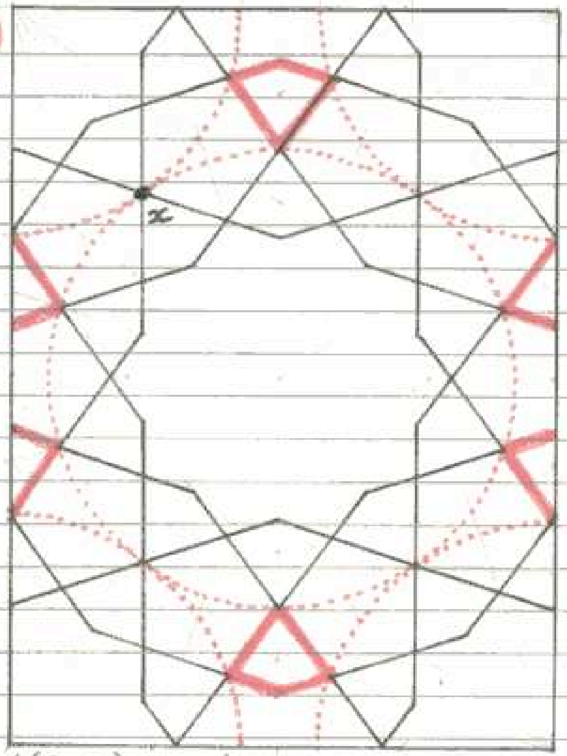
An examination of pattern types and their more elaborate variations may conveniently begin with the (3x2) rhomb series, and specifically with the central member of the series, when __m__ = __n__ = 10 (see p. 12). The greatest variety seems to occur with arrangements of cells derived from type II patterns, so we may begin by examining this series, using Rp1(3x2)10,10 as a basis.

All the cells in type II patterns have precisely defined shapes, and if we limit the interstitial patterns to these cells, then it is obvious that the distance between the centres of two stars, such as C, D in fig. B, may be similarly defined by the ~~cells~~ sequence of interstitial cells which lie on, or touch the line CD. It is convenient to be able to refer to all the cells which may occur by means of code letters, or some other equally simple system. Consequently the cells occurring in fig. B are labelled __a__, __b__, __c__, __d__ and __e__, as indicated in fig. C, opposite.

The centre to centre distance CD of the stars shown in

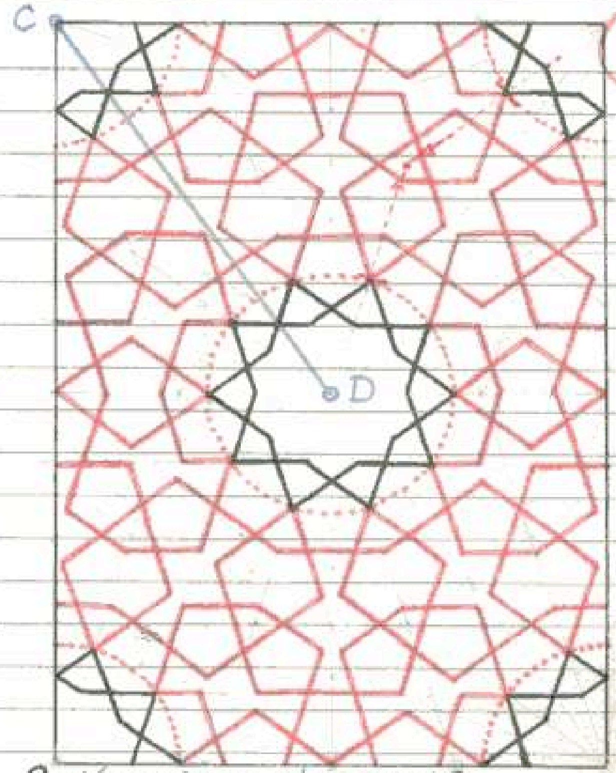
* These are not of course the only interstitial __cells__ in this pattern, which are in fact of three different kinds, __b__, __c__ and __d__ as coded in fig. C, opposite. In primary patterns

A



Rp1(3x2)10,10/II

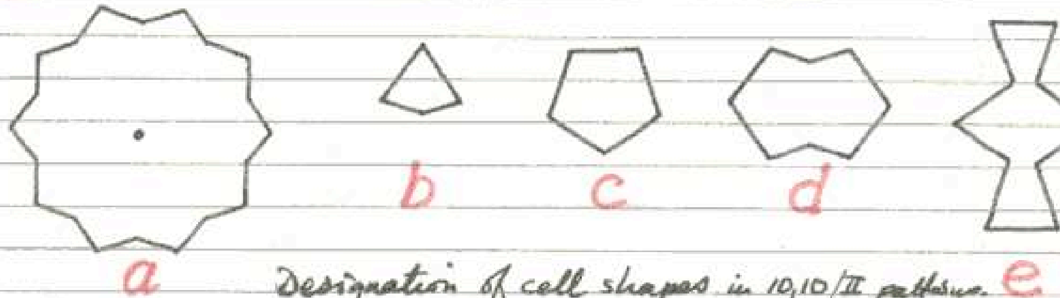
B



Rp1(3x2)10,10/II → 3AB

N.B. we could define "interstitial cells" as any cells entirely enclosed by interstitial pattern lines (red in patterns above. Cells only partly enclosed by interstitial lines are peripheral cells.

C



Designation of cell shapes in 10,10/II patterns.

fig. B may then be precisely fixed by noting that the stars are separated by three interstitial cells, c, e, c. As derivative patterns become more complex, and the distance between the primary stars becomes greater, it will be found that the number of different ways of rearranging the cells forming the interstitial pattern increases. For this reason, the foregoing method for coding the distance CD does not necessarily allow one to complete the remainder of the pattern in a unique manner. Line CD in fig. B is the side of

of types I and II the term "interstitial cell" is used in a particular sense to refer to the pair of cells which are congruent to the outer cells of stars or rosettes.

Fri 22 Sept 1978

PATTERN VARIATION

the rhomb in the rhombic pattern Rp1(3x2)10,10, and the same sequence of interstitial cells occurs on every such side. It is of course also possible to designate the sequence of interstitial cells occurring on both major and minor axes of the rhomb in the same pattern, and for the simpler varieties this method would certainly enable the remainder of the interstitial pattern to be uniquely determined.

It will be noted that interstitial cells are always arranged, when they lie on the side of the basic rhombus, or on its two axes, so that each such line coincides with a symmetry axis (mirror axis) of the cell. In other cases a cell merely touches the line at one of its points, or it does not touch it at all. When a cell lies on the line, its code letter may be accompanied by a single prime, e.g. a', b', c', etc. When the cell touches the line only at one of its points it may bear a double prime, e.g. b", c", etc. Thus, for the pattern shown in fig. B on p. 58 the rhomb side may be coded c', e', c'; the minor axis b', d', b', c", b', d', b' (one could also add a c" at beginning and end of this sequence); and the major axis c', e', c', b", c', e', c'. For even greater precision, one could also code the cells occurring on specified radii within the rhombus (see pp. 25, 26), but the code for any particular pattern becomes already quite cumbersome. Note also that in this last case some of the cells will not be symmetrically disposed each side of the radius.

However, it so happens that the commoner variants in the decagonal series may be derived from ~~the~~ simpler patterns in their series, and may consequently be given a somewhat simpler code designation (as for example Rp1(3x2)10,10/II-3AB for the pattern shown in fig. B, p. 58). Referring to fig. A on p. 58, note that the short axis of the basic rhombus has a single interstitial cell lying on it, and may therefore be given the designation d' (the long axis would become b'd'b'). Now, suppose we take this sequence of a single interstitial cell, d', and use it as the side of the basic rhombus in an Rp1(3x2) 10,10 pattern. The result is as shown in fig. A on p. 60, opposite. The remaining interstitial cells are then completed by continuing

Fri 22 Sept 1978.

Rp1(3x2)10,10/II–2A

rhomb side = d′

short axis = c″, e′

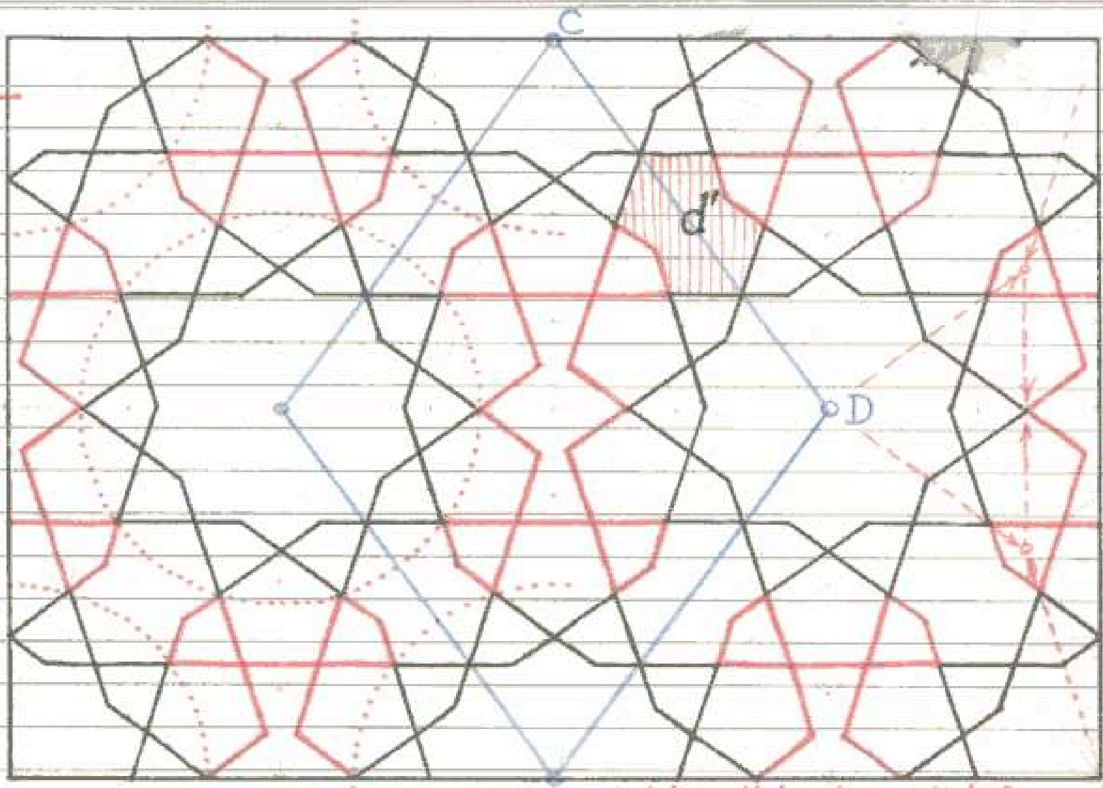
long axis = c′, e′, c′

Rhomb edge
= i(–)d′ or id′

A better convention would be a single prince if the line touches a cell at one point, two primes if it intersects a cell at two points.

Mon 28 May 1984

A



interradial collinear links } see p.157
radial collinear links

Rp1(3x2)10,10/II–2B

rhomb side = b′d′b′

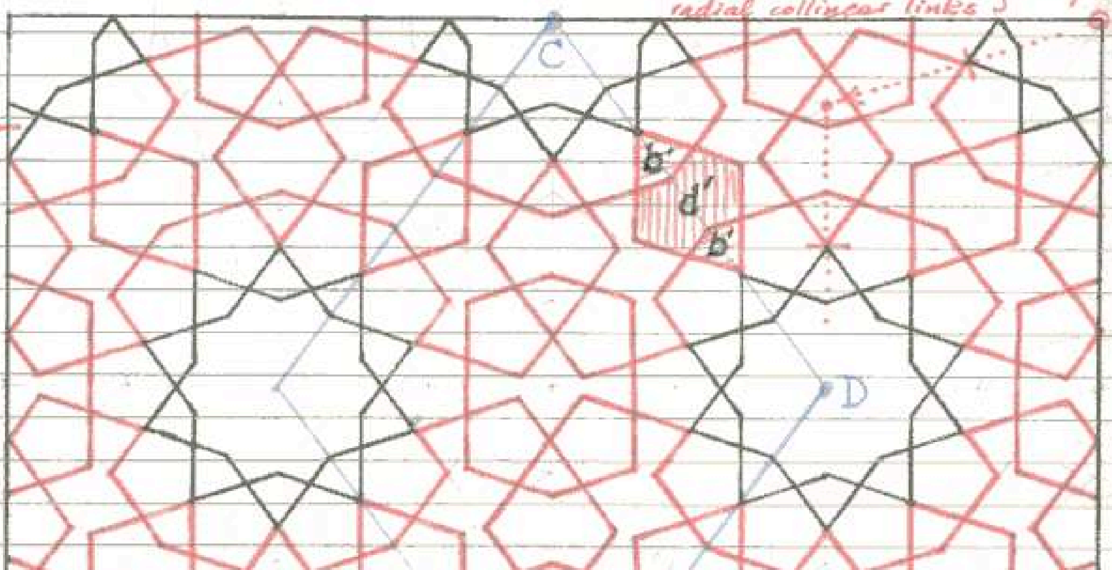
short axis = c′e′c′

long axis =
b′d′b′c″e′c″b′d′b′

Rhomb edge
= r(c″b′)d′.

Fri 15 Feb 1985

B



N.B. Care must be taken to make all pentagons appear regular; some of those drawn above are unsatisfactory

Fri 22 Sept 1978

PATTERN VARIATION

pattern lines which are already present until they meet other lines similarly produced. In this way it will be seen that an e-cell is automatically formed at the centre of the basic rhombus. Had we used the long axis of the initial rhombus with its sequence of interstitial cells b'd'b', and used this as the side of the basic rhombus in an Rp1 (3x2)10,10 pattern then the result is as shown in fig. B on p.60.

Let us distinguish the short axis and the long axis, respectively by the letters A and B, referring to the initial decagonal pattern in which the primary stars are in contact on the side of the basic rhombus (i.e. the pattern Rp1(3x2)10,10/I. Now, the pattern resulting when the short axis of the latter pattern is used to form the side of the rhombus in a new pattern may be designated

Rp1(3x2)10,10/II−2A.

The "A" signifying that the short axis of the initial pattern becomes the side of the new pattern, while the "2" indicates that the pattern concerned is a second-stage derivative. (The figure 2 may be omitted without loss of clarity).

The pattern resulting when the long axis of the initial pattern is used in this way may similarly be designated

Rp1(3x2)10,10/II−2B

The figure "2" again indicating a second-stage derivative.

A third-stage derivative may then be formed from such second-stage patterns by creating new rhomb sides from either axis of a second-stage rhombus. For example, if the short axis of II−2A is used as the new rhomb edge the resulting third-stage pattern may be designated II−3AA or in abbreviated form simply II−3A^2. If the long axis of II−2A is used instead, the resulting third-stage pattern bears the designation II−3AB, or, in full, Rp1(3x2)10,10/II−3AB.* Fourth, or even fifth-stage derivatives will be found possible, although many of the higher derivatives result in awkward groups of interstitial cells which inevitably lead to ugly, non-authentic cell shapes however

* see fig. B, p.58.

Side-axis-side (SAS) series

much rearrangement is carried out.

This "side-axis-side" series is important, since it accounts for most of the commoner variants in authentic decagonal patterns. It is not of course suggested that any of the original designers of these patterns used such methods to ~~[illegible]~~ obtain derivative patterns; it is unlikely that they did so, and it is probable that any variation in this series was obtained by manipulating rearrangements of ceramic, wooden or paper tile shapes, jigsaw-puzzle fashion.

However, not every linear arrangement of interstitial cells can be obtained by sides or rhombic axes in the SAS series by any means, so the designations of patterns in this series, although conveniently compact, are not, unfortunately, universally applicable to all derivative decagonal patterns.

Derivative patterns in the decagonal series are of course possible with other varieties than the type II patterns just discussed, and the same designations 2A, 2B etc may be used in SAS series patterns. Here, however, it will often be found that other types cannot be carried beyond the second or third stage without extremely awkward cell shapes resulting.

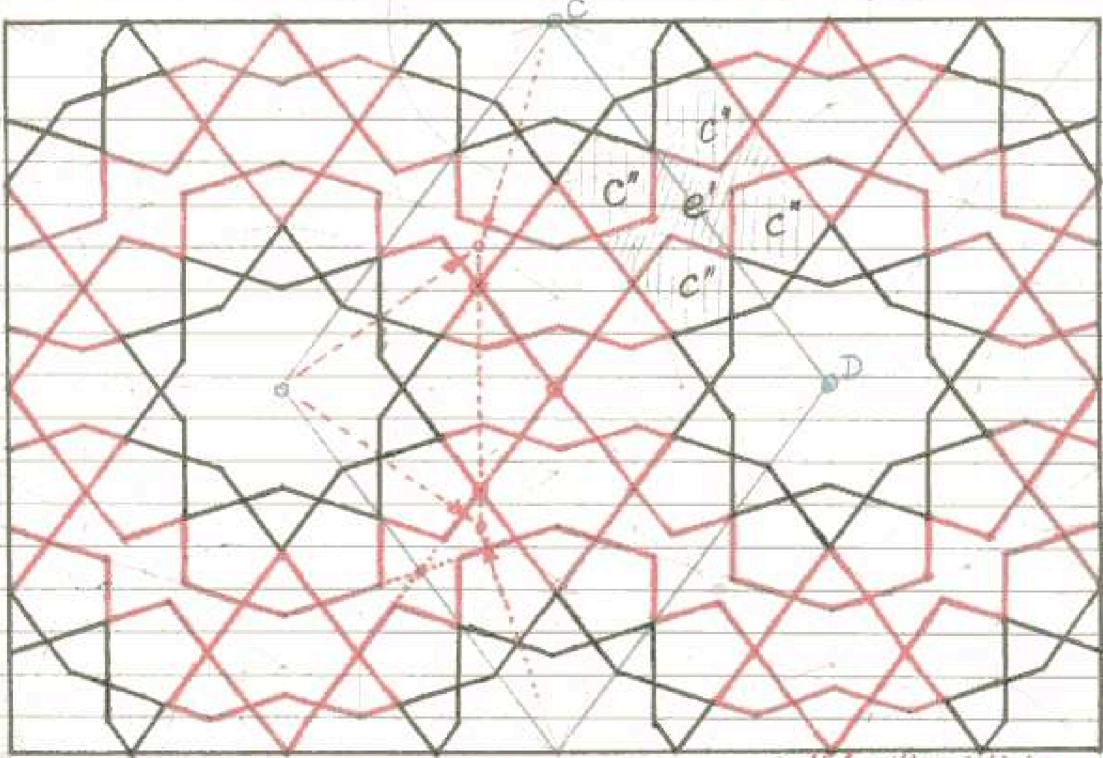
Not all authentic patterns place their stars on the vertices of a rhombic tessellation. Many common patterns use a grid of rectangles (a true square is not possible in exact decagonal patterns*) and we can specify the lengths of the sides of the rectangle in the same way as has been outlined above, either by noting all interstitial cells lying on the line joining the centres of a pair of separated stars, or by the abbreviated designations of the SAS series.

* Some of Bourgoin's (1879) Plates in his decagonal series are constructed "sur plan carré" but this is an error. If a square is used as a basis, the result is more or less distorted.

Sat 23 Sept 1978

PATTERN VARIATION

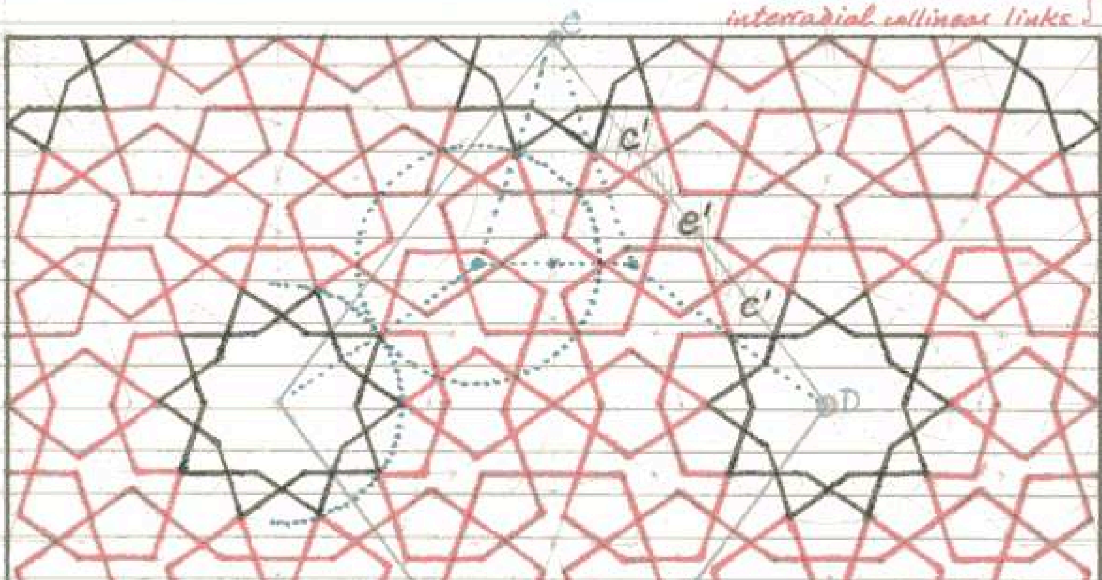
Thursday, MARCH 17, 1966



Rp1(3x2)10,10/II-3A
rhomb side
= c"e'c"
short axis
= d'b"d'
long axis
= c"b'd'b'd"-
-b'd'b'c"

Rhomb edge:
i(c")e'.

radial collinear links
interradial collinear links } see p.157



Rp1(3x2)10,10/II-3AB
rhomb side
= c'e'c'

short axis
= c"b'd'b'c"b'd'b'c

long axis
= c'e'c'b"c'e'c'

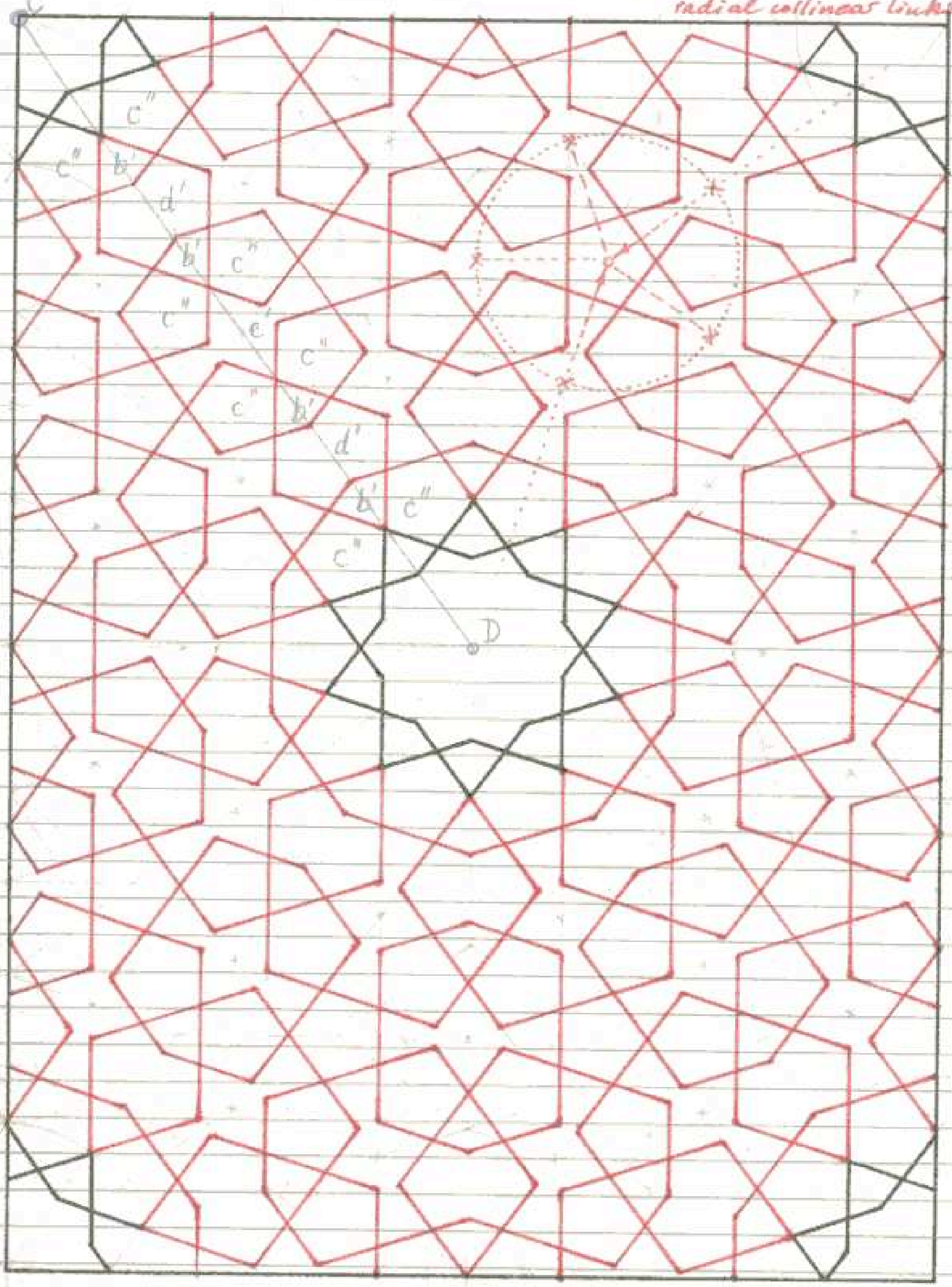
Rhomb edge: i(c')e'.

Sun 24 September 1978

Friday, MARCH 18, 1966

PATTERN VARIATION

radial collinear links – see p. 157



Radius of primary star = half distance from m1.n2 to centre n. Note that lines of (3×2)10,10/Ia can be traced in this pattern. The primary stars have superimposed on the central stars of the type Ia rosette.

Rp1(3×2)10,10/II – 3B² Rhomb edge = c″b′d′b′c″e′c″b′d′b′c″

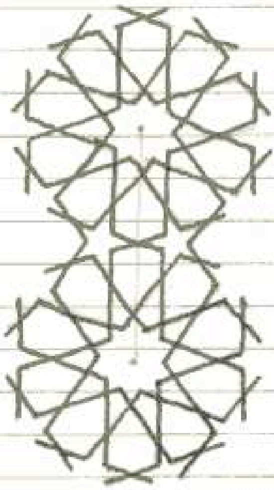
The rhomb could be abbreviated as r(c″b′d′b′c″)e′ i.e. the link is radial ("r"), and the last mentioned cell (e′) is on the centre of the rhomb edge. If link is interradial prefix "i" is used.

15 Feb 1985

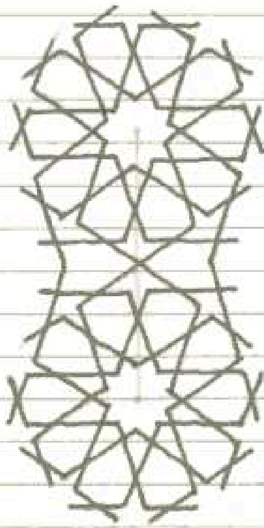
[illegible] Wed 27 Sept 1978

PATTERN VARIATION

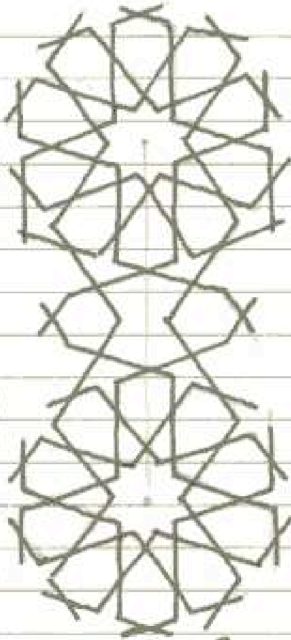
Saturday, MARCH 19, 1966



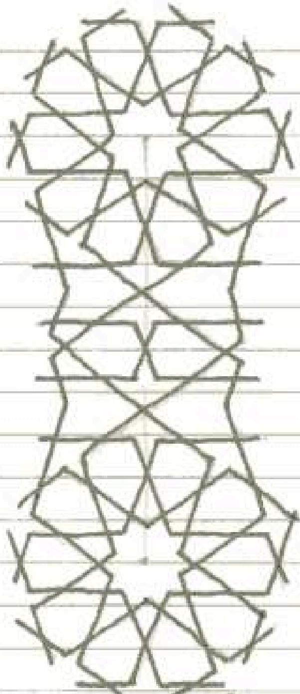
Ia – 1
radial



Ia – 2A
interradial



Ia – 3A^2
radial

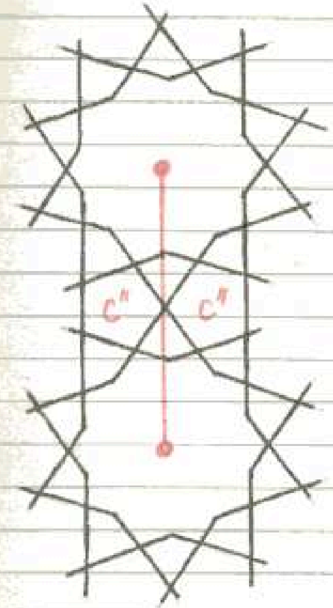


Ia – 4A^3
interradial

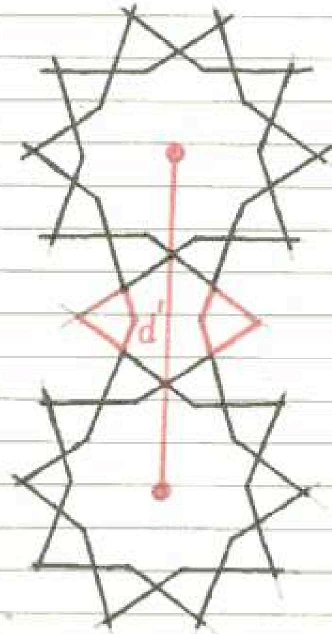
Sunday, MARCH 20, 1966

Monday, MARCH 21, 1966

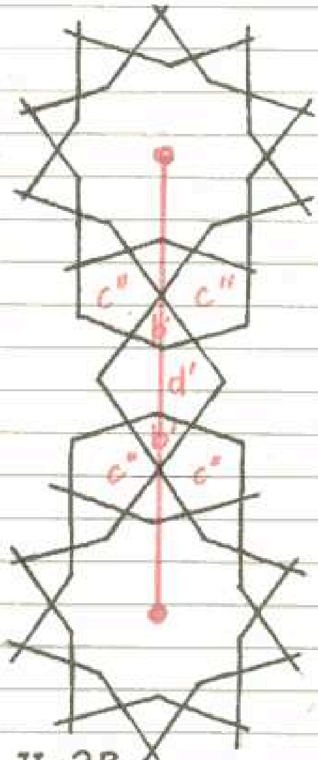
Links: r.c″ i.d′ r(c″b′)d′ r(c″)e



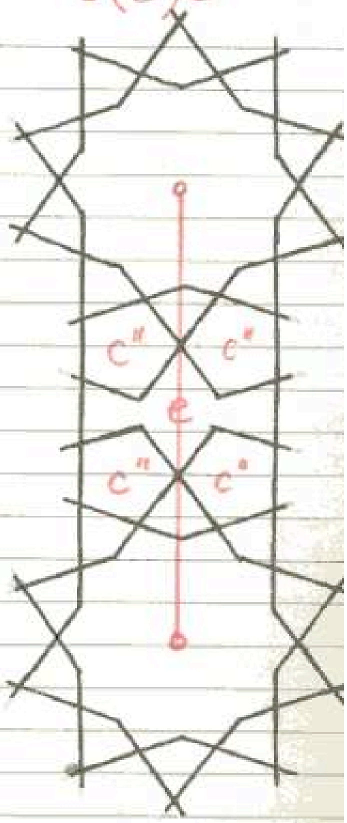
II-1
radial



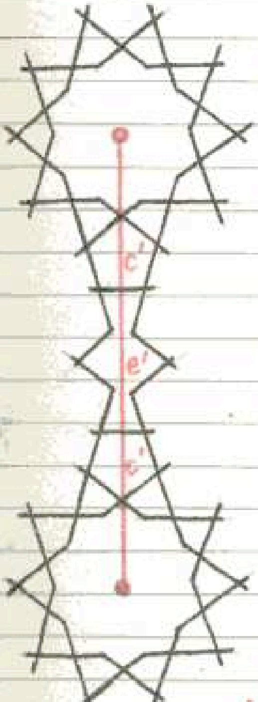
II-2A
interradial

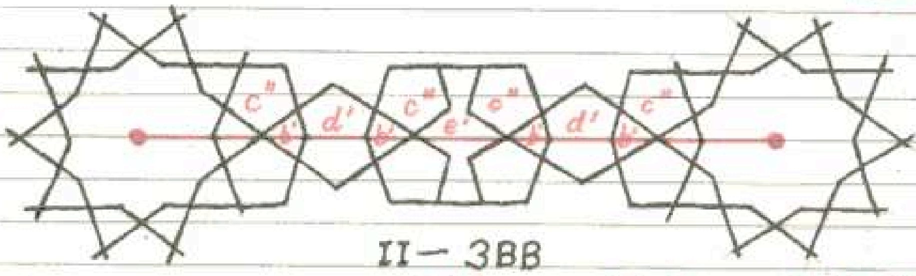


II-2B
radial



II-3AA
radial





II-3BB
radial

Link: r(c″b′d′b′c″)e′.

i(c′)e′

II-3AB
interradial
(≈ 3BA)

Thu 12 April 1984

Sizes of angles in the general type I patterns, in terms of m, n, p & q. see diagram opposite on p. 68

$$a = \frac{2}{m} + \frac{2}{n} = \frac{m-2p+2q}{mq} \quad \text{or} \quad \frac{n+2p-2q}{np}$$

$$b = 1 - a - \frac{2}{n} = \frac{m(q-2)+2(2p-q)}{mq} \quad \text{or} \quad \frac{n(p-1)-2(2p-q)}{np}$$

$$c = 2-a-2b = \frac{2q+3m-6p}{mq} \quad \text{or} \quad \frac{n-2q+6p}{np}$$

$$d = 1-\frac{2}{m}-\frac{2}{n} = \frac{m(q-1)+2(p-q)}{mq} \quad \text{or} \quad \frac{n(p-1)-2(p-q)}{np}$$

$$e = a+\frac{2}{n} = \frac{2m-4p+2q}{mq} \quad \text{or} \quad \frac{n+4p-2q}{np}$$

$$f = a+\frac{2}{m} = \frac{m-2p+4q}{mq} \quad \text{or} \quad \frac{2n+2p-4q}{np}$$

$$g = 1-a-\frac{2}{m} = \frac{m(q-1)+2(p-2q)}{mq} \quad \text{or} \quad \frac{n(p-2)-2(p-2q)}{np}$$

$$h = 2-a-2g = \frac{m-2p+6q}{mq} \quad \text{or} \quad \frac{3n+2p-6q}{np}$$

$$i = \frac{1}{2}-\frac{1}{n} = \frac{m(q-1)+2p}{2mq} \quad \text{or} \quad \frac{n-2}{2n}$$

$$j = \frac{1}{2}-\frac{1}{m} = \frac{m-2}{2m} \quad \text{or} \quad \frac{n(p-1)+2q}{2np}$$

$$k = 1-d = a$$

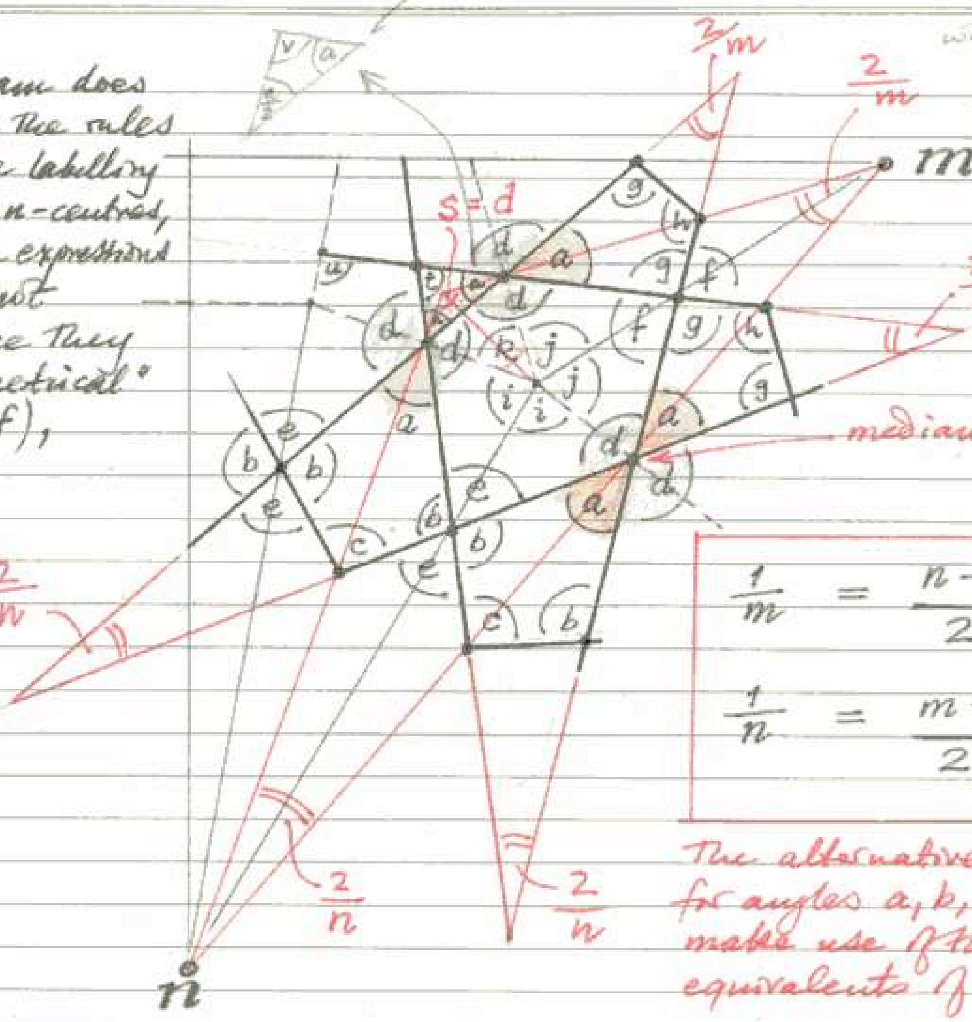
RH Tue 12 April 1984

Wednesday, MARCH 23, 1966

$$v = 1 - \frac{3a}{2} = \frac{m(2q-3)+6(p-q)}{2mq} \quad \text{or} \quad \frac{n(2p-3)-6(p-q)}{2np}$$

when $p = 3$ $q = 2$, $v = \frac{1}{2}h$ and this triangle is congruent to half the order m-cell.

N.B. This diagram does not agree with the rules on p.22 for the labelling of the m- and n-centres, but the angle expressions resulting are not affected, since they are in "symmetrical" pairs, e.g. (e,f), (c,h).



$$\boxed{\begin{aligned} \frac{1}{m} &= \frac{n-2q}{2np} \\ \frac{1}{n} &= \frac{m-2p}{2mq} \end{aligned}}$$

The alternative expressions for angles a, b, c, d, etc make use of the above equivalents of $1/m$ & $1/n$.

$$t = 1 - 2a = \frac{m(q-2)+4(p-q)}{mq} \quad \text{or} \quad \frac{n(p-2)-4(p-q)}{np}$$

$$u = 1 - a - \frac{e}{2} = \frac{m(q-2)+4p-3q}{mq} \quad \text{or} \quad \frac{n(2p-3)-2(4p-3q)}{2np}$$

In (3×3) rhombs the t, a, a triangle becomes equilateral, since $t = a = 1/3$.

General expressions for angle sizes in other pattern types are not possible as a rule, since many types have non-determinate slopes of their line segments, but the degree to which this is the case varies widely.

Thu 12 April 1984

It is useful for some purposes to label the pattern line segments of type I patterns (and the interlace segments of their topological equivalents) as on the diagram on p. 70, opposite.

It is also convenient to identify the different sectors delimited by the radii from the m- and n-centres. These sectors are labelled as 1, 2, 3 etc (first, second and third, etc) and colour coded as shown.

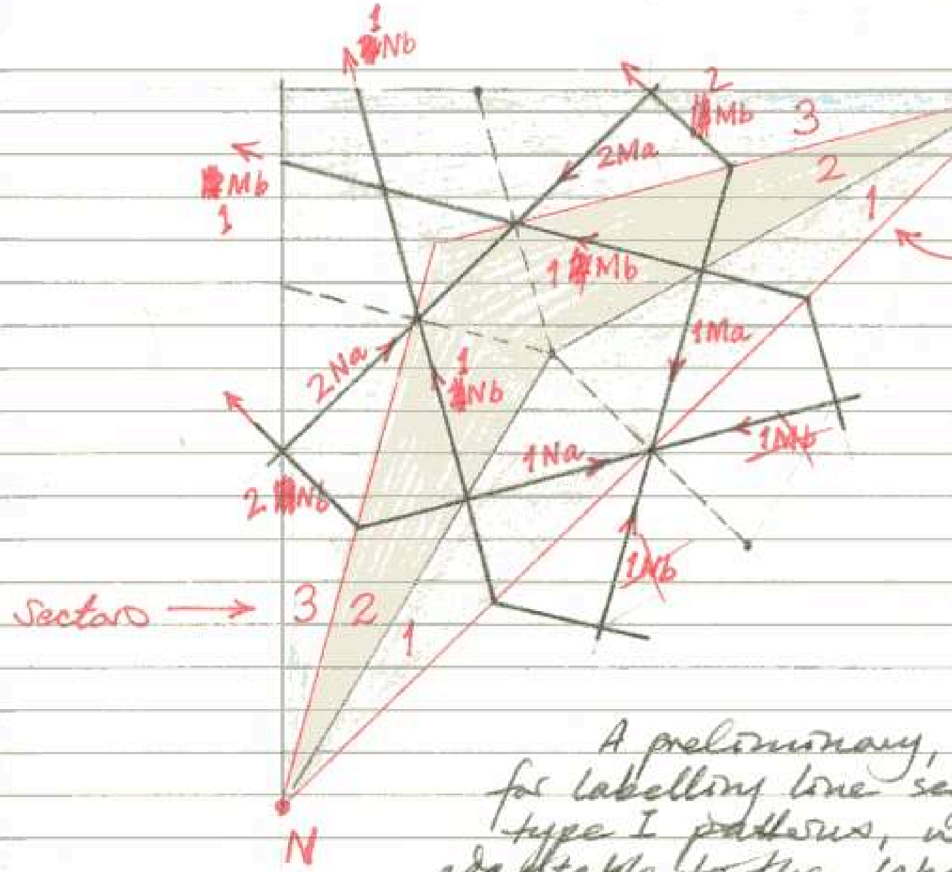
For all type I patterns in $(p \times q)$ rhombs the pattern of lines in each sector is always topologically equivalent, whatever the values of p and q. Thus sector 1 always contains parts of 1Nb + 1Na + 1Ma + 1Mb; sector 2 always contains 1Na + 1Nb + (2Na + 2Ma) + 1Mb + 1Ma; and so on. As can be seen, each sector can be characterised by the order in which line segments are encountered when proceeding from n to m, or the reverse. In a topological sense the sectors are theoretically symmetrical for any rhomb in which $p = q$, but when $p \neq q$ at least one of the highest numbered sectors becomes asymmetrical.

The following sequences are to be found in successive symmetrical sectors:

Sector		No. of segments
1	1Mb 1Ma 1Na 1Nb	4
2	1Ma 1Mb 2Ma+2Na 1Nb 1Na	5
3	2Mb 2Ma 1Mb 1Nb 2Na 2Nb	6
4	2Ma 2Mb 3Ma+3Na 2Nb 2Na (1Mb 1Nb)	7
5	3Mb 3Ma 2Mb 2Nb 3Na 3Nb (1Mb 1Nb)	8

segments in parentheses in sectors 4, 5 occur theoretically, but in practice they would normally be modified or omitted.

[initials] Thu 12 April 1984



This is drawn as a (2, 4, 4) triangle, so M = N = 12.

[signature]

A preliminary, suggestive scheme for labelling line segments within type I patterns, which is also adaptable to the labelling of segments of interlacing bands in topologically equivalent patterns.

As in the analogous case of radii within the general (p×q) rhombus, the angles of intersections may be labelled by using the above symbols. Thus the intersection

1Na. 1Nb is equivalent to angles e + b on p. 68. However,

this is obviously of no use in identifying individual angles, for which the letters given of p. 68 are more appropriate.

It is debatable whether it is preferable to reverse the designations "a" and "b" for the M and N segments.

Thu 12 April 1984

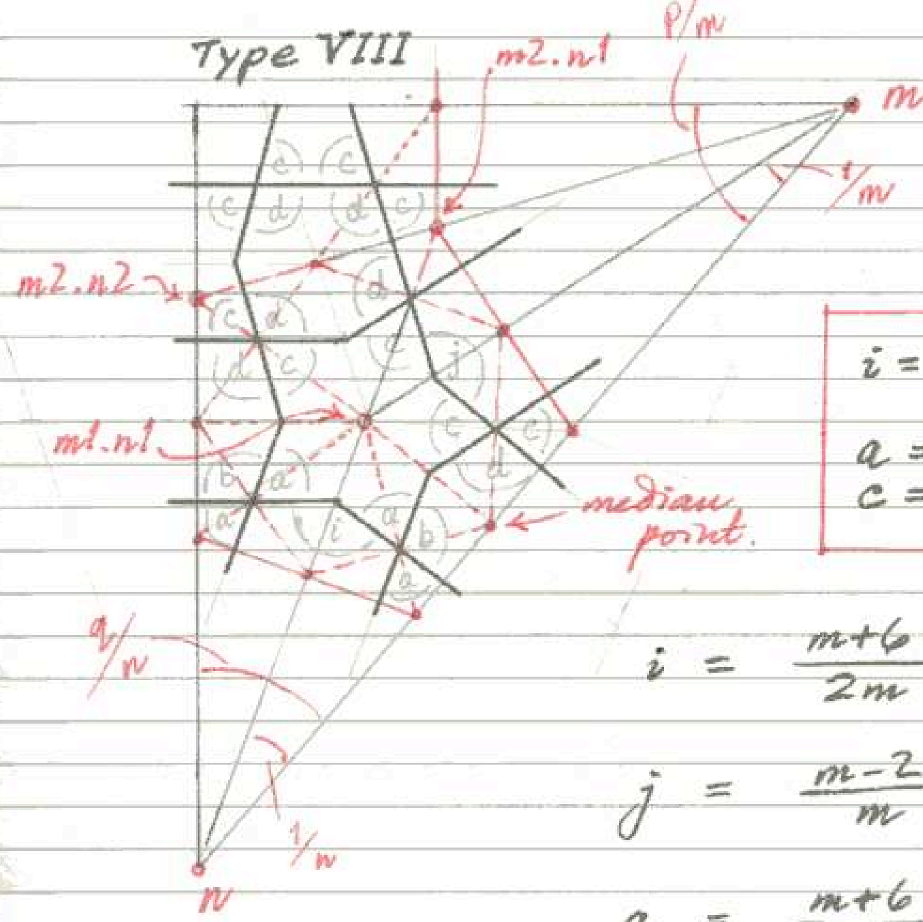
Saturday, MARCH 26, 1966

General expressions for angles between crossing straight line segments are possible for type I patterns in any size of (p x q) rhomb, since these patterns are completely determined, and are not dependent on the special geometry of any particular rhomb size, as for example the (3x2) or (2x1) rhombs, where interstitial cells are possible (with certain 'types'), congruent to the outer cells of the m-motifs.

Within the (3x2) type series, general expressions for angle sizes would be possible for types VII, VIII, IX and X since at least the essential lines of these types are determinate

Sunday, MARCH 27, 1966

Type VIII



$$\boxed{\begin{array}{ll} i = 1-\frac{2}{n} & j = 1-\frac{2}{m} \\ a = \frac{1}{2}i & b = 1-a \\ c = \frac{1}{2}j & d = 1-c \end{array}}$$

$$i = \frac{m+6}{2m} \quad \text{or} \quad \frac{n-2}{n}$$

$$j = \frac{m-2}{m} \quad \text{or} \quad \frac{2n+4}{3n}$$

$$a = \frac{m+6}{4m} \quad \text{or} \quad \frac{n-2}{2n}$$

$$b = \frac{3m-6}{4m} \quad \text{or} \quad \frac{n+2}{2n}$$

$$c = \frac{m-2}{2m} \quad \text{or} \quad \frac{n+2}{3n}$$

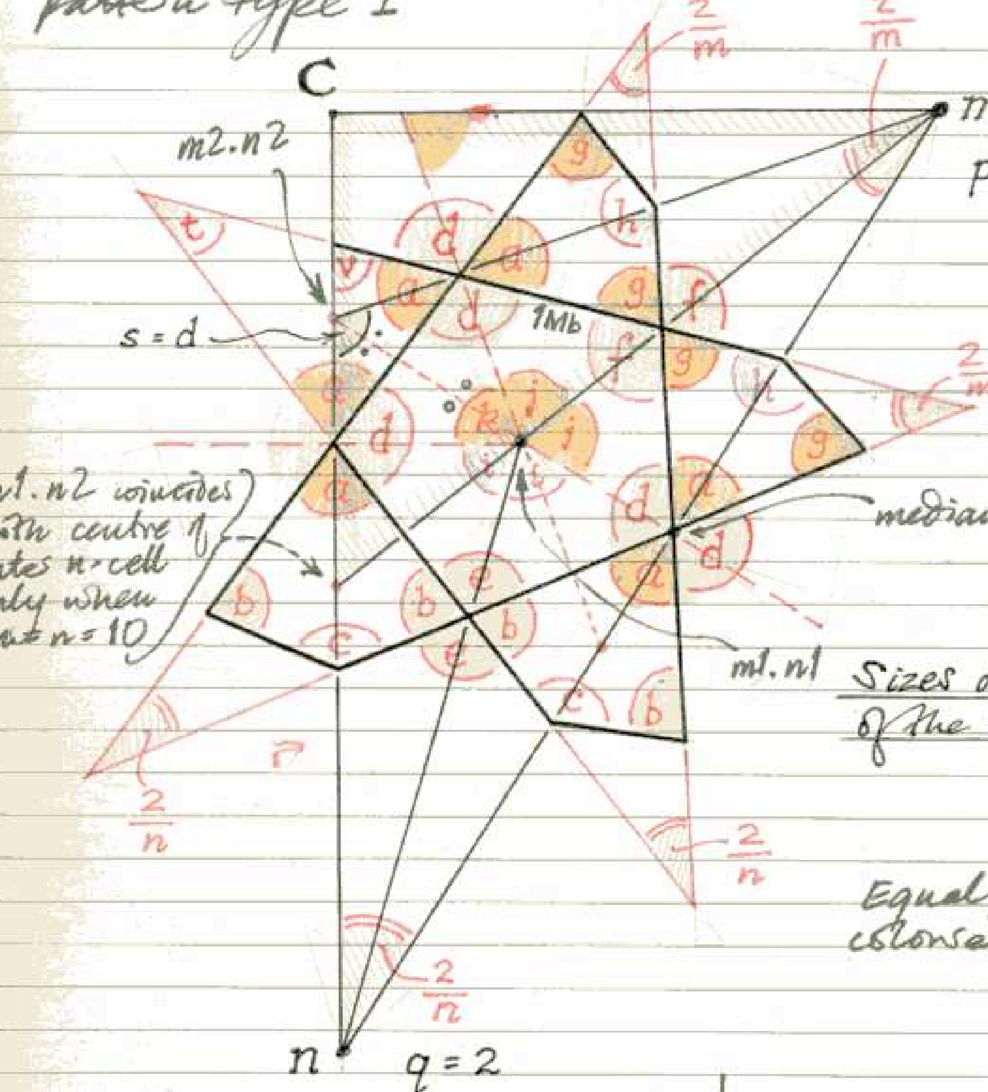
$$d = \frac{m+2}{2m} \quad \text{or} \quad \frac{2(n-1)}{3n}$$

3

Apr Mon 16 April 1984

Tuesday, MARCH 29, 1966

(3 x 2) rhombs
pattern type I



$$\frac{1}{m} = \frac{n-2q}{2np} = \frac{n-4}{6n}$$

$$\frac{1}{n} = \frac{m-2p}{2mq} = \frac{m-6}{4m}$$

Sizes of angles in patterns of the type $(3\times 2)m,n/I$

Equal angles are similarly coloured.

$a = \frac{m-2}{2m}$ or $\frac{n+2}{3n}$		$d = \frac{m+2}{2m}$ or $\frac{2n-2}{3n}$
$b = \frac{8}{2m}$ or $\frac{2n-8}{3n}$		$e = \frac{2m-8}{2m}$ or $\frac{n+8}{3n}$
$c = \frac{3m-14}{2m}$ or $\frac{n+14}{3n}$		$f = \frac{m+2}{2m}$ or $\frac{2n-2}{3n}$

Mon 16 April 1984

$g = \frac{m-2}{2m}$ or $\frac{n+2}{3n}$

$j = \frac{m-2}{2m}$ or $\frac{n+2}{3n}$

$h = \frac{m+6}{2m}$ or $\frac{n-2}{n}$

$k = a$

$i = \frac{m+6}{4m}$ or $\frac{n-2}{2n}$

$t = \frac{2}{m}$ or $\frac{n-4}{3n}$

$v = \frac{m+6}{4m}$ or $\frac{n-2}{2n}$

$(i = \frac{1}{2}h = v)$

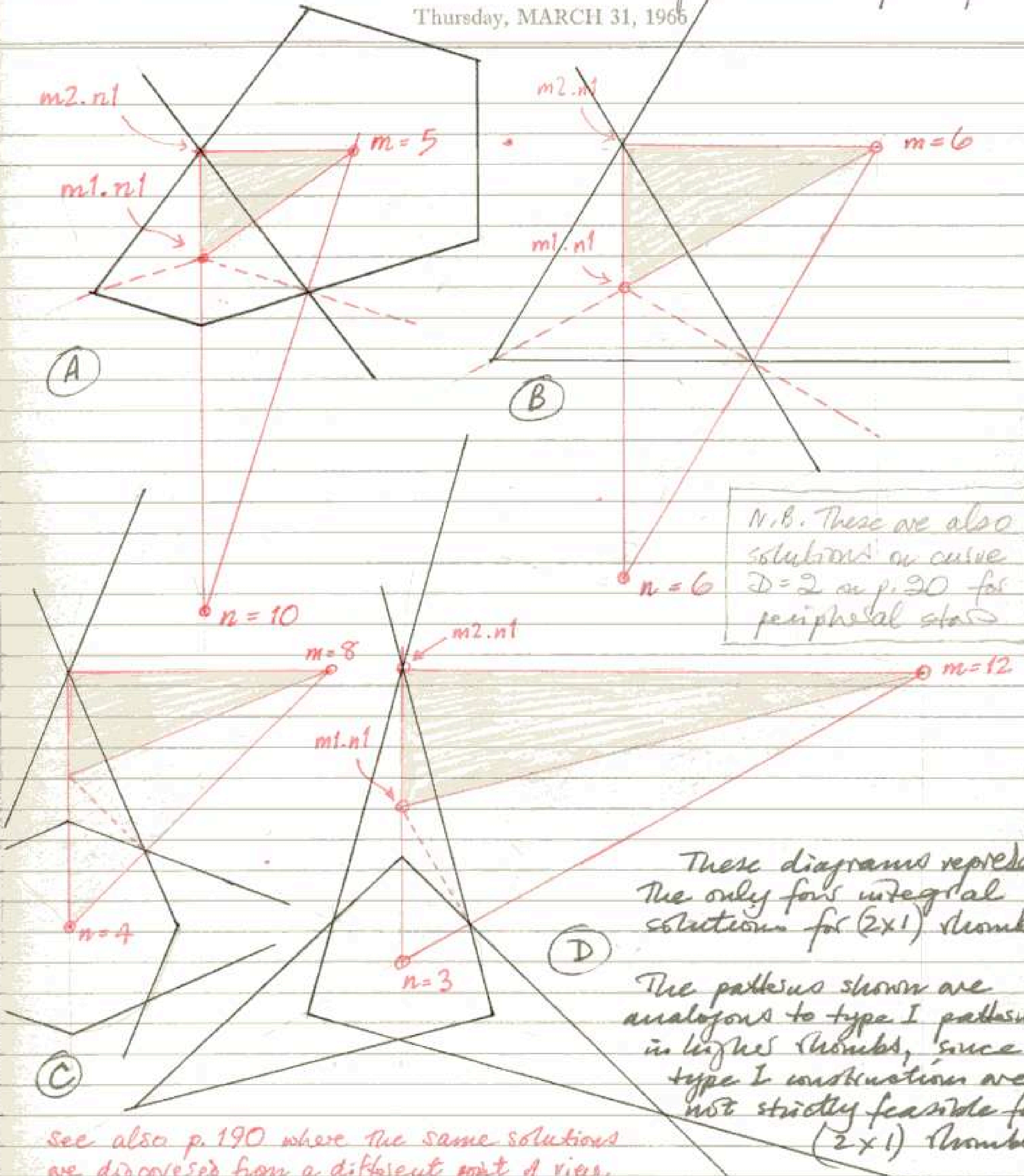
Comparison of these general expressions for angle sizes in (3×2) rhombs shows that $a = g = j = k$ and $v = i = \frac{1}{2}h$. Therefore the cell centred on m2.n2 is similar (and is indeed congruent) to the outer m-cells, and pattern segment 1Mb is one edge of a regular n-gon centred on n. However, the latter observation clearly does not hold for segment 1Nb, except when $m = n = 10$. For other values (i.e. integral values) of m, n in (3×2) rhombs see pp.12,14

Actual values for (3×2) rhombs are as follows:—

m,n:	7,28	8,16	9,12	10,10	12,8	14,7	18,6	30,5
a	64° 17.14′	67.5°	70°	72°	75°	77° 8.57′	80°	84°
b	102° 51.43′	90°	80°	72°	60°	51° 25.71′	40°	24°
c	90°	112.5°	**130°**	144°	165°	180°	200°	228°
d	115° 42.86′	112.5°	110°	108°	105°	102° 51.43′	100°	96°
e	77° 8.57′	90°	100°	108°	120°	128° 34.29′	140°	156°
f	115° 42.86′	112.5°	110°	108°	105°	102° 51.43′	100°	96°
g	64° 17.14′	67.5°	70°	72°	75°	77° 8.57′	80°	84°
h	167° 8.57′	157.5°	150°	144°	135°	128° 34.29′	120°	108°
i	83° 34.29′	78.75°	75°	72°	67.5°	64° 17.14′	60°	54°
j	64° 17.14′	67.5°	70°	72°	75°	77° 8.57′	80°	84°
t	51° 25.71′	45°	40°	36°	30°	25° 42.86′	20°	12°

75) (2×1) rhombs

Mon 16 April 1984



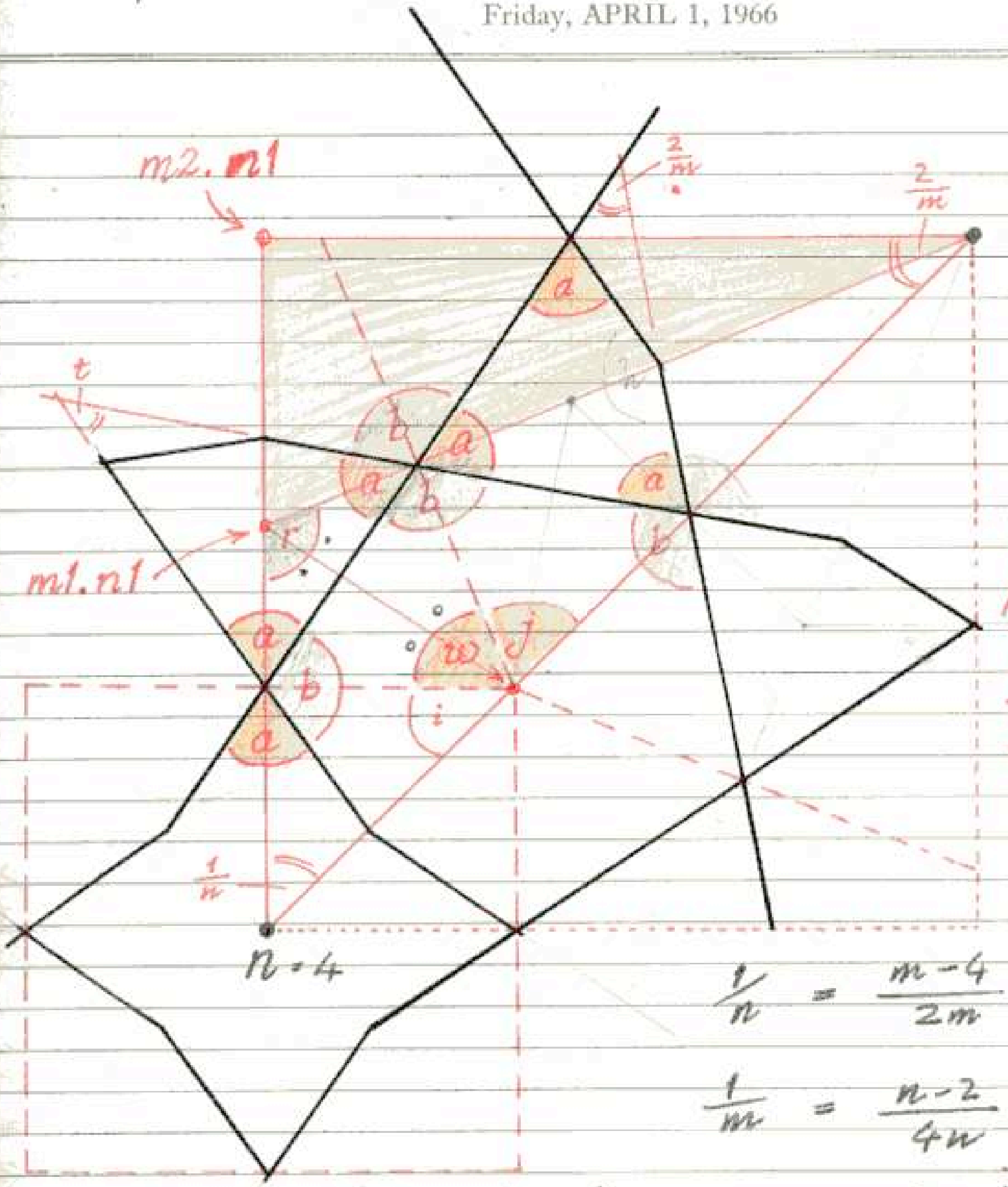
N.B. These are also solutions on curve D = 2 on p. 90 for peripheral stars

These diagrams represent the only four integral solutions for (2×1) rhombs.

The patterns shown are analogous to type I patterns in higher rhombs, since type I constructions are not strictly feasible for (2×1) rhombs.

See also p. 190 where the same solutions are discovered from a different point of view, together with $m = 4$, $n = \infty$.

Mon 16 April 1984 (2x1) Rhombs (76



Specifically, this right triangle may be derived directly from the (m, C, m1.n2) triangle of (3x2)8, 16/I.

$$\frac{1}{n} = \frac{m-4}{2m}$$

$$\frac{1}{m} = \frac{n-2}{4n}$$

$$r = 1 - \frac{1}{m} - \frac{1}{n} = \frac{m(2-1)+2}{2m} \quad \text{or} \quad \frac{n(4-1)-2}{4n}$$

$$w = 1 - r = \frac{1}{m} + \frac{1}{n} = \frac{m-2}{2m} \quad \text{or} \quad \frac{n+2}{4n}$$

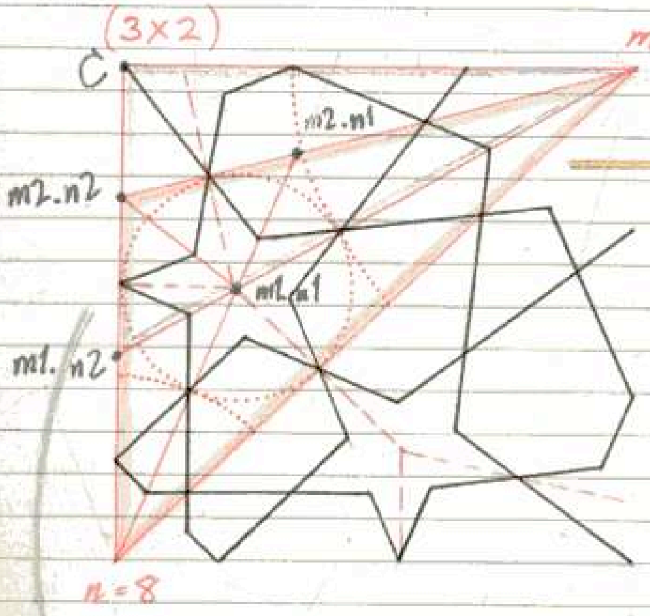
$$a = 1 - r = w$$

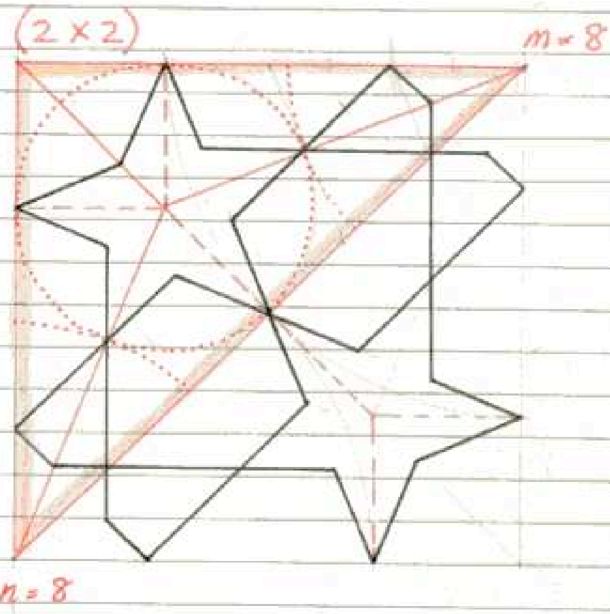
$$b = 1 - w = r$$

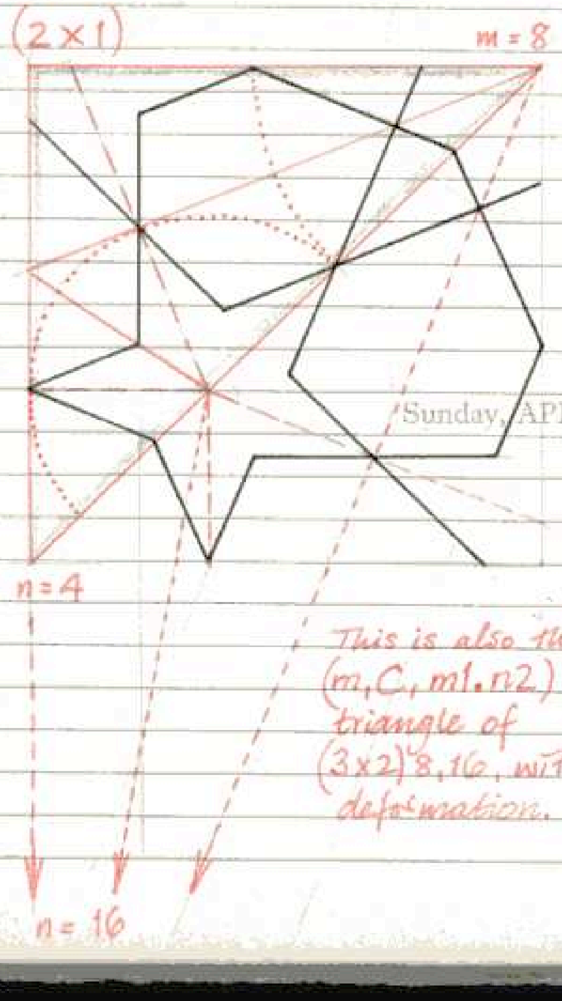
N.B. The above pattern is topologically equivalent to sectors 2+3 of (3x2)m, n/I OR MORE ACCURATELY, TO THE m, C, m1.n2 TRIANGLE OF (3x2)m, n/I pattern.

77) Topological Invariance between different rhomb sizes. Tue 17 April 1984

Saturday, APRIL 2, 1966



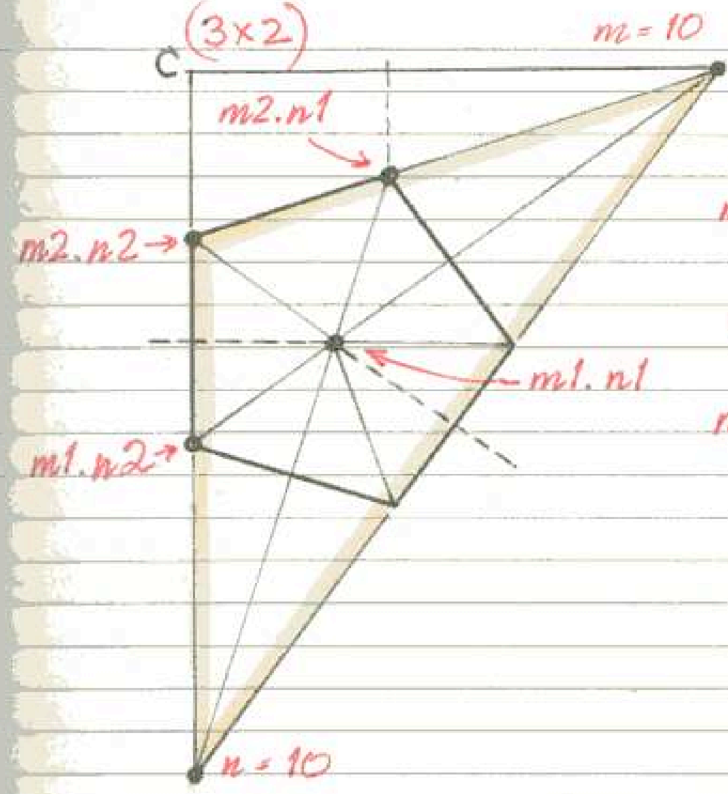


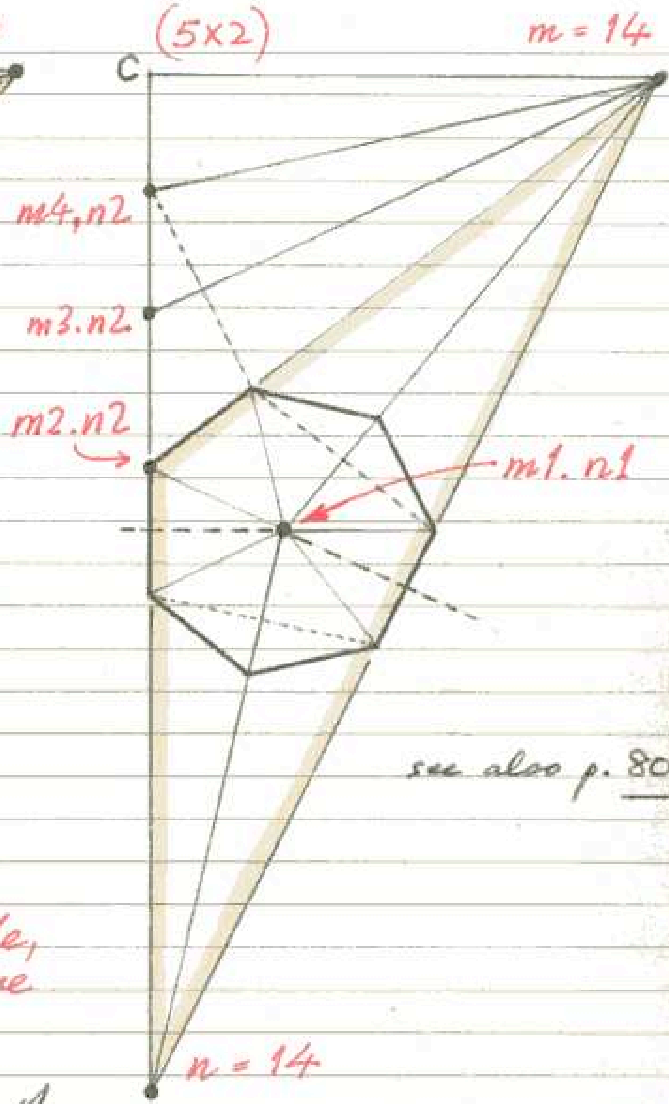


These diagrams illustrate transformations under topological invariance. In the first case the 2nd collateral triangle (m, n, m2.n2) is enlarged until the angle at m2.n2 is a right angle, thus producing one quarter of a (2×2) rhomb – since of course the 2nd collateral triangle has all its angles bisected. The second case takes the pattern lines within the (m, C, m1.n2) triangle of a (3×2) right triangle and makes this a new (2×1) right triangle. In the 2nd case the original point m1.n1 remains, but is now the intersection of the bisector of the angle at m1.n1 of the new rhomb.

Sunday, APRIL 3, 1966

Wed 18 April 1984





see also p. 80, A.

= ~~1st~~ 2nd collateral triangle, and its in-centre, the point m1.n1.

These two form the first two of a series of (p×2) rhombs, (p must be odd), where the inscribed M-gon in the 2nd collateral triangle is a {p+2}, and where m = n = 2(p+2).

The (5×2) solution is extremely rare, the only example known to me occurring in the East Porch of the Sunghur Bey mosque at Nigde, Turkey (see A. Gabriel 1931 "Monuments turcs d'Anatolie" Vol. I Pl. XI.). The (3×2) solution is of course common and widespread, as the basis for numerous different patterns using (3×2) rhombs. No other (5×2) on the above basis seem to occur as authentic Islamic patterns, but (5×2)15,12 realized as the same design as the original Nigde pattern forms an excellent geometrical pattern.

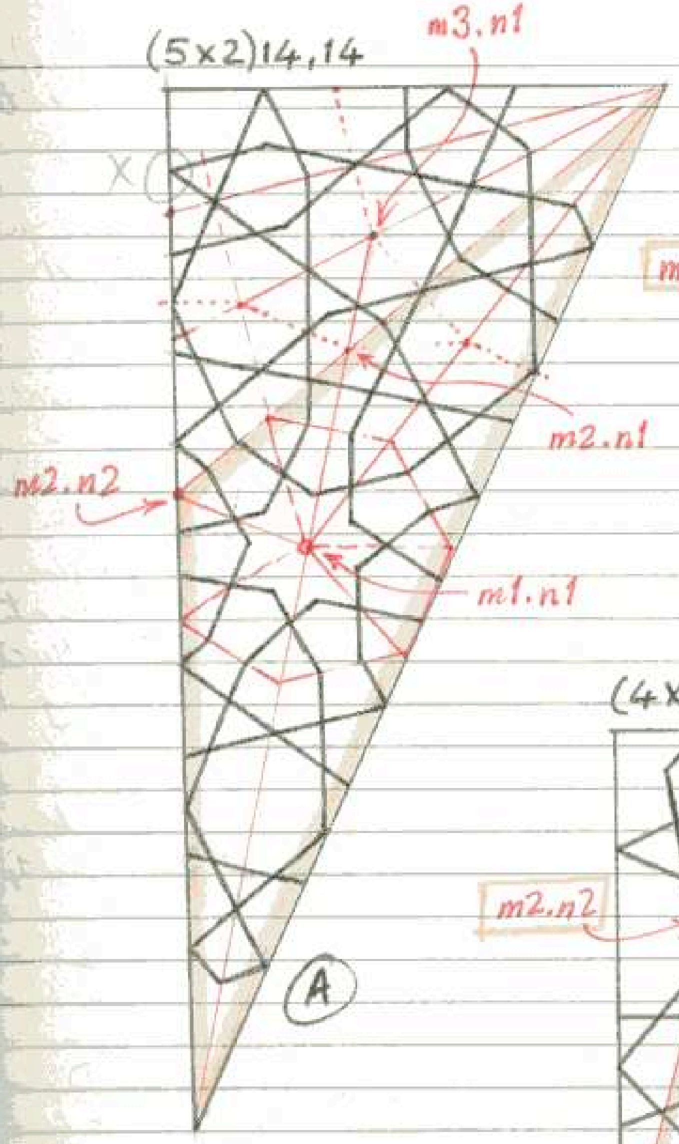
SERIES OF PATTERN TYPES

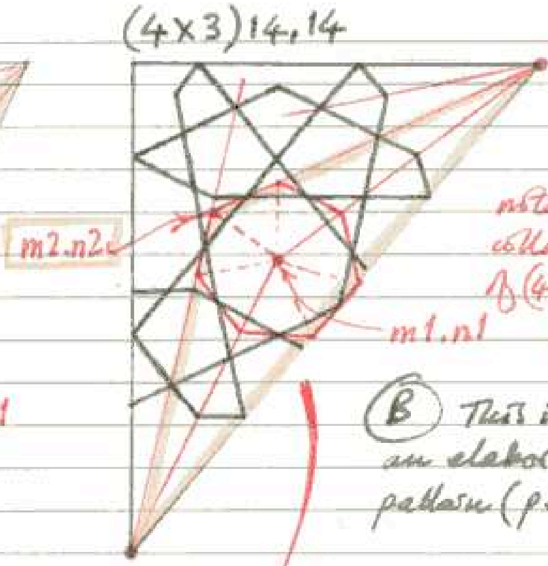
Thu 19 April 1984

Theoretically each set of (p×q) rhombs has a series of pattern types derived in each case from the "central" pair of the set, for which $m = n = 2(p+q)$. See for example pp. 29–56 of this notebook, where a number of pattern types for (3×2) rhombs are illustrated and discussed. However, in practice very few such sets are encountered, and these are limited to low values of p, q. The theoretical background was certainly not appreciated by the original muslim pattern designers, who would often apply a given pattern type to a rhomb size for which it was geometrically quite inappropriate. One of the most frequently used of these realizations was based on the interstitial pair group of the (3×2) rhomb series (types I, II, III, IV, V, VI – blue labels – on pp. 46–56), but since other sizes have an analogous interstitial pair of cells, e.g. (2×1) rhombs, the muslim artists unavoidably gained a confused picture of the geometrical relationships between the different rhomb sizes. The interstitial pair configuration was even attempted in the case of (4×3) rhombs, to which it cannot be satisfactorily adapted. In fact, (4×3) and other rhomb sizes have their own sets of pattern types, quite distinct from the familiar (3×2) series of pattern types, but this is still largely unexplored territory. Some pattern types first met with in low-valued (p×q) rhombs have what we may term almost general applicability, e.g. the ubiquitous rosettes or simple stars in contact (types I & II – blue labels – pp. 46, 48) but higher rhombs often have special underlying geometries not possible in lower rhombs. The series of (p×2) rhombs suggested on p. 78 is a case in point. The geometry illustrated of the (5×2) rhomb has probably many possibilities, but seems to occur only as an elaborated version of the construction lines shown; even this is adaptable to other (5×2) rhombs, but many other types on this basis are probably waiting to be discovered. Fig. A on p. 80 opposite, is a design of my own on this basis*, which also makes use of certain additional intersections (cf. pp. 25, 26), for example, m3.n1 and m2.n1.

*although discovered quite independently of the Sunghur Bey design.

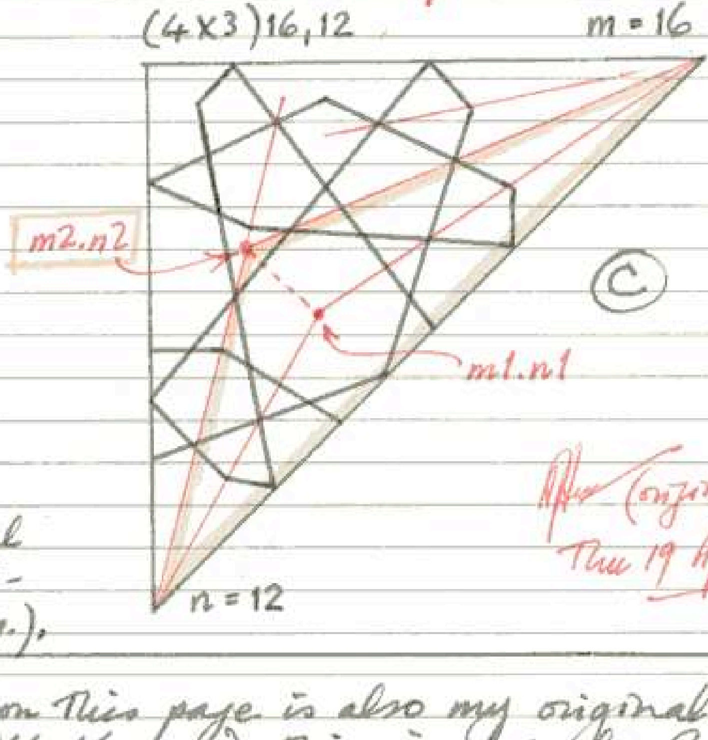
Thu 19 April 1984





note that the 2 collateral triangl[es] of (4×2) & (5×2) a[re] similar.

(B) This is essentially an elaborated type I pattern (p. 48, top).



The important 2nd collateral triangle (m, n, m2.n2) is outlined in colour (see p. 25 et seq.).

(original)
Thu 19 April 1984

The design in fig. B on this page is also my original design for the central (4×3)14,14, and this is also clearly adaptable to other rhombs in the (4×3) series, as shown in fig. C.

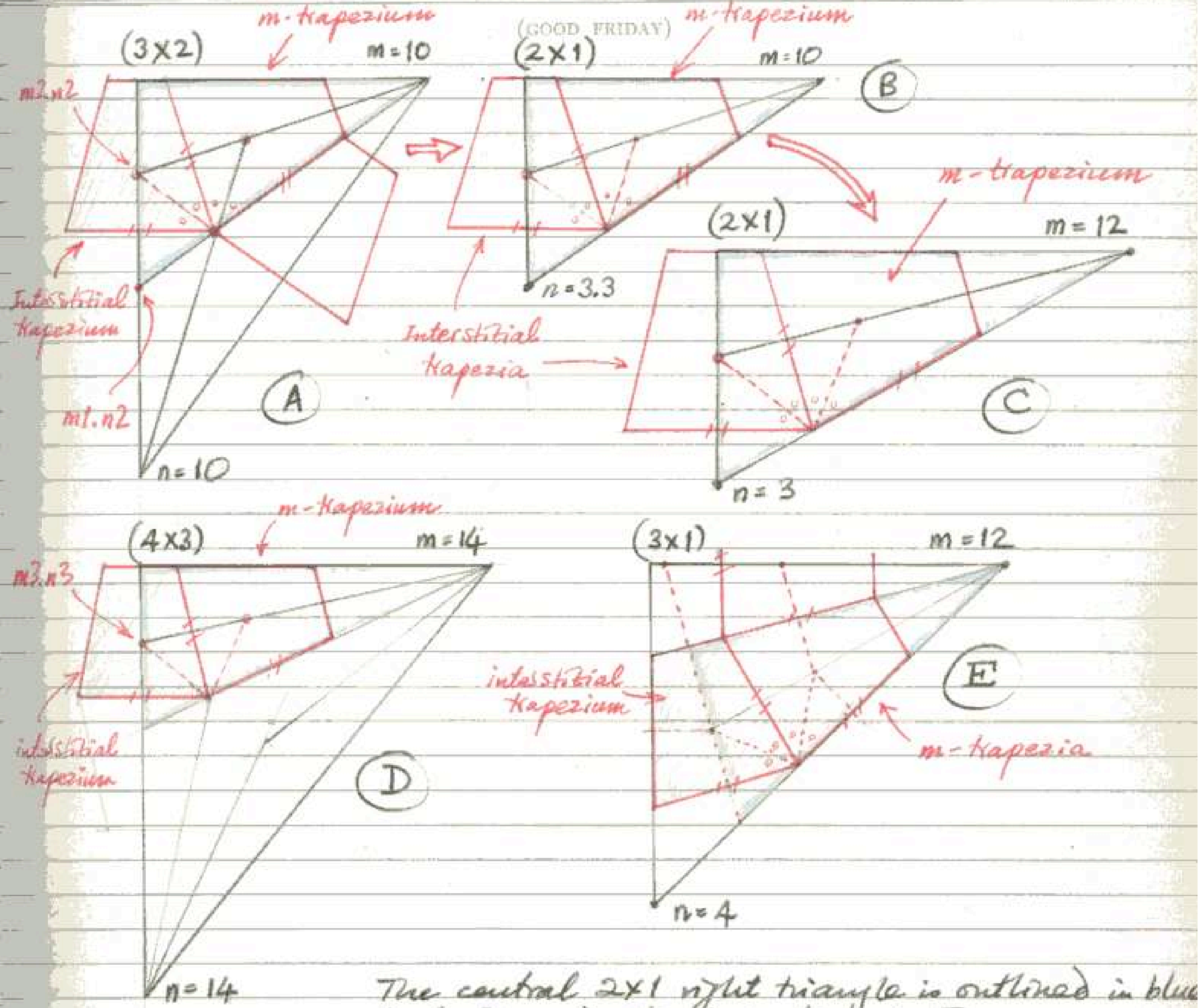
INTERSTITIAL PAIR CONFIGURATION

[signature] Fri 20 April 1984

An "interstitial pair" configuration consists of a pair of congruent cells, at or near the rhomb centre, congruent to the outer cells of one or both of the principal stars or rosettes centred on the rhomb vertices. In (3×2) rhombs the interstitial cells can only be congruent to the outer cells of the m-centre motif, not to those of the n-motif, unless m = n = 10. The underlying basis allowing this congruence consists of m-trapezia (pink) and interstitial trapezia (green) within which are inscribed the pattern cells concerned, on the mid-points of the longer sides of the trapezia (see fig. A, p. 82). The three longer sides of these trapezia are equal in length. Note that this construction is a property of the (m, C, m1.n2) triangle alone – C having the right angle at the rhomb centre – and is not dependent on the structure of the rest of the rhomb. It is therefore immediately transposable to a (2×1) rhomb structure, as shown in figs. B and C. The first does not of course lead to an integral rhomb, since n = 3.3, but reference to the table on p. 14 shows four possible pairs of values for (2×1) rhombs, one of which, (12, 3), is illustrated in fig. C.

Note that, since the central 2×1 right triangle can give rise to congruent m- and interstitial trapezia, independently of the rest of the rhomb, this construction can be achieved in any size of rhomb whatsoever (fig. D, p. 82). However, for any rhomb size greater than (3×2) it is not possible to achieve this construction and complete m- and n-motifs, that is, with m or n complete outer cells, respectively. It is for this reason that we conclude that the interstitial pair configuration is not suited to rhomb sizes higher than (3×2).

A rather different interstitial pair configuration is based on the construction shown in fig. E opposite. This is ultimately derived from the same source, using the 2×1 right triangle as before, but its right angle now no longer coincides with the rhomb centre. It is suitable for (3×1) rhombs. A type II pattern (p. 46 blue label) on this basis occurs in Egypt.



The central 2×1 right triangle is outlined in blue in A–D, and its derivative in E.

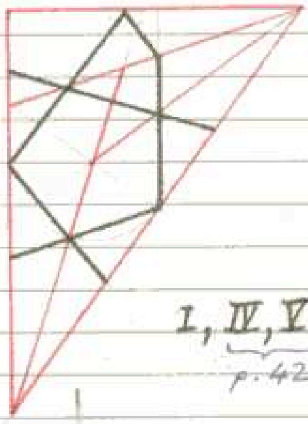
In authentic Islamic patterns the correct congruence of m- and interstitial outer cells was rarely achieved where $m \neq n$, and either no congruence was attempted at all, or occasionally an attempt was made to produce congruent interstitial and n-outer cells, a procedure which leads to all kinds of irregularities.

83 | Pattern Types in (3×2) rhombs   Pfx Tue 24 April 1984

* Those type designations are the ones I have used in a paper "Islamic star patterns" * to be published in Muqarnas, but they should still be regarded as provisional only, pending a definitive and logical classification.

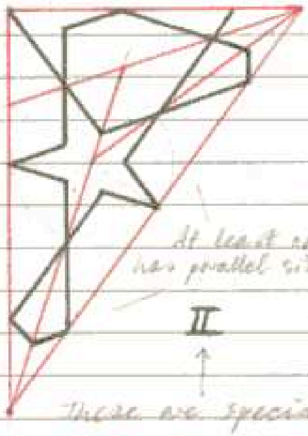
Saturday, APRIL 9, 1966

"Type" designations on pp. 83, 84 are those in blue on pp. 46–56.



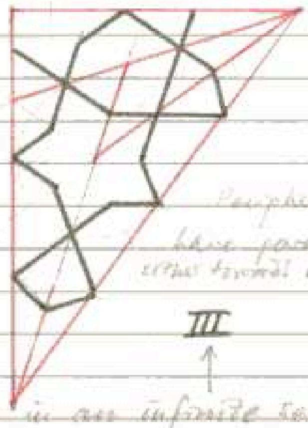
I, IV, V.

$\underbrace{}_{p.\,42}$



At least one rosette has parallel sides

II



Peripheral stars have parallel sides

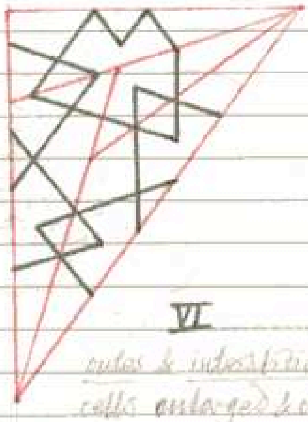
III

These are special cases in an infinite series

GROUP "A" PATTERNS

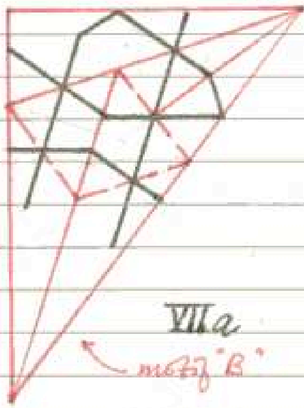
motif "A"

motif "B"



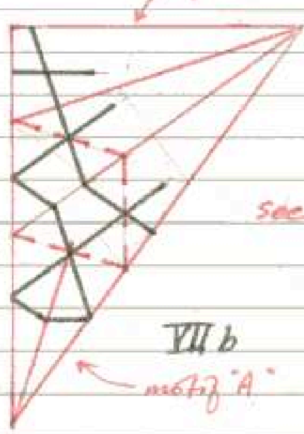
VI

outer & interstitial cells enlarged & overlap



VIIa

motif "B"



see pp. 191–192

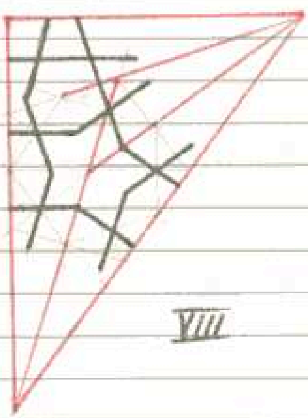
VIIb

motif "A"

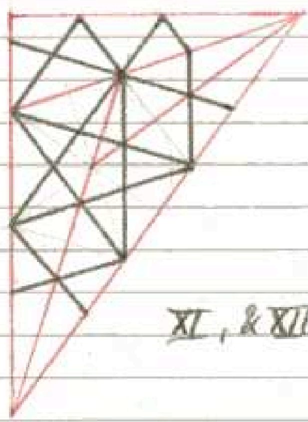
Rpl(3×2)10,10 / VII forces two kinds of rhomb: VIIa, VIIb.

GROUP "B" PATTERNS

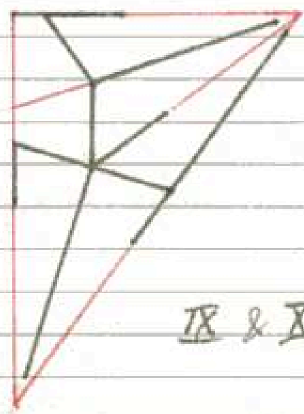
Easter Sunday, APRIL 10, 1966



VIII



XI, & XII



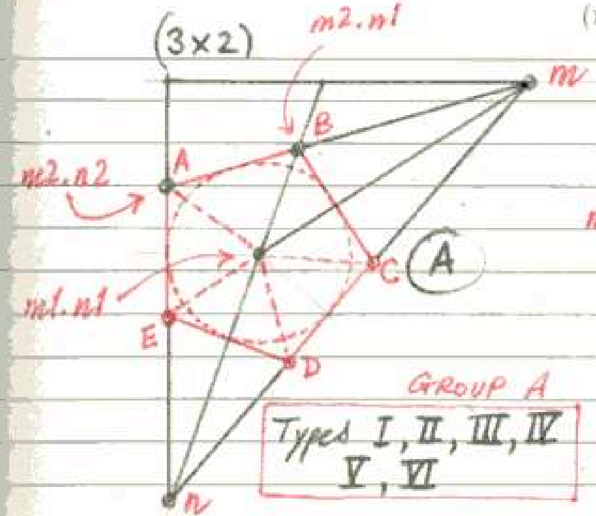
IX & X

* "Islamic star patterns", Muqarnas 4: 182–197, 34 figs. (1987)

Apr Sun 22 April 1984

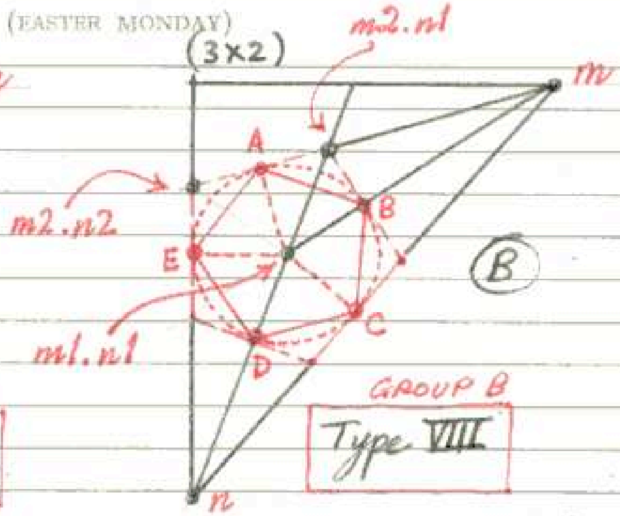
Monday, APRIL 11, 1966

(EASTER MONDAY)



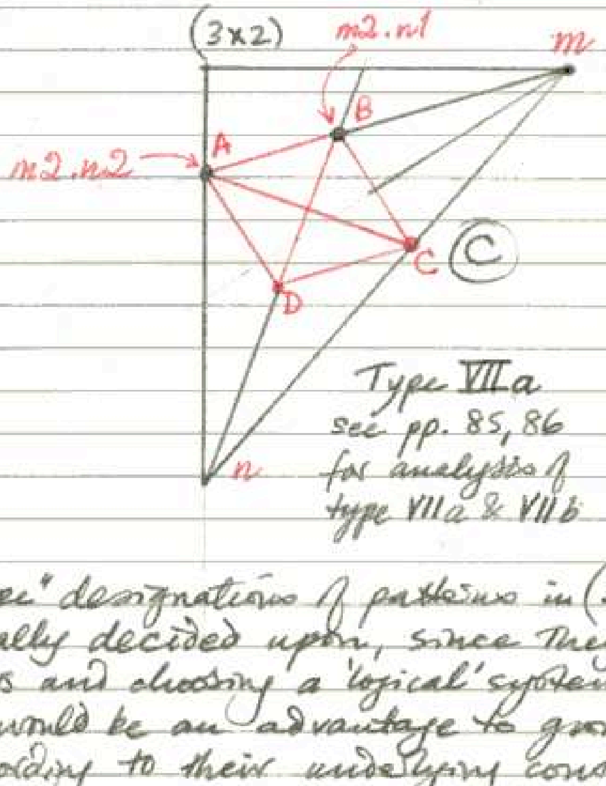
GROUP A

Types I, II, III, IV
V, VI



GROUP B

Type VIII



Type VIIa
see pp. 85, 86
for analysis of
type VIIa & VIIb

Type VII is essentially a "mixed" variety – one motif being a partially realized type III* motif, the other a type VIII motif, although the latter in types VII and VIII is differently related to the underlying (3×2) rhomb structure. Thus, while in most cases we may speak of the motif of a certain pattern type as a type I motif, this is not so in the case of type VII.

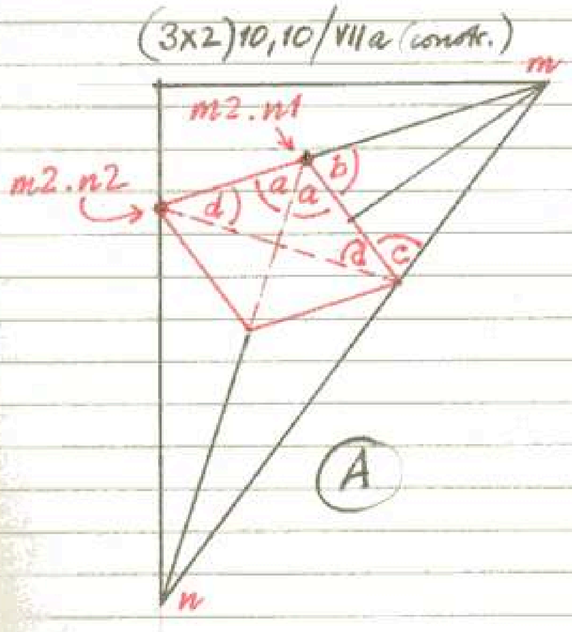
* or type I – see pp. 191–192

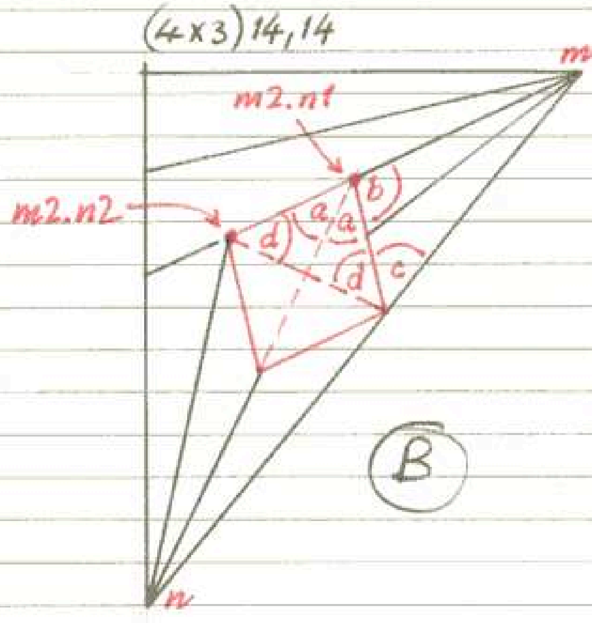
"Type" designations of patterns in (3×2) rhombs have not yet been finally decided upon, since they are related to one another in various ways and choosing a 'logical' system of labelling is to some extent arbitrary. It would be an advantage to group patterns into different categories according to their underlying constructions, that is, the radii, intersections and other points made use of in drawing them. Thus, group A, above, is clearly quite distinct from group B which at present contains only type VIII. – But A & B are related in that both use peripheral elements m1.n1

The specific pattern types chosen are primarily those encountered in authentic Islamic ornament, but a more complete geometrical treatment would require an investigation of any additional possibilities

85] Generalized Construction for (3×2)/VIIa

Wed 25 April 1984





In the general case (represented here by a (4×3) rhomb) the following angle values may be found quite easily (cf. p.67 et seq.).

a) $\dfrac{m-2p+4q}{2mq}$ or $\dfrac{n+p-2q}{np}$

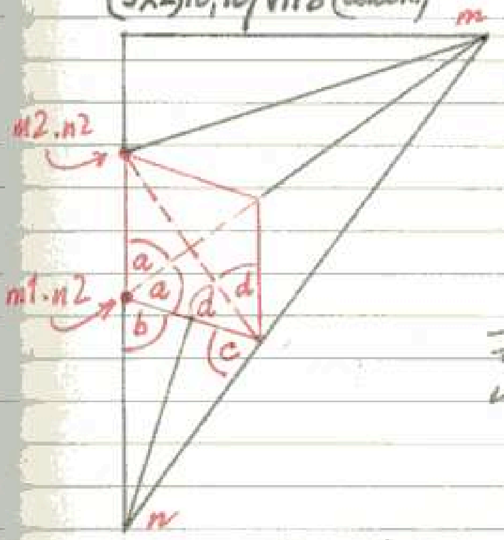
b) $\dfrac{m(q-1)+2(p-2q)}{mq}$ or $\dfrac{n(p-2)-2(p-2q)}{np}$

c) $\dfrac{m-2p+2q}{mq}$ or $\dfrac{n+2p-2q}{np}$

When $p=3$, $q=2$ we find that $b = c = \dfrac{m-2}{2m}$

Thus only in (3×2) rhombs can that side of the small constructional rhombus facing point $\underline{m}$ be the side of a regular m-gon centred on $\underline{m}$.

(3x2)10,10/VII b (contd.)



$$a = 1/m + 2/n$$

$$b = 1 - 2a = \frac{mn - 4m - 2n}{mn}$$

$$c = 1 - b - \frac{2}{n} = \frac{4}{n}$$

From which we easily discover that only when $m = n = 10$ does

$$b = c = \frac{4}{10}.$$

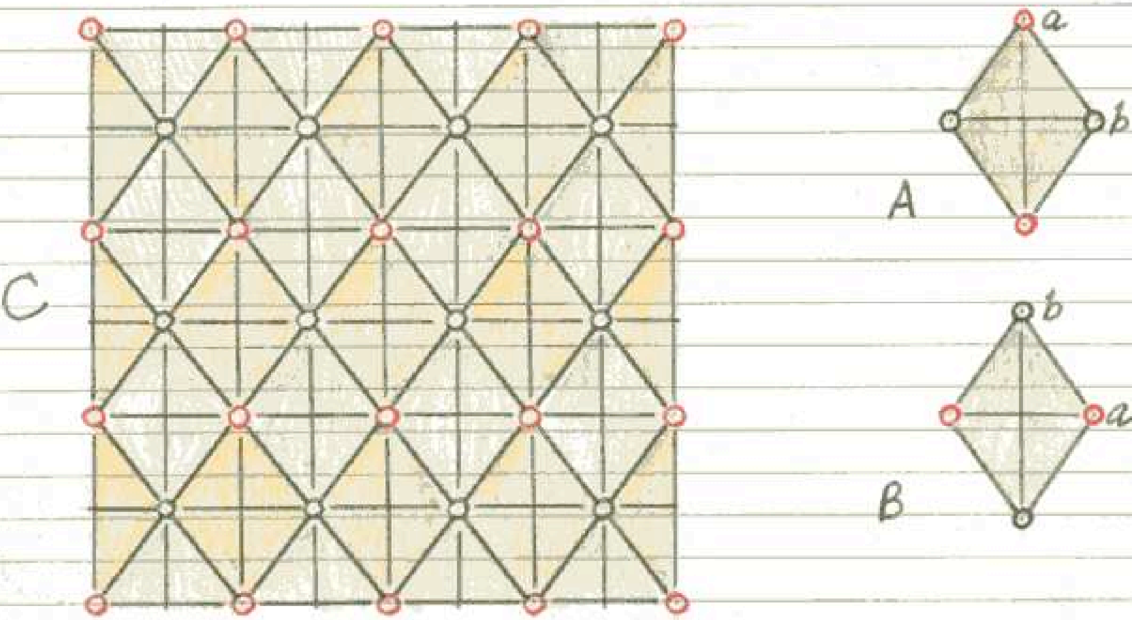
Therefore only type VII a will give an exact result for any pair of values (m, n) in the (3x2) rhomb. It is possible to draw type VII b patterns when $m \neq n$, but the results become less satisfactory as m and n differ more widely in value. Indeed both types VII a and VII b are extant in authentic Islamic ornament, but it is necessary to try to disguise the inaccuracies of VII b as much as possible.

The recognition of two varieties of type VII, i.e. a & b, is necessary since the motifs at the m- and n-centre are of different kinds (see the top figure on p. 50), therefore the parent pattern in which $m = n = 10$ necessarily contains two distinct realizations of this type, even though it is not possible to differentiate between m & n in this case.
There is one other group of patterns recognized among authentic variations, in which two distinct varieties and two distinct kinds of rhomb are evident, viz. that comprising types IX & X. Here, however, both varieties can be constructed as exact patterns; indeed these latter types can be drawn in any size of $(p \times q)$ rhomb.

In case these notions may not be entirely clear, a brief explanation is given over the page (p. 87). →

[illegible] Wed 25 April 1984

$(p \times q)m, n$ where $m = n$ but m, n motifs are of different kinds



In any Rp1 rhombic tessellation, in which for each rhomb $m = n$, if the motifs are of two alternating kinds – a, b – i.e. the style of representation is different, then the interiors of the rhombs of the Rp1 tiling will be of two kinds, depending on the relation of the two kinds of motif to the major and minor axes of the rhombs (see figs. A and B, above). We have in effect a dichromatic colouring of the original rhombic tiling, as represented at fig. C above. If $m = n$ then the star centres cannot be consistently labelled m or n, but if $m \neq n$, that is, if a and b represent motifs with different numbers of rays, then clearly an Rp1 tiling such as that shown above is no longer possible.* But any pattern which can incorporate one of the kinds, A or B, of rhombs may also be attempted with the alternative kind, although as we have shown (p. 86) only one variety may give an exact result.

If $p/m = q/n$ the original rhomb is a square and these remarks do not apply.

*This statement is incorrect: such patterns produce the "dichromatic [illegible]" shown above, but with dissimilar star centres.

INTEGRAL POLYGONS

Friday, APRIL 15, 1966 see p. 24 of Book 2.

The central notion behind a great deal of the numerical analysis of repeating star patterns in these notes is that of a convex polygon (which can tile the plane either by itself or with other polygons) in which the interior angles are integral multiples of $1/n$, where n is any integer greater than 2.* Here, as elsewhere, angles are expressed as fractions of 180° or π, so $1/n$ for example is to be understood as $180°/n$ (cf. pp. 23, 24 with reference to definition of "star-centre"). Star motifs are centred on the vertices of the various polygons constituting the tiling, and they form collinear links between each pair of adjacent vertices, that is, each edge of the tiling forms a collinear link between a pair of adjacent star motifs. Motifs may or may not form collinear links along diagonals of a given polygon of the tiling. Generalizing this notion, we allow non-collinear ("parallel") links along the edges of the tiling.

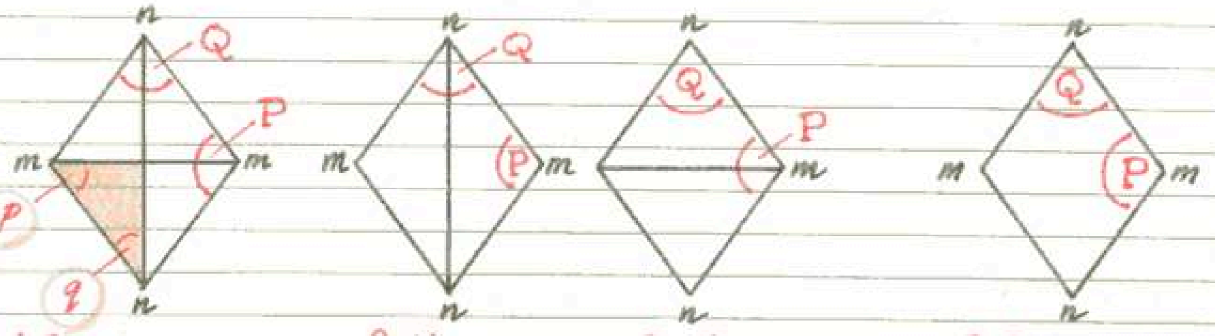
The rhombus holds a special place in these analyses, both because of its prevalence in a great deal of authentic Islamic ornament, and also because a rhombus with m and n centres alternating on the vertices allows the important relationship $p/m + q/n + 1/2 = 1$ to be used in enumerating all possible values for m, n, p and q (see pp. 11, 12 et seq.). The quantities p and q refer to the number of divisions of $1/m$ and $1/q$ respectively in the right triangle forming one quarter of a rhombus, which can thus be referred to as a $(p \times q)$ right triangle, and by extension the whole rhomb can be referred to as a $(p \times q)$ rhombus. Obviously in such cases the number of divisions in the whole internal angle at the m or n vertices is always an even number, and both rhomb diagonals form collinear links through the rhomb centre. It is possible however to consider rhombs in which the number of equal divisions at either the m or n vertex, or both, is an odd number, and such rhombs can form repeating patterns, although of course the total symmetry is lower.

* n may vary for each vertex.

89 INTEGRAL RHOMBS.

Wed 25 April 1984

In the case of rhombic configurations of star motifs – i.e. motifs of two sizes, or of two different numbers of rays, m, n, alternating on the vertices of a rhombus – the following definitions may be adopted.



P, Q both even   P odd   Q odd   P, Q both odd.

A. symmetrical   B. semi-symmetrical   C. asymmetrical

1. An integral rhomb $(p \times q)_{m,n}$ is one in which m and n are both integers. Only in integral rhombs can star motifs at the rhomb vertices be completed to form m- or n-fold regularly formed stars. Non-integral rhombs may be used to illustrate methods of construction of specific pattern types within the rhomb itself, but since the motifs cannot close up such rhombs are of no use in producing repeating patterns in the two-dimensional plane.

Semi-integral rhombs can however be used in certain radially symmetrical arrangements where only one of the two sizes of motif need close up. When all sides of a rhomb coincide with parallel links (see p. 197), the rhomb angles are non-integral multiples $1/m$ or $1/n$.*

Previous rhomb analyses have concentrated on the values p, q in the $(p \times q)$ right triangle which is one quarter of the whole rhomb (see coloured triangle at the bottom left of fig. A above). This is acceptable if the number of divisions in the whole of each interior rhomb angle is an even integer, but in generalizing the original notions we may wish to deal with cases in which

* This does not seem to be necessarily true in every case Tue 22 Jan 1985
see, for example, p. 194.

The whole interior angle is divided into an odd number of equal angles. Clearly, this would result in non-integral values for p, q, so in the general situation we consider the number of divisions P, Q in the whole interior angles at m and n, respectively, and such rhombs are distinguished as $[P \times Q]m,n$ rhombs, enclosing P, Q in square brackets instead of in parentheses as in the case of p, q. Obviously, $(p \times q)m,n$ is equivalent to $[P/2 \times Q/2]m,n$. When P or Q is an even number, then the rhomb is symmetrical about the diagonal joining such a pair of vertices. That is, the axis or diagonal joining vertices at which the interior angle has an even number of divisions coincides with a mirror axis of the whole rhomb (but not, of course, necessarily of the whole periodic pattern of which the rhomb may form a part). Depending on whether both, either or neither P or Q are even numbers we distinguish three principal categories (figure of page 89, opposite).

2. Symmetrical rhombs — P and Q both even numbers, both rhomb diagonals form mirror axes in the rhomb itself: fig. A.
3. Semi-symmetrical rhombs — either P or Q even, but not both; only one rhomb axis forms an axis of symmetry.
4. Asymmetrical rhombs — P and Q both odd numbers. The rhomb has no axes of symmetry.

Note that categories 2–4 may refer to integral or non-integral rhombs as defined on p. 89.

The relationship between m, n, P, Q is as follows:—

we have $\dfrac{P}{m} + \dfrac{Q}{n} = 1$ from which

$$m = \frac{nP}{n-Q} \quad \text{and} \quad n = \frac{mQ}{m-P}.$$

cf. p.11 et seq.

 Appx Thu 26 April 1984

Asymmetrical rhombs have not so far been investigated, but they would in any case prove difficult to incorporate in repeating patterns. None are known from authentic Islamic ornament.

Semisymmetrical rhombs (sensu str.) are not known from authentic sources, but there are many possibilities waiting to be discovered, some of which have been drawn elsewhere (see Rp1[7×4]11,11 on p. 92). Mirror axes are present in only one direction, which would of course bar them from use as typical Islamic star patterns. However, a number of square arrangements are extant which satisfy the definition of semisymmetrical rhombs, and some use differently numbered centres at opposite ends of the diagonal which coincides with the mirror axis, together with a tendency to transform an "interstitial" element into a major motif forming collinear links with all four vertices. A strict classification of many of these is difficult.* In Bourgoin's (1879) collection plates 154 and 159 would belong here, as does the point (14, 8) on the curve on p. 92, although the latter has not been realized as a pattern. A selection of integral solutions for semisymmetrical rhombs [P×Q] is given below.

[P×Q]	m, n pairs	Semi-symmetrical rhombs.							
4×1	$\boxed{8,2}$	6,3	5,5						
3×2	9,3	$\boxed{6,4}$	5,5	4,8					
5×2	15,3	$\boxed{10,4}$	7,7	6,12					
7×2	21,3	$\boxed{14,4}$	9,9	8,16					
9×2	27,3	$\boxed{18,4}$	15,5	12,8	11,11	10,20			
4×3	16,4	10,5	$\boxed{8,6}$	7,7	6,9	5,15			
6×3	24,4	15,5	$\boxed{12,6}$	9,9	8,12	7,21			
5×4	25,5	15,6	$\boxed{10,8}$	9,9	7,14	6,24			
7×4	35,5	21,6	$\boxed{14,8}$	11,11	9,18	8,32			
9×4	45,5	27,6	21,7	$\boxed{18,8}$	15,10	13,13	12,16	11,22	10,40

* They can be grouped with the kites - see p93 et seq. Or better still, they can be treated as 2-kite patterns: $K_1 + K_2$. See note on p. 292

Thur 26 April 1984

Friday, APRIL 22, 1966

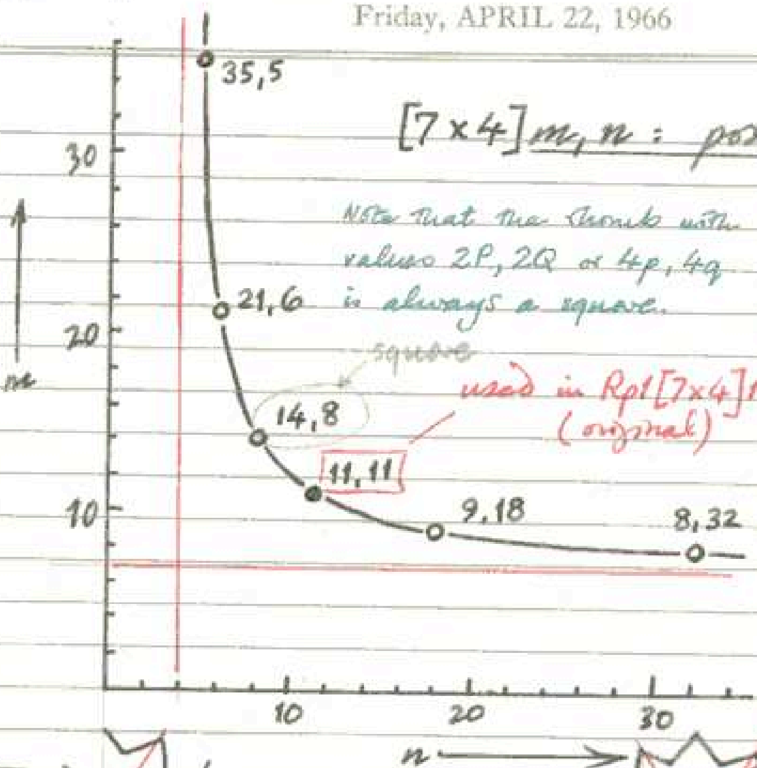
[7 x 4] m, n : positive, integral solutions

$$\frac{7}{m} + \frac{4}{n} = 1$$

$$m = \frac{7n}{n-4}$$

$$n = \frac{4m}{m-7}$$

Note that the rhomb with values 2P, 2Q or 4p, 4q is always a square.



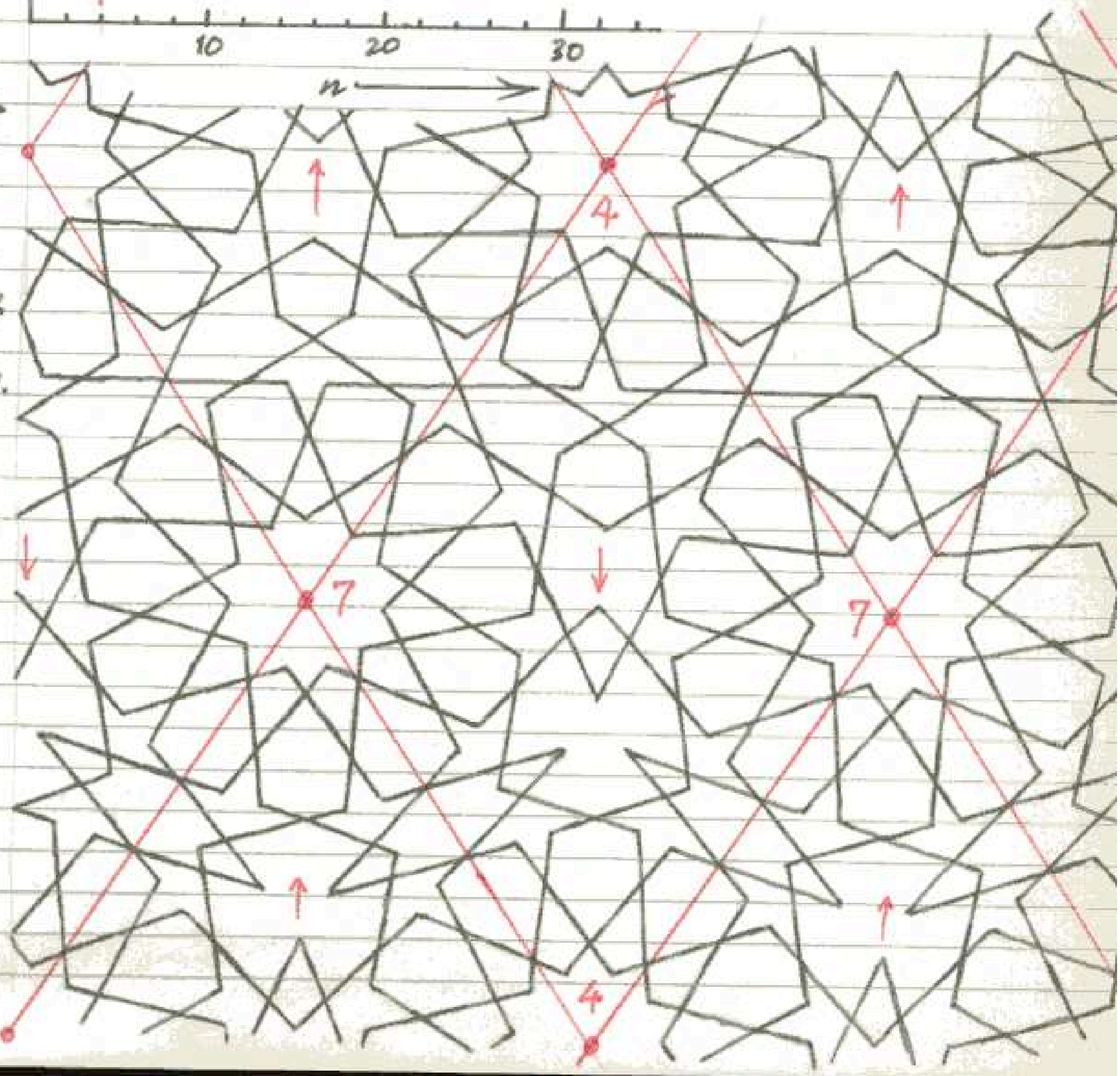
used in Rp1[7x4]11,11/II (original)

Part of Rp1[7x4]11,11/II

mirror axes vertically, but not horizontally. Glide reflection axes horizontally.

Sat 1 Feb 1969 (original)

The rhombs are all identical but are in alternating orientations ↓↑



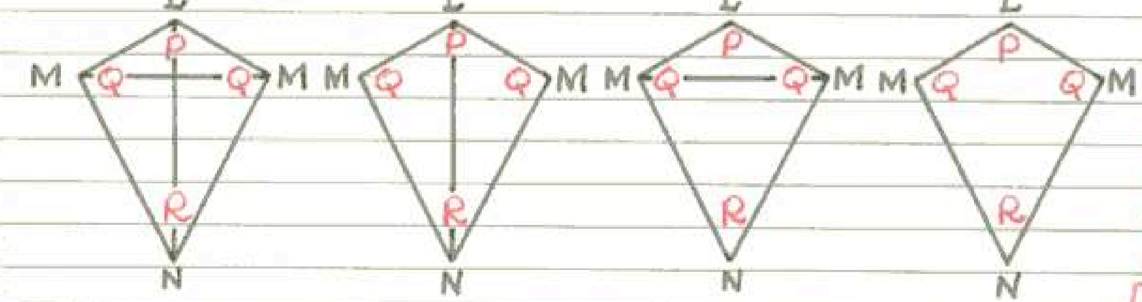
93] KITES

After Thu 26 April 1984

see further discussion, Book 2 pp.13,14,15.

Saturday, APRIL 23, 1966

After rhombs, kites are the next most obvious choice of polygon which can be subjected to numerical analysis of various kinds. A similar classification to that of rhombs (p.89) can be adopted here also,



Note that if $P = R$ kite is topologically equivalent to a [P x Q] rhomb.

A. "symmetrical"   B. "semi-symmetrical"   C. "asymmetrical"

but only the L-N axis can coincide with a mirror axis. The marked axes in figs. A-C above correspond strictly to collinear links rather than mirror axes so the designations here borrowed from rhomb categories are not entirely appropriate. In addition, the presence of a marked M-M axis does not indicate that Q is even, neither does its absence indicate that Q is odd. The presence of collinear links along the L-N axis can be determined by inspection, i.e. if P and R are even. A collinear link M-M will be present if

$$\frac{P}{2L} + \frac{x}{M} = \frac{1}{2} = \frac{R}{2N} + \frac{y}{M} \quad \text{where } x + y = Q.$$

Sunday, APRIL 24, 1966

A suitable general notation for kites is K[P x Q x R]L,M,N and of course the following relation holds

$$\frac{P}{L} + \frac{2Q}{M} + \frac{R}{N} = 2$$

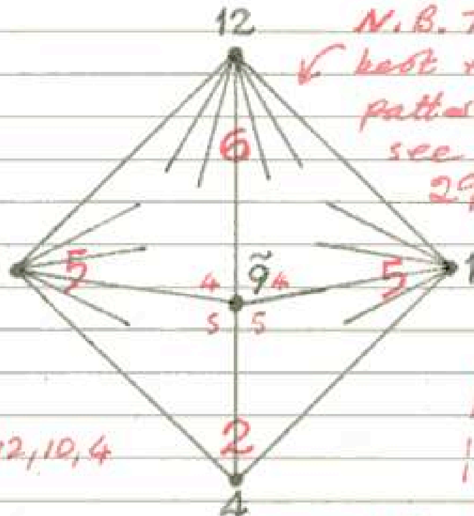
If P/L is a right angle the latter becomes $\frac{2Q}{M} + \frac{R}{N} = \frac{3}{2}$

which makes enumeration easier. If N is a multiple of 4 this type of kite can sit in a square with N at the centre and L at a vertex. A repeating pattern can then be completed with the addition of rhombs whose angles are $\frac{2(M-Q)}{M}$ and $\frac{N-2R}{N}$.

Not many patterns of this type are found in authentic Islamic ornament but repeating patterns are extant using the kite K[2×5×4]4,8,16 together with the rhomb R[6×4]8,16 (the latter equivalent to (3×2)8,16 of course) in which 8-stars are centred on the vertices of t{4,4}, with 16-stars at the centres of the octagons and effectively 4-fold octagons or "Khatams" at the centres of the squares.

Some of the semi-symmetrical squares referred to on p.91 can be formally regarded as 'semi-symmetrical' kites if they include stars of 3 different sizes, as follows:—

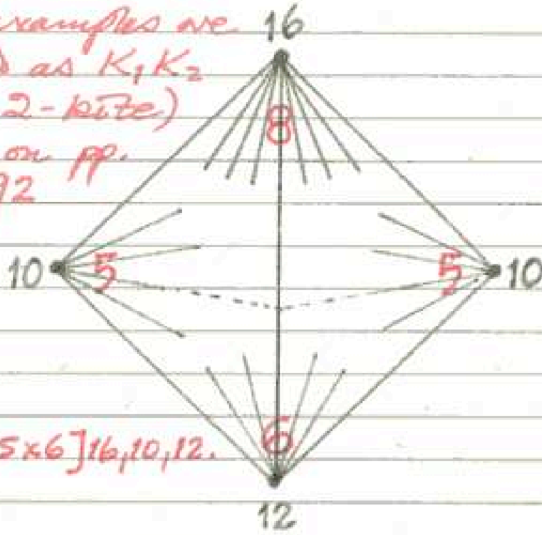
N.B. These examples are best regarded as K_1K_2 patterns (= 2-kite) see notes on pp. 291-292



[6×5×2]12,10,4

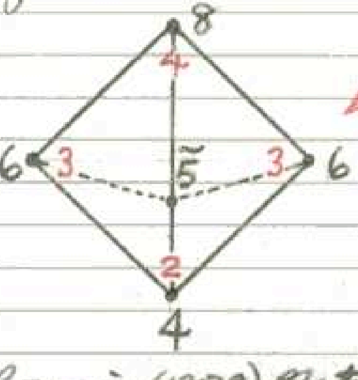
see p. 292: type VI

A. Fairly common in Asia Minor and elsewhere. See also Bourgoin (1879) Plate 159.



[8×5×6]16,10,12.

B. Cairo, minbar see pp. 289-29
Type VI with inscribed type II rosettes. On the same basis are Bourgoin's (1879) Pls. 160, 162 but his plates are very clumsily drawn.



C. Bourgoin (1879) Plate 154

Doubling all numbers produces the basis for the main stone carved border round the entrance: the Sultan Han, Konya/Aksaray road, Turkey. (see Hill & Grabar, 1964 fig. 461) – see p. 125 of these notes.

 KITES + RHOMBS. AFfM Thu 26 April 1984

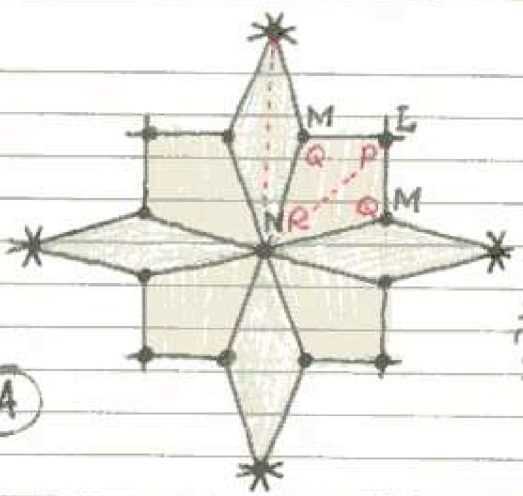
As mentioned on p. 93 kites in which P/L is a right angle are special cases, since they can become incorporated in repeating patterns on a square basis if N is a multiple of 4. It is necessary to complete the repeating pattern with rhombs whose interior angles are $\frac{2(M-Q)}{M}$ and $\frac{N-2R}{N}$.

Fig. A on p. 96 shows the general scheme, with rhombs coloured green, kites light orange. Those cases in which points M lie on the vertices of the semiregular tessellation t{4,4} form a special subset of this group of patterns.*
A selection of solutions where P/L = 90° and N is a multiple of 4 are given below:- (L is a subsidiary centre and the number of rays depends largely on the pattern style adopted.)

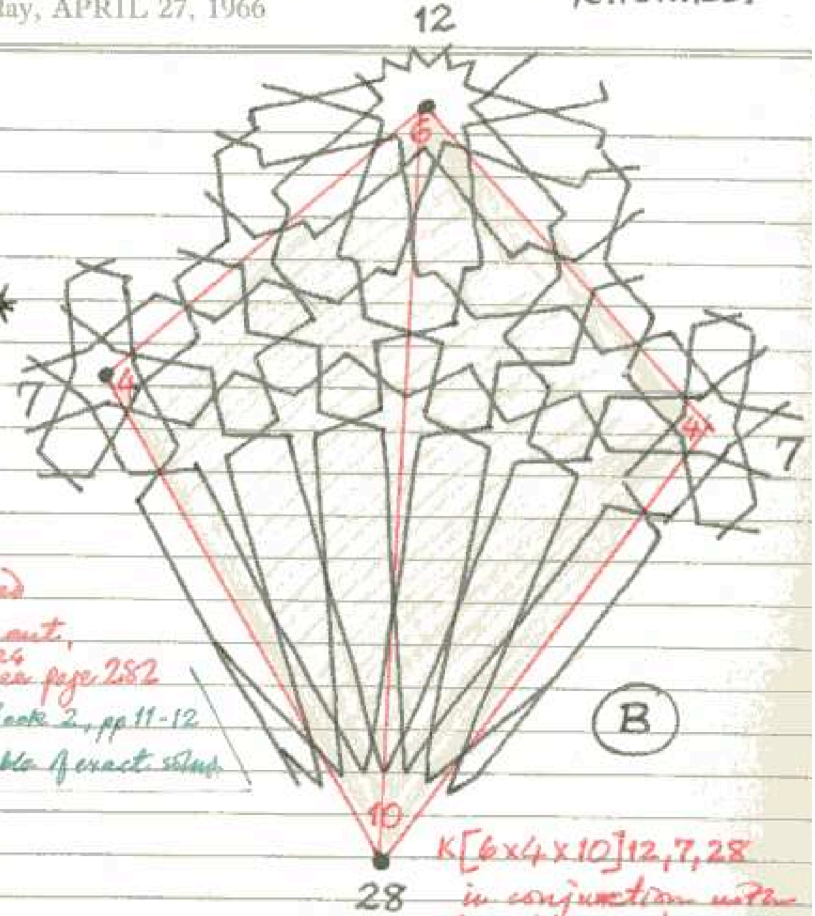
L=90°	M	N	Q	R	
12_6	7	28	4	10	= Fig. B, p. 96 opposite. + (3×2) 7,28
	6	60	4	10	
	8	40	5	10	
	7	140	5	10	A complete set of exact solutions
	6	48	4	8	is given in Book 2 of these notes,
	8	32	5	8	p. 3. AFfM Fri 19 April 1985
	7	112	5	8	
	6	36	4	6	
8	8	24	5	6	= Bourgoin (1879) Plate 150
	7	84	5	6	
12_6	10	20	6	6	= A.J.Lee 18 May 1965 + (4×2) 10,20
	9	36	6	6	
	12	18	7	6	
	10	60	7	6	
	6	24	4	4	
4	8	16	5	4	= Bourgoin (1879) Plates 146, 149, 152
	7	56	5	4	
	9	24	6	4	
	12	12	7	4	
	10	40	7	4	
	12	24	8	4	
	11	88	8	4	
4	9	12	6	2	= Fig. C, p. 96 opposite + (3×2) 9,12

* t{4,4} solutions are in red. see also the figure on p. 126.

Thu 26 April 1984



(A)

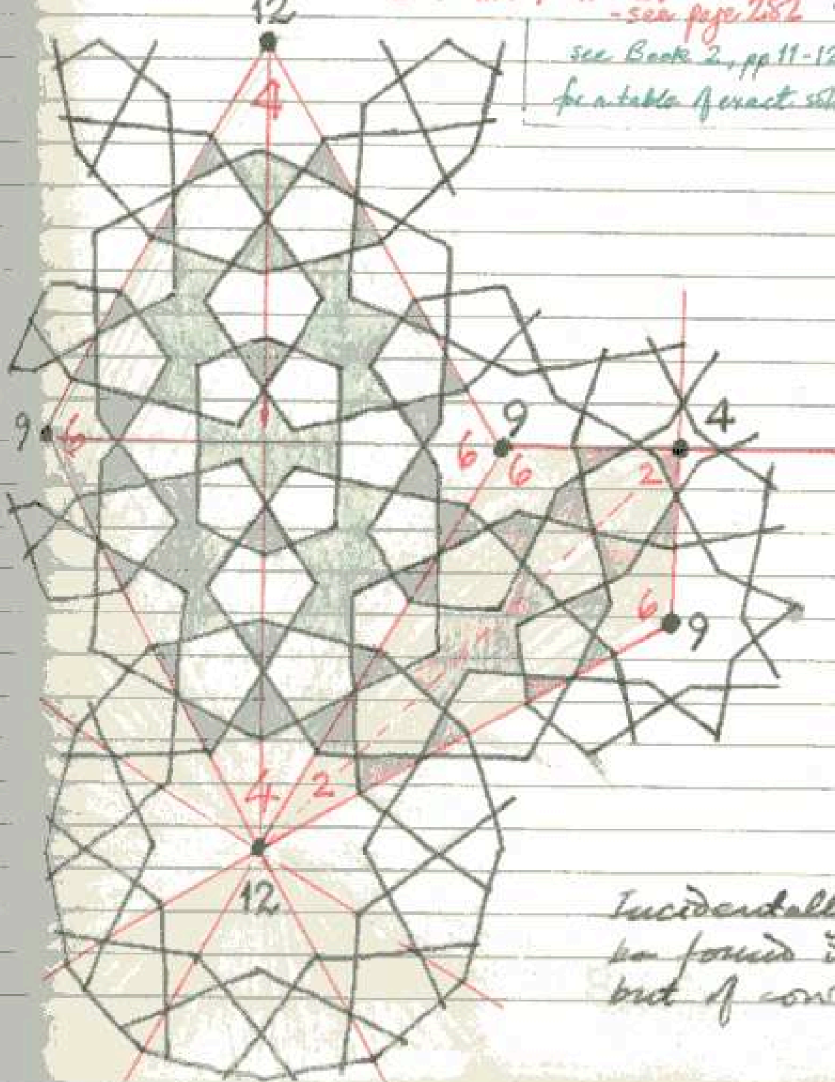


(B)

K[6x4x10]12,7,28
in conjunction with
(3x2)7,28/II
(original)

A hexagonal version of this scheme, based on t{6,3}, is extant in Turkish ornament.
L = 3 M = 7 N = 24
– see page 282

see Book 2, pp 11-12 for a table of exact values



(C) K[2x6x2]4,9,12 in conjunction with
(3x2)9,12/I–2B
(original)

There is actually no collinear link between the 9-stars in the kite, but it is close enough for an approximate pattern to be drawn.

Incidentally, this use of (3x2)9,12 is to be found in the Alhambra in Spain, but of course in a different pattern style.

 Thu 26 April 1984

Thursday, APRIL 28, 1966

Continuing the scheme of fig. A on p. 96, a number of solutions are possible in which M is represented by a nearly-regular centre, although in some cases it is a moot point whether to regard the latter as an interstitial element, in which case the pattern is classifiable in a different way. A number of solutions of this kind are given below, both authentic Islamic patterns and some of my own invention.

KITE						RHOMBOS				
L	M	N	P	Q	R	M	N	P	Q	
4	$\widetilde{10}$	12	2	6	4	$\widetilde{10}$	12	8	2	A.J.L. 22 Mar 1977
4	$\widetilde{7}$	20	2	4	8	$\widetilde{7}$	20	6	2	Morocco
8	$\widetilde{10}$	16	4	6	4	$\widetilde{10}$	16	8	4	Morocco, Tlemcen* (see also fig. 126)
4	$\widetilde{5}$	8	2	3	2	$\widetilde{5}$	8	4	2	Bourgoin (1879) Plate 148
12	$\widetilde{7}$	16	6	4	6	$\widetilde{7}$	16	6	2	Bourgoin (1879) Plates 133, 134, 135

* Pattern shown on p. 138 →

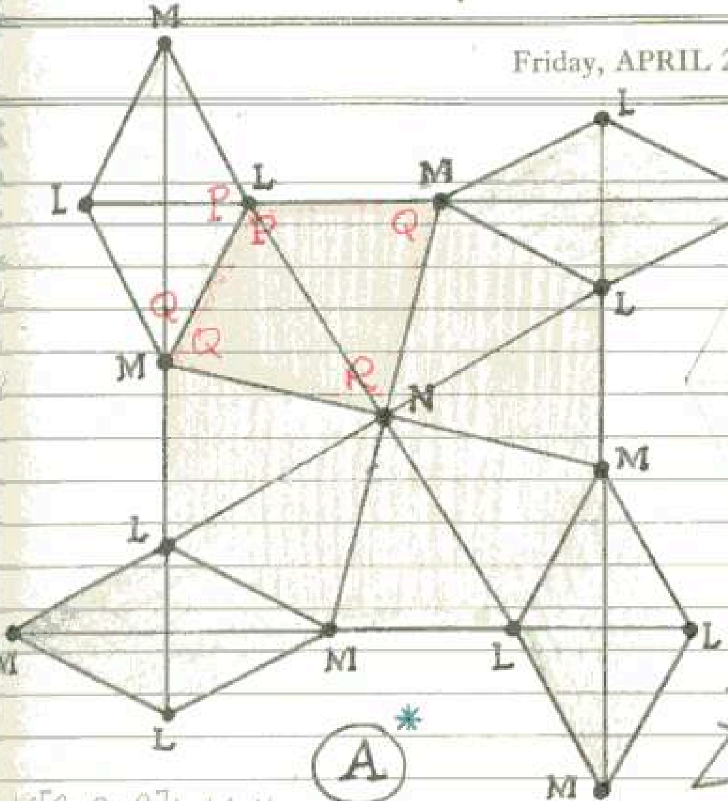
A different Kite + Rhombus scheme, but on the same square basis, is shown in fig. A on p. 98 opposite. Again, N must be a multiple of 4. Only ?two* examples of this scheme are known at present, both of them my own invention, figs. B & C opposite, p. 98. No authentic Islamic patterns on this basis are known. In fig. A it will be seen that kites could be formed in two ways from the eight scalene triangles surrounding point N, but we may adopt the convention that distance L-N is greater than M-N. Under this convention it will be seen that fig. C was incorrectly drawn at first.

* see note opposite

KITE						RHOMBUS				
L	M	N	P	Q	R	L	M	P	Q	
12	$\widetilde{14}$	16	8	6	8	12	$\widetilde{14}$	8	4	: fig. B opposite, p. 98
$\widetilde{9}$	15	20	6	6	10	$\widetilde{9}$	15	6	6	: fig. C opposite, p. 98.
$\widetilde{6}$	10	8	4	4	4	$\widetilde{6}$	10	4	4	: original (24 March 1976)**
10	$\widetilde{9}$	12	6	4	6	10	$\widetilde{9}$	8	4	: original (1 April 1985)

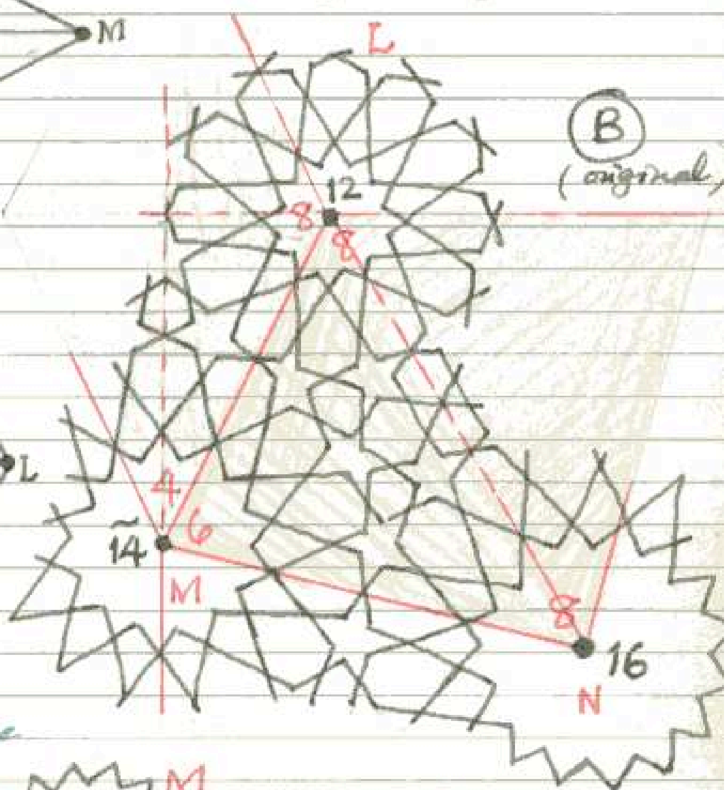
** see p. 140 for a drawing of this.

Friday, APRIL 29, 1966

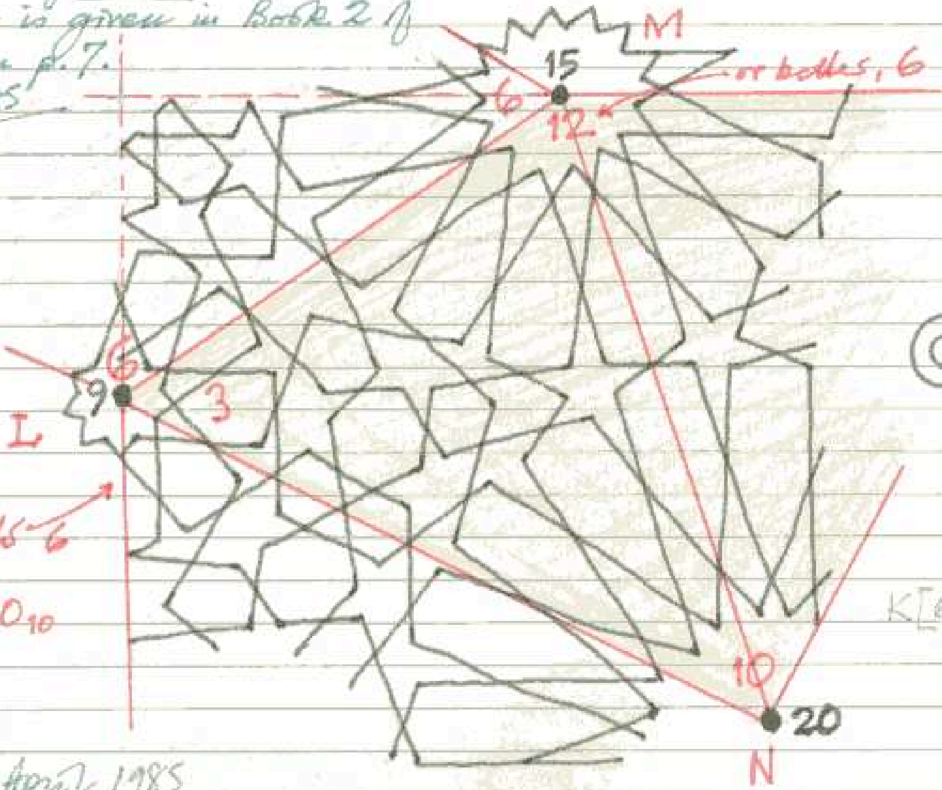


K[P×Q×R]L,M,N.

K[8×6×8]12, 14, 16



* A complete set of exact solutions to the above pattern is given in Book 2 of these notes, on p. 7.
Fri 19 April 1985



$L9_6$ $M15_6$ $N20_{10}$

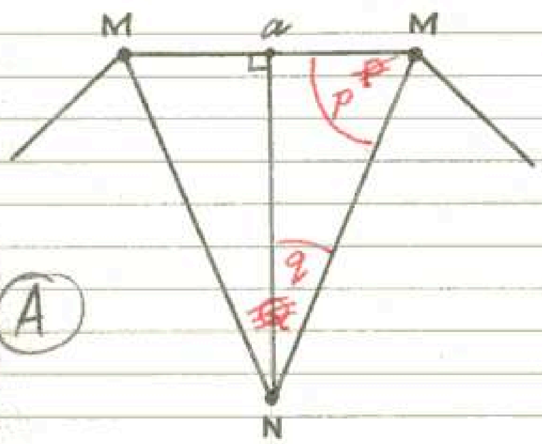
K[6×6×10]9,15,

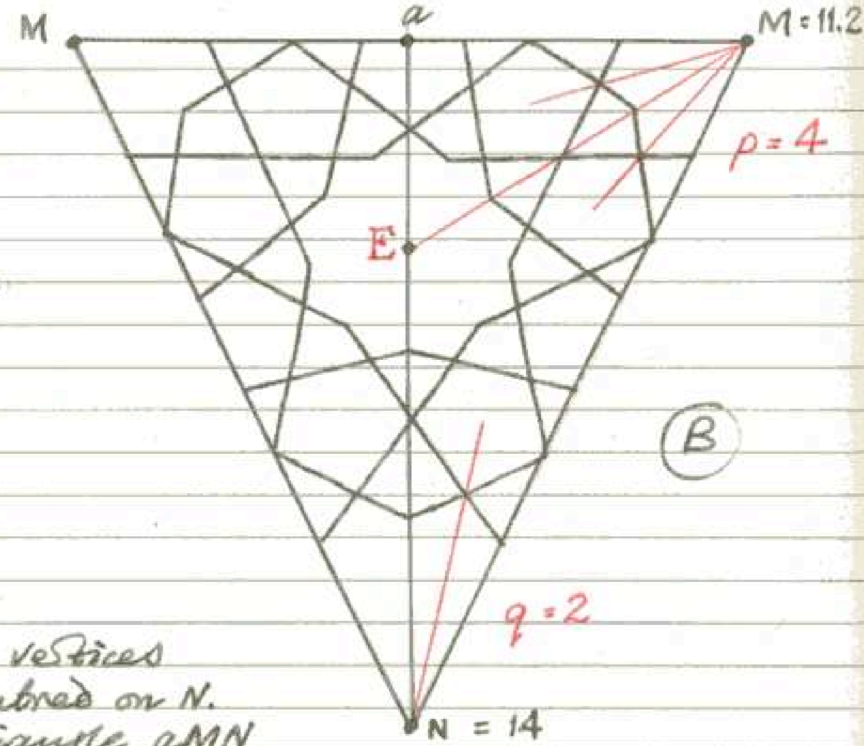
* Note:—

Sun 14 April 1985

Many more examples, exact & non-exact, of this group have since been found.

99 | Radial Compositions within Regular polygonal outlines. Fri 27 April 1984



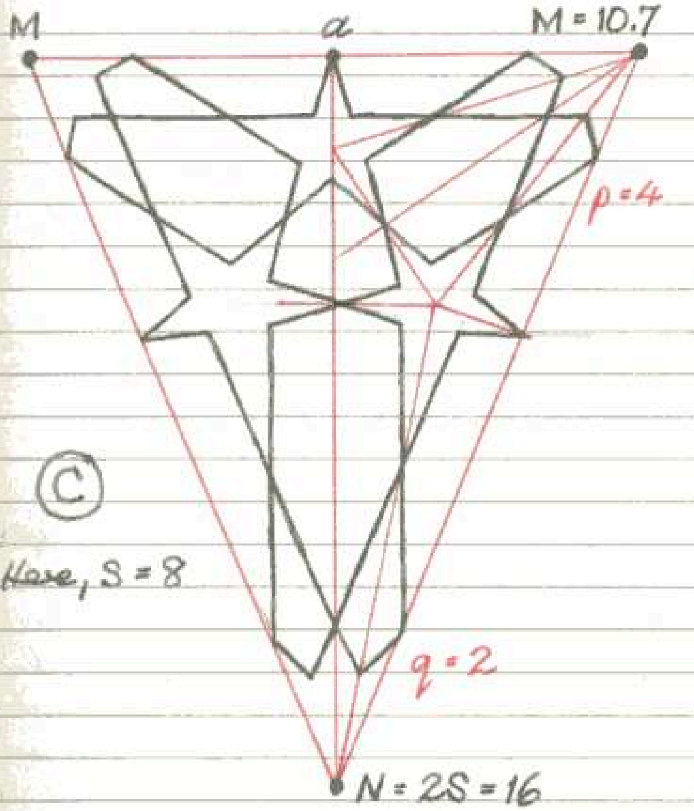


In fig. A M, M are the vertices of a regular polygon centred on N. In the right angled triangle aMN angle M is divided into p equal angles, angle N into q equal divisions. This triangle may then be regarded as one quarter of a $(p \times q)$ rhombus, or as a $(p \times q)$ right triangle, and a suitable pattern constructed inside it. In general this right triangle will effectively be a quarter of a symmetrical, semi-integral rhombus. Centre N will automatically be an integral division of 180°, but this is not always so for angle M, but if the pattern is confined to the interior of the original regular polygon this is of no importance. If the regular polygon has S sides then the star at N will have qS points, i.e. $N = qS$. In fig. B above we have one sector of a regular heptagon, triangle aMN is divided to form a (4×2) right triangle, $N = 14$. The value of M is not in general integral, but in order to calculate its actual value we note that

$$\frac{p}{M} + \frac{1}{S} + \frac{1}{2} = 1$$

from which $M = \dfrac{2Sp}{S-2}$. In fig. B, M thus equals 11.2

which suggests that an approximately regular 11-rayed star could be completed at centre M if it were required to extend the pattern beyond the boundaries of the original heptagon. Indeed, this kind of observation can often suggest a new pattern using nearly-regular stars and should always be carried out in situations of this kind. Fig. C on this page shows another example, this time in one sector of a regular octagon, the right triangle aMN again divided as a (4×2) right triangle. The value of M is lower, and at 10.7 and again could be completed as an 11-star, but of slightly lower accuracy. (Note that $8/11 + 4/16 = 43/44$). Since the rosette in fig. C are all parallel sided, the pattern could be drawn in Moroccan style, with the full width of the M-rays all round the octagonal border. No further examples need be illustrated, since enough has been suggested to show that this sort of radially symmetrical pattern has many possibilities.



Any existing (p×q) rhomb pattern may be incorporated in the right triangle aMN and repeated 2S times round centre N. Instead of a regular polygonal outline, regular star outlines may be used. A number of such compositions are encountered in authentic Islamic ornament; the two examples shown here are, however, my own inventions.

The insertion of an approximately regular 9-star at E in fig. B opposite suggests yet another possible line of inquiry which can be investigated by numerical calculation. In choosing an initial polygonal outline it is better to restrict the choice to fairly low values, say $S \leq 12$. Long, narrow right triangles are unsatisfactory.