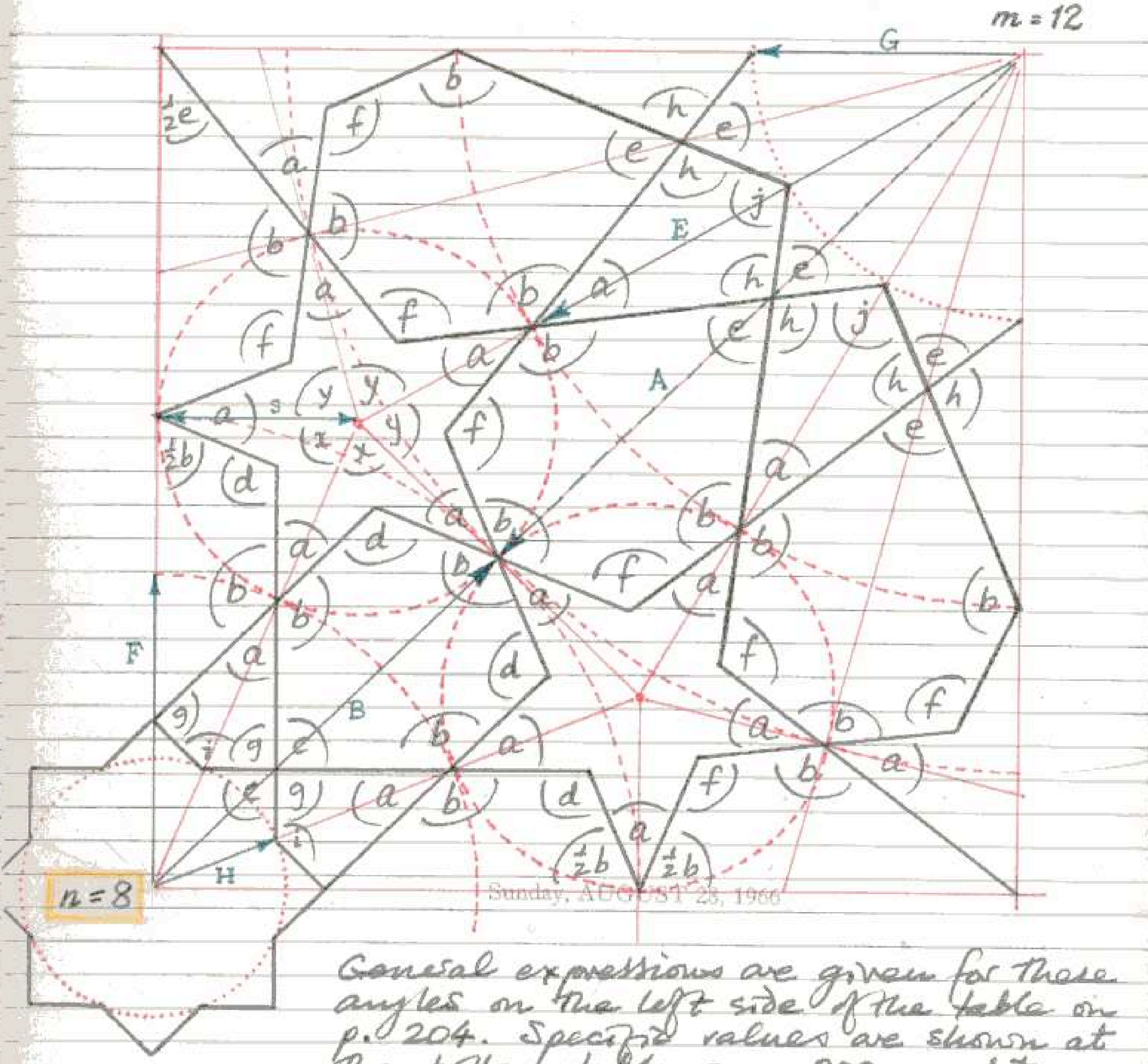
201 | (3x2)m, n Type II, where m > n — Thur 14 Feb 1985

"n-parallel"



General expressions are given for those angles on the left side of the table on p. 204. Specific values are shown at the bottom table on p. 202, opposite.

In the primary STANDARD CONSTRUCTION for $m \geq n$, i.e. the smaller, n-rosette is drawn with parallel sides & collinear terminal segments, the inner points of the interstitial cells always meet at the centre of the rhomb (see pp. 205, 206, 208). – see p. 207.

[illegible] Wed 13 Feb 1985

SOLUTION OF (3X2) TYPE II PRIMARY STANDARD CONSTRUCTION | 202

Monday, AUGUST 29, 1966 N.B. A + B = 1.0

~~segment~~ radii lengths:—

(BANK HOLIDAY)

		7,28	8,16	9,12	10,10	12,8	14,7	18,6	30,5		
	S	0.0913	0.13438	0.15433	0.162460	0.16270	0.1548	0.1351	0.0918	S	
m	A	0.1896	0.32442	0.42402	0.5	0.60721	0.678	0.76604	0.87362	A	m
m	E	0.1191	0.21677	0.29691	0.36327	0.46593	0.541	0.6428	0.7866	E	m
m	G	0.0530	0.08979	0.11726	0.13876	0.22474	0.3002	0.4195	0.6222	G	m
n	B	0.8104	0.67558	0.57598	0.5	0.39279	0.322	0.23396	0.12638	B	n
n	F	0.7242	0.55443	0.44196	0.36327	0.26246	0.202	0.1351	0.0644	F	n
n	H	0.5040	0.30006	0.19733	0.13876	0.10871	0.090	0.0675	0.0398	H	n

Angular values:—

"m-parallel"* "n-parallel"*

	7,28	8,16	9,12	10,10	12,8	14,7	18,6	30,5	
a	51° 25.71′	45°	40°	36°	45°	51° 25.71′	60°	72°	a
b	128° 34.29′	135°	140°	144°	135°	128° 34.29′	120°	108°	b
c	64° 17.14′	67.5°	70°	72°	90°	102° 51.43′	120°	144°	c
d	135°	123.75°	115°	108°	112.5°	115° 42.86′	120°	126°	d
e	102° 51.43′	90°	80°	72°	75°	77° 8.57′	80°	84°	e
f	115° 42.86′	112.5°	110°	108°	120°	128° 34.29′	140°	156°	f
g	115° 42.86′	112.5°	110°	108°	90°	77° 8.57′	60°	36°	g
h	77° 8.57′	90°	100°	108°	105°	102° 51.43′	100°	96°	h
i	77° 8.57′	90°	100°	108°	135°	154° 17.14′	180°	216°	i
j	154° 17.14′	135°	120°	108°	105°	102° 51.43′	100°	96°	j
x	83° 34.29′	78.75°	75°	72°	67.5°	64° 17.14′	60°	54°	x
y	64° 17.14′	67.5°	70°	72°	75°	77° 8.57′	80°	84°	y

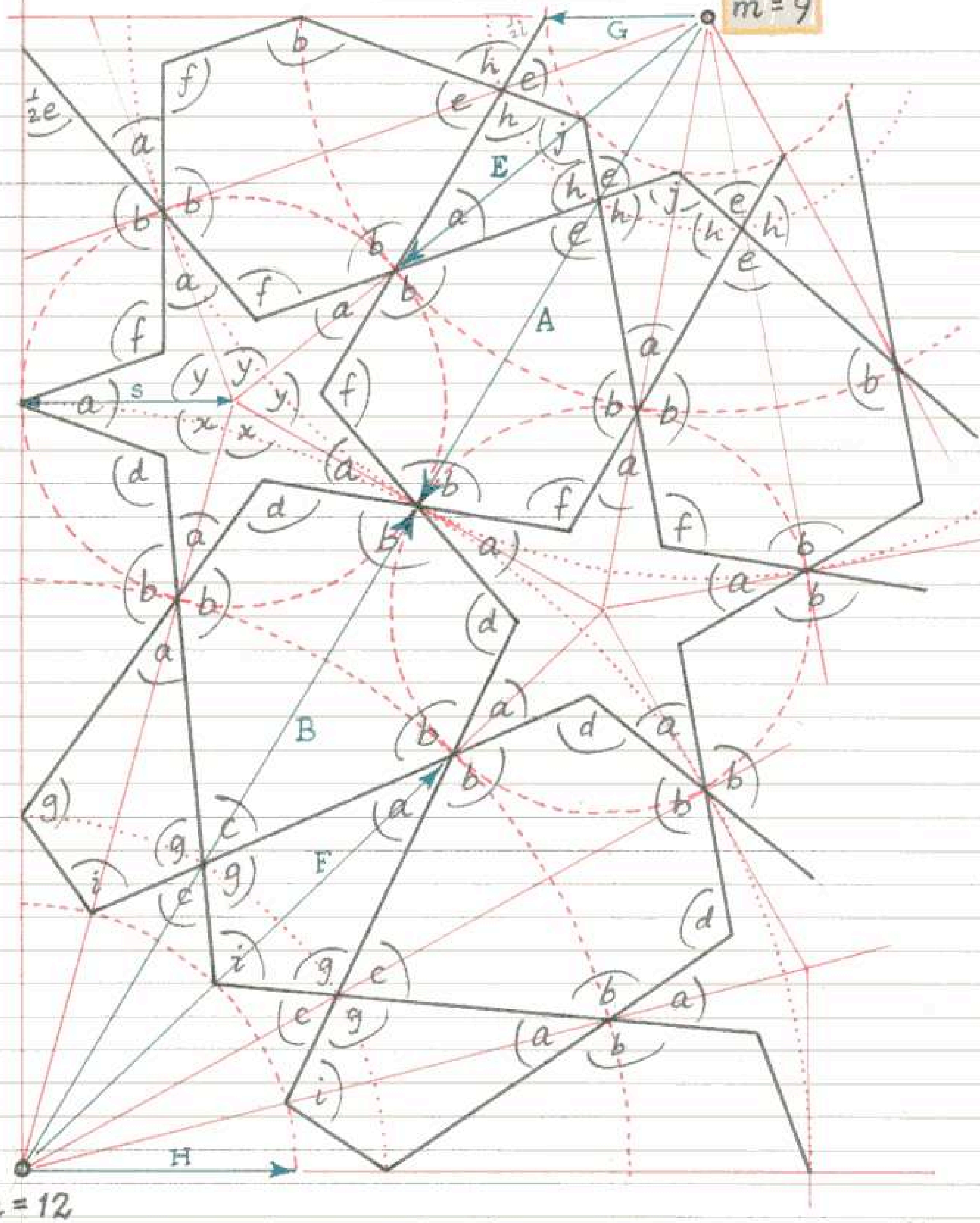
* "m-parallel" and "n-parallel" means that the m- or n-rosette respectively has parallel sides and collinear terminal segments (terminology from 1976 notes). When applied to type III rhombs these terms refer to peripheral stars — see p. 215 →

(3×2) m, n Type II where m < n

"m-parallel"

Tuesday, AUGUST 30, 1966

Thur 14 Feb 1985



General expressions for angles are shown opposite, right. Specific angular values are given on p. 202, bottom.

Thu 14 Feb 1985.

SOLUTION OF (3x2) m, n / II
PRIMARY STANDARD SOLUTION*
GENERAL EXPRESSIONS FOR ANGLES*

	"n - parallel"		"m - parallel"		
		$m > n$		$m < n$	
a		$\frac{2}{n}$		$\frac{2}{m}$	a
b		$\frac{n-2}{n}$		$\frac{m-2}{m}$	b
c	$= 2a$ =	$\frac{4}{n}$		$\frac{m-2}{2m}$	c
d	$= 1 - \frac{1}{2}b$ =	$\frac{n+2}{2n}$	$= a + x$ =	$\frac{m+14}{4m}$	d
e		$\frac{n+2}{3n}$	$= 2a$ =	$\frac{4}{m}$	e
f	$= a + y$ =	$\frac{n+8}{3n}$	$= 1 - \frac{1}{2}b$ =	$\frac{m+2}{2m}$	f
g	$= 1 - c$ =	$\frac{n-4}{n}$	$= 1 - c$ =	$\frac{m+2}{2m}$	g
h	$= 1 - e$ =	$\frac{2n-2}{3n}$	$= 1 - e$ =	$\frac{m-4}{m}$	h
i		$\frac{6}{n}$		$\frac{m-4}{m}$	i
j		$\frac{2n-2}{3n}$		$\frac{6}{m}$	j
x		$\frac{n-2}{2n}$		$\frac{m+6}{4m}$	x
y	$= e$ =	$\frac{n+2}{3n}$	$= c$ =	$\frac{m-2}{2m}$	y

* In the primary standard construction, based on peripheral circles of radius S, the smaller of the two rosettes, m or n, is drawn as parallel sided with collinear terminal segments. The two halves of the table above show expressions for all angles in terms of the centre which has the smaller rosette. This becomes a necessary procedure in order to avoid confusion.

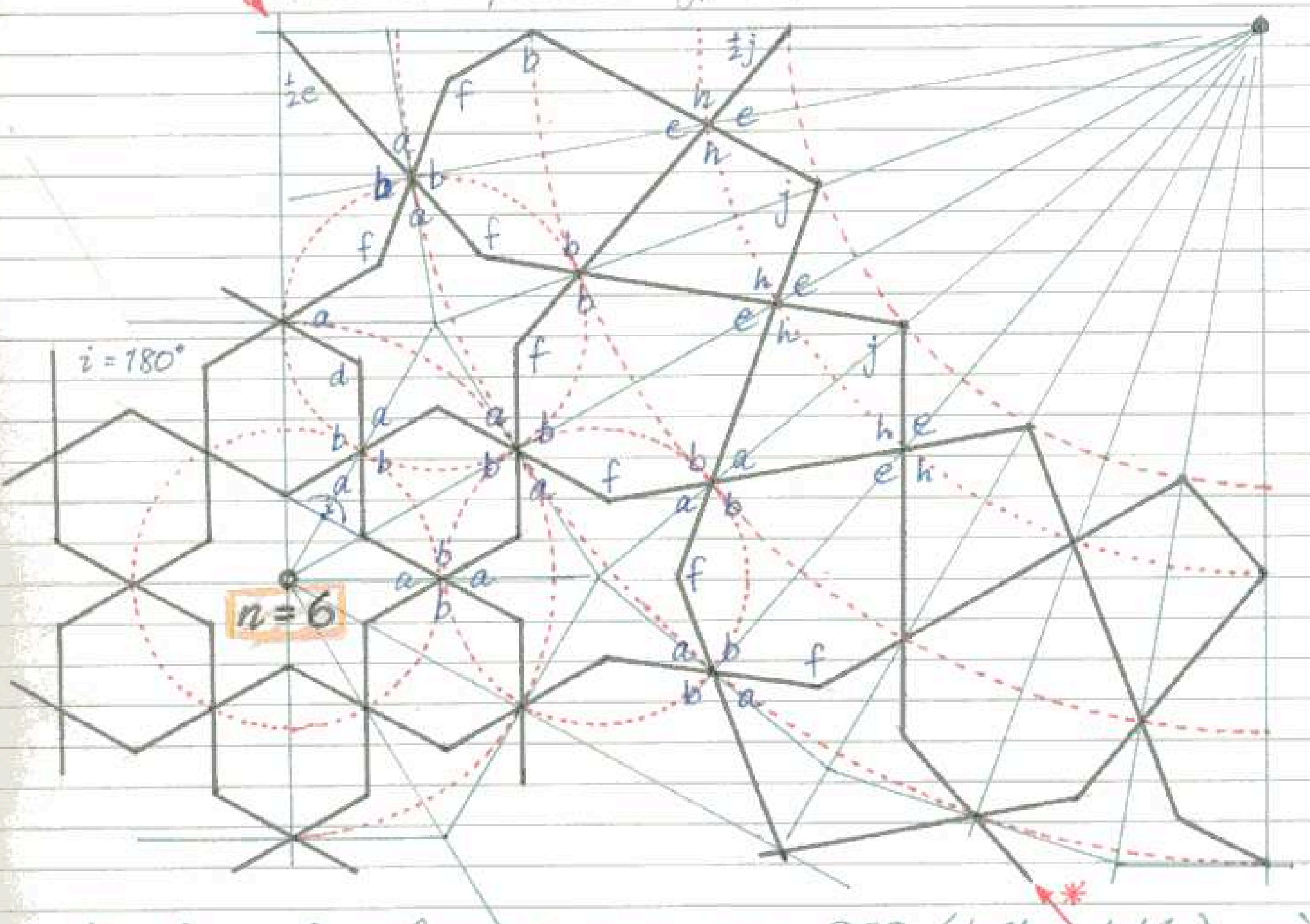
$$E = \frac{A}{\cos \pi/m} - S \qquad F = \frac{B}{\cos \pi/n} - S \qquad A = 1 \Big/ \left(\frac{\cot \pi/n}{\cot \pi/m} + 1\right)$$

$$G = \frac{E \sin \frac{1}{2}a}{\sin \frac{1}{2}j} \qquad H = \frac{F \sin \frac{1}{2}a}{\sin \frac{1}{2}i} \qquad B = 1 - A$$

205 (3x2) Type II   m > n   ("n-parallel")   Fri 15 Feb 1985.

Primary STANDARD CONSTRUCTION

* Note that this version is indistinguishable from an n-parallel type III



Actual angular values are given on p. 202 (bottom table), and lengths of radii (relative to rhomb edge, which equals unity) p. 202 (top table). Proof that the interstitial cells meet at the rhomb centre is given on p. 207.

A version of this pattern, with the 6-rosette replaced by a star of David, occurs in Herat (Afghanistan) at Gazur Gah, Sanctuary of Khwajah Abd Allah Ansari (1428-29), and nearly contemporary derivative versions are known from Iran. The pattern does not otherwise appear to be common.

* Note alignment of outer segments of larger rosette when the rosette number is exactly divisible by the smaller number. Here the cells concerned are 18/6 = 3 main divisions apart.

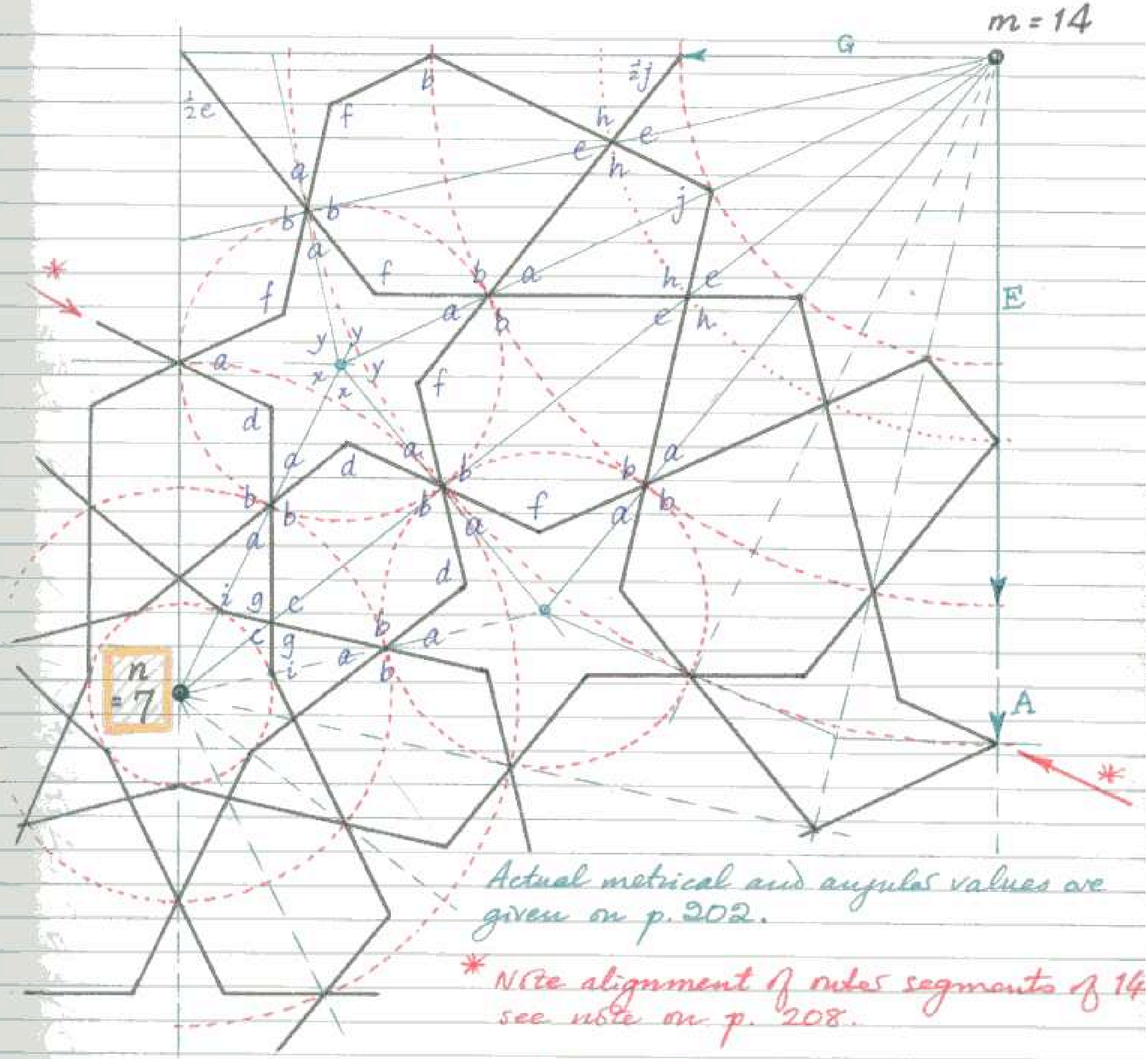
Fri 15 Feb 1985

(3x2) Type II   m > n

Primary STANDARD CONSTRN

("n - parallel")

Friday, SEPTEMBER 2, 1966



Actual metrical and angular values are given on p. 202.

* Note alignment of outer segments of 14 see note on p. 208.

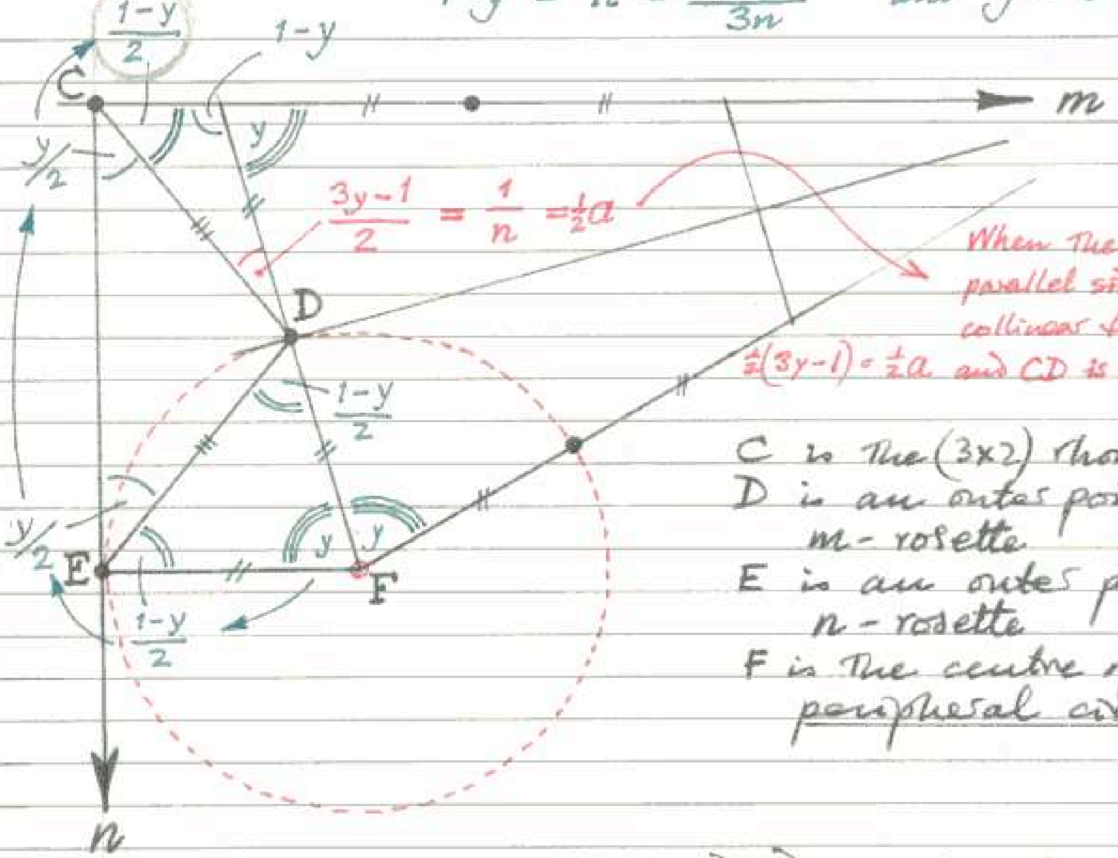
The most common use of this rhomb in authentic Islamic ornament is as an H2 rhombic tessellation, together with the rhomb (3x1) 7,14. There is however an inexactitude in the authentic use of the latter rhomb, since the outer stars of the 7-rosettes are made to meet across the shorter rhomb axis, which is not correct on the above construction.

N.B. A similar difficulty occurs in the rhomb (4x2) 10,20 where, in a "standard" construction the 10-rosettes do not quite exactly meet at the rhomb centre.

207 (3x2) Type II
Primary STANDARD CONSTRUCTION

RR Fri 15 Feb 1985

$1-y = h = \frac{2n-2}{3n}$ and $y = e$



When the n-rosette is parallel sided and has collinear terminal segments, $\frac{1}{2}(3y-1) = \frac{1}{2}a$ and CD is a pattern line.

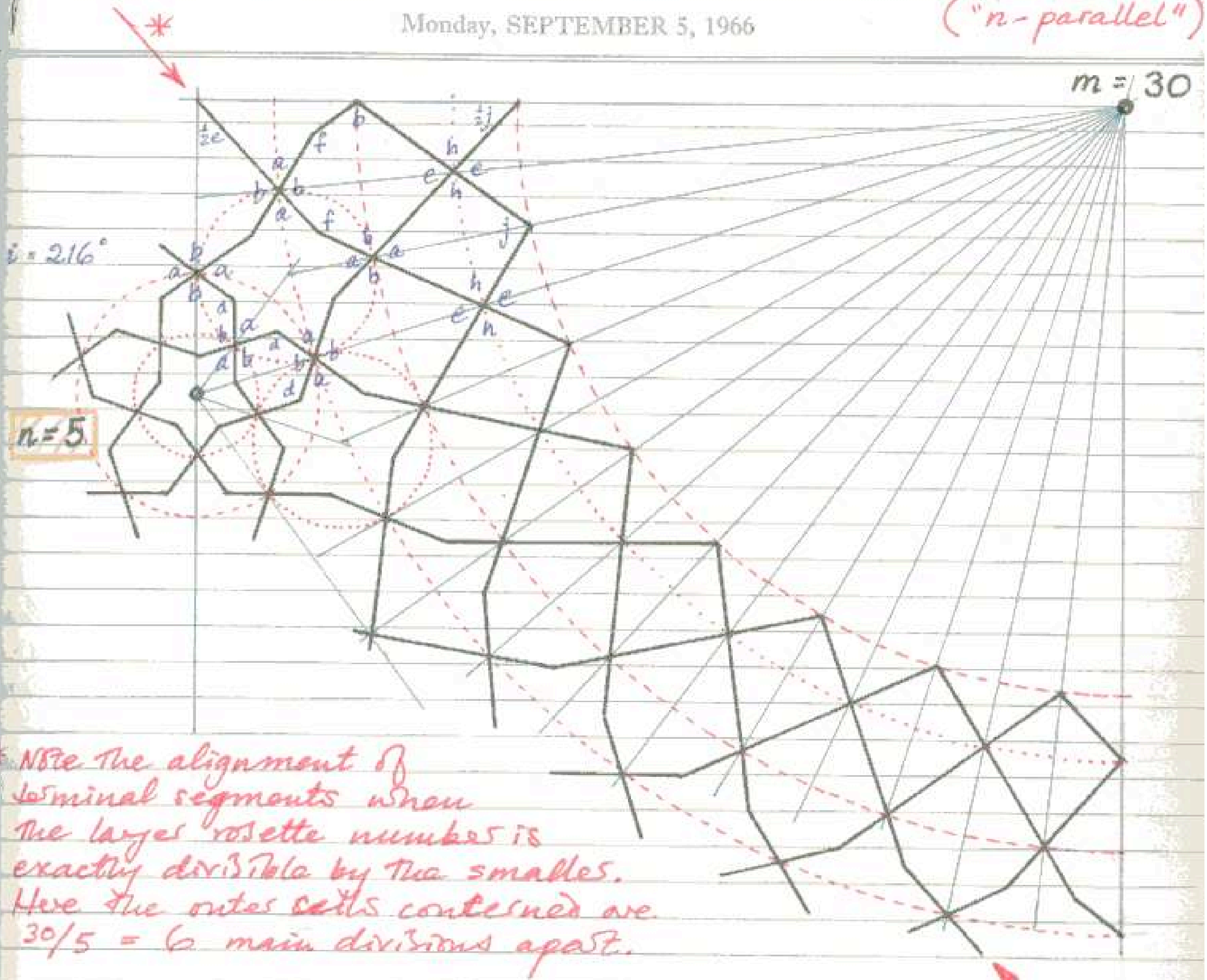
C is the (3x2) rhomb centre
D is an outer point of the m-rosette
E is an outer point of the n-rosette
F is the centre of the peripheral circle

In the primary standard construction method for (3x2) type II patterns, i.e. when the smaller rosette for m>n is drawn with parallel sides and collinear terminal segments*, it is found that the inner points of the interstitial cells, in addition to congruence of these cells with the m-outer cells, meet at the centre of the rhomb. In cases where m<n, if the primary standard construction is used, i.e. the smaller m-rosette has parallel sides and collinear terminal segments*, the interstitial cells no longer meet at the rhomb centre (see p. 203, (3x2)9,12/II). In this case, if the interstitial cells are made to meet at the rhomb centre, then the m-rosette will have obtuse terminal segments and divergent sides (pp. 3, 4). Such a construction for (3x2)9,12/II is known from Cairo; see p. 212.

* i.e. "n-parallel"   * i.e. "m-parallel"

Fri 15 Feb 1985

(3x2) Type II m > n | 208
Primary STANDARD CONSTRUCTION
("n-parallel")



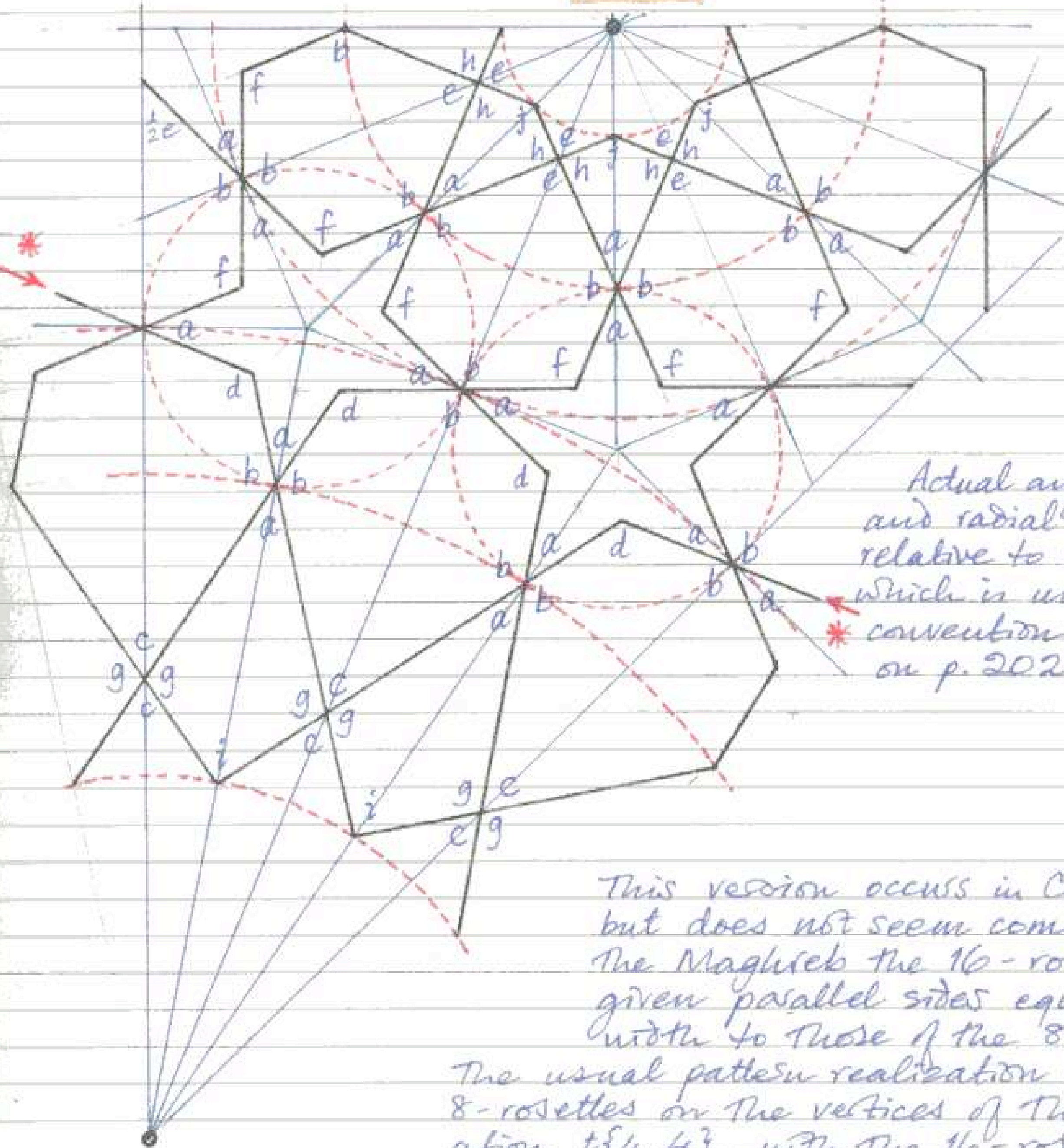
* Note the alignment of terminal segments when the larger rosette number is exactly divisible by the smaller. Here the outer cells concerned are 30/5 = 6 main divisions apart.

This rhombus does not appear to be used in authentic Islamic ornament, and would be difficult to incorporate in a repeating pattern. In any case it is unsatisfactory as it stands; the peripheral stars are very ugly, and there is too much disparity in size between the m- and n-outer cells.

209 (3x2) Type II "m - parallel" Primary STANDARD CONSTRUCTION

RH Fri 15 Feb 1985

m = 8



Actual angular values and radial lengths, relative to rhomb edge which is unity by convention, are shown on p. 202.

n = 16

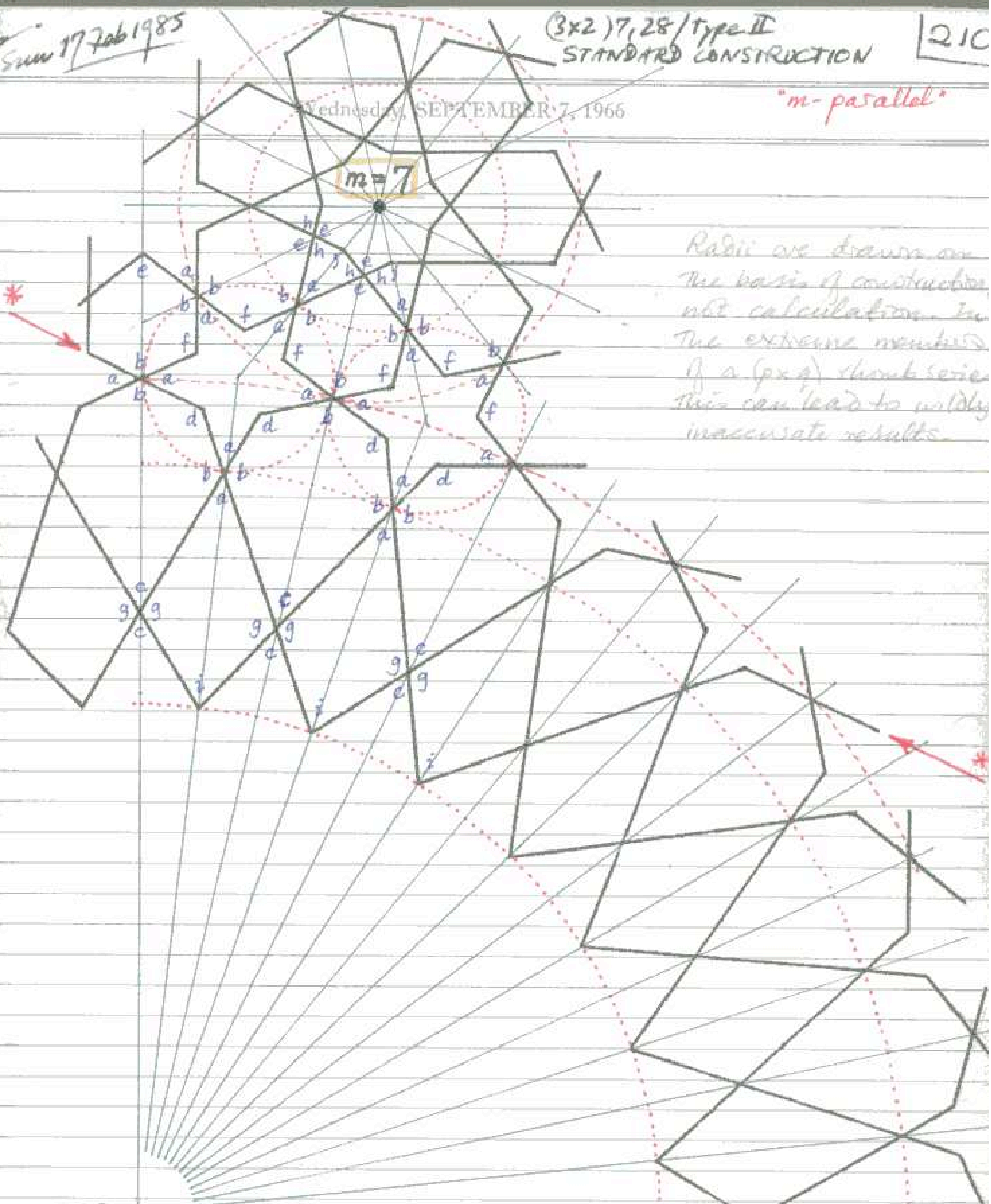
This version occurs in Central Asia, but does not seem common. In the Maghreb the 16-rosettes are given parallel sides equal in width to those of the 8-rosette. The usual pattern realization has the 8-rosettes on the vertices of the tessellation t{4,4}, with the 16-rosettes centred on the octagons. Thus, the (3x2) rhombus is used with the kite [2x5x4]4,8,16. (cf. pp. 93-96).

* Note alignment of outer segments - see note on p. 208

Sun 17 Feb 1985

(3×2)7,28/Type II
STANDARD CONSTRUCTION

"m-parallel"



Radii are drawn on the basis of construction not calculation. In the extreme members of a (p×q) rhomb series this can lead to wildly inaccurate results.

* Note alignment of outer segments – see note on p. 208

(3×2) Type II STANDARD CONSTRUCTION

After Sun 17 Feb 1985

In the usual version of the "standard" type II construction the smaller rosette is drawn with parallel sides and collinear terminal segments, and this realization is termed m- or n-parallel, according to which centre has the parallel sided rosette. However, this construction is not obligatory, and the version shown opposite (p. 212) shows the larger rosette with parallel sides! This is an authentic version from Cairo, on bronze doors to the Khāngā and Mausoleum of Baibars al-Gāshankīr (1306–10). Measurements on photos (in Creswell, Musl. Arch. Egypt, vol II Pl. 95) agree remarkably well with the theoretical construction shown opposite, suggesting that the same construction was used. It seems likely that the reason for the appearance of the divergent-sided 9-rosette was an initial desire to have the interstitial cells meet at the centre of the rhomb. This in turn suggests that the standard construction was being rigidly followed, leading to the divergent sided 9-rosettes, which type appears usually to be avoided.

A summary of the results of choosing the m- or n-parallel version according to whether <u>m</u> is greater, lesser or equal to <u>n</u> is given below.

Summary of (3×2) Type II Standard Constr.

	$m < n$ (7,28 8,16 9,12)	$m = n$ (10,10)	$m > n$ (12,8 14,7 18,6 30,5)
m-parallel	n-convergent central cells* do <u>not</u> meet at rhomb centre	n-parallel also central cells* meet at rhomb centre	n-divergent central cells* overlap at rhomb centre
n-parallel	m-divergent central cells meet at rhomb centre	m-parallel also central cells meet at rhomb centre	m-convergent central cells meet at rhomb centre

* "central cells" = interstitial cells.

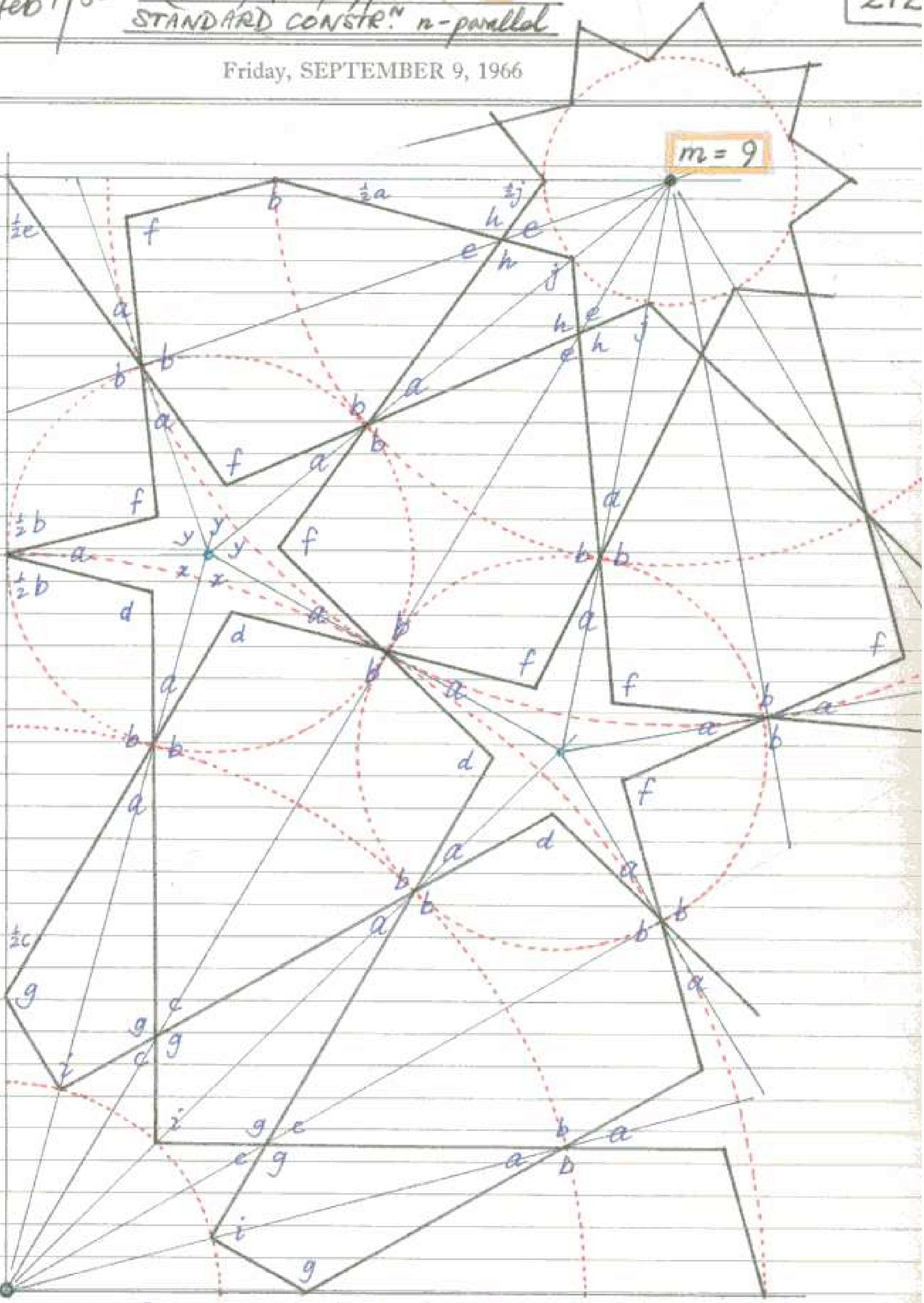
Sun 17 Feb 1985 (3×2)9,12/Type II m < n
STANDARD CONSTR.N n-parallel

G = 0.0938
H = 0.1618

a = 30°
b = 150°

j = 110°
i = 90°

Angle between divergent sides of 9-rosette is 10°.

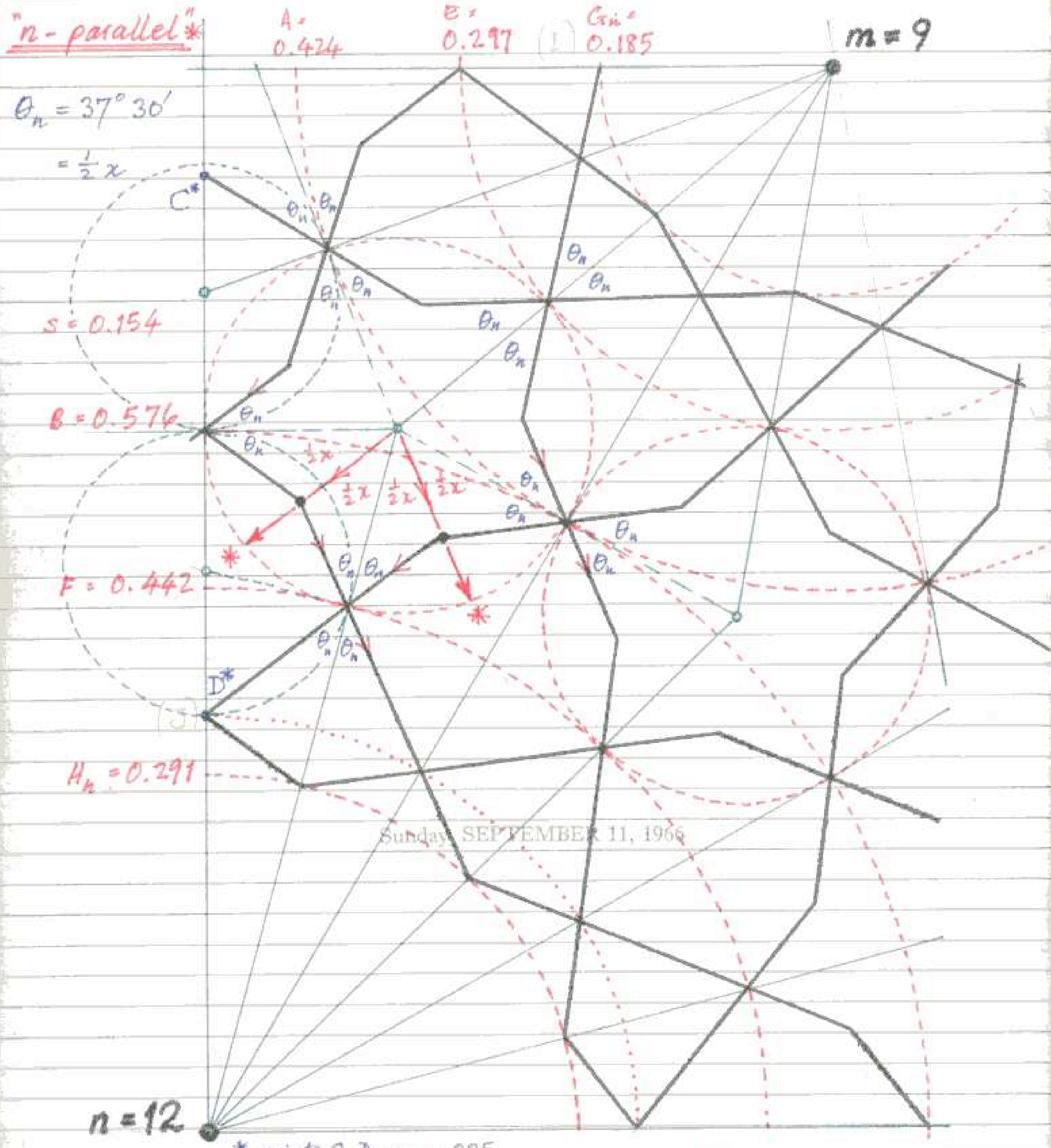


Cairo: Bronze plated doors of Khānqā & Mausoleum of Baybars al-Gāshankīr (1306-10).

213 | $(3\times 2)\,9,12/\mathrm{III}$ n-parallel
STANDARD CONSTRUCTION

Wed 20 Feb 1985

Saturday, SEPTEMBER 10, 1966



Sunday, SEPTEMBER 11, 1966

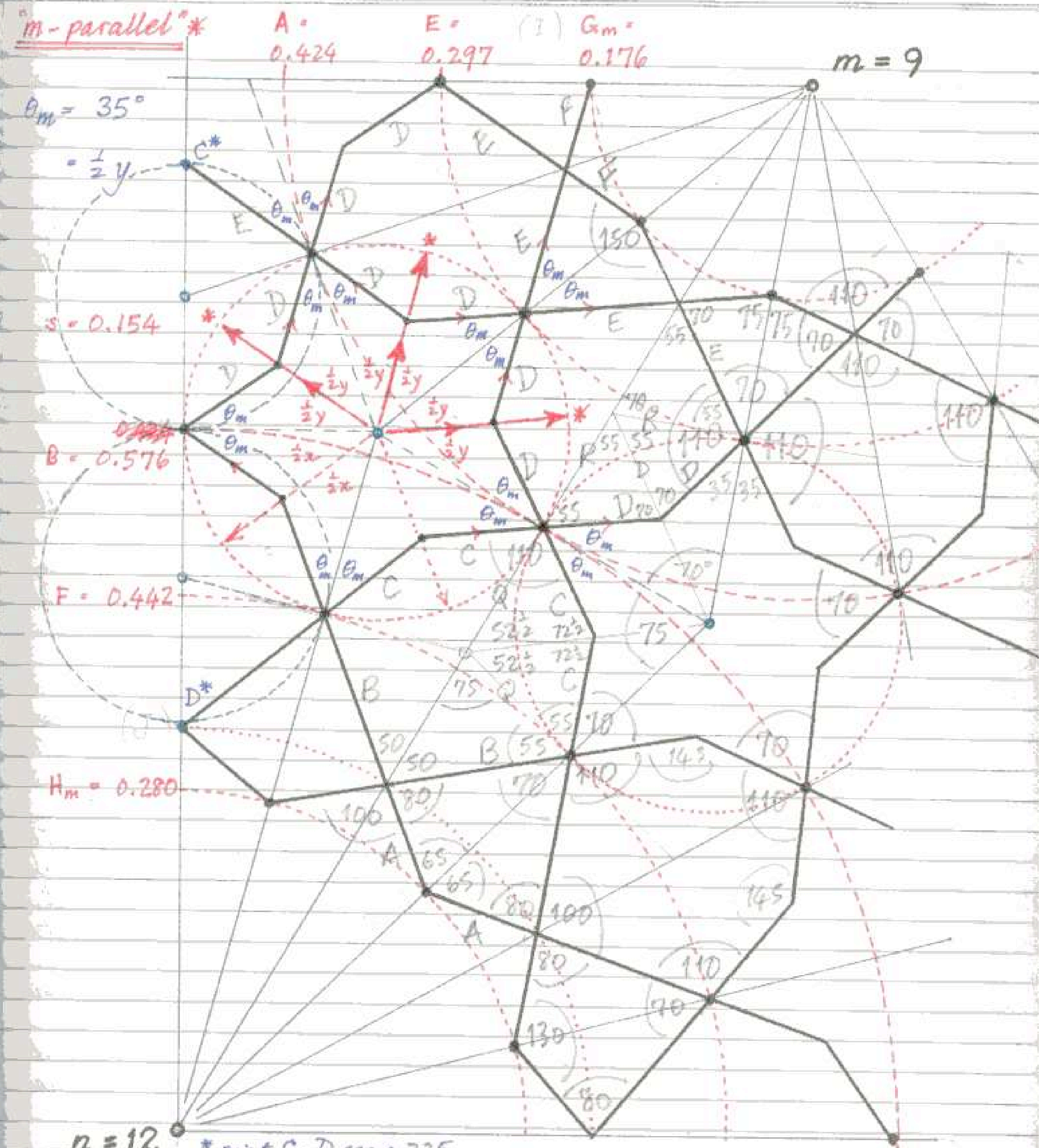
* points C, D see p. 225

* sides of peripheral stars parallel to x-bisectors

Wed 20 Feb 1985

(3x2) 9,12/III m-parallel

STANDARD CONSTRUCTION



* point C, D see p. 225

* sides of peripheral stars parallel to y-bisectors.

(3X2) Type III
STANDARD CONSTRUCTION

A Plam Wed 20 Feb 1985

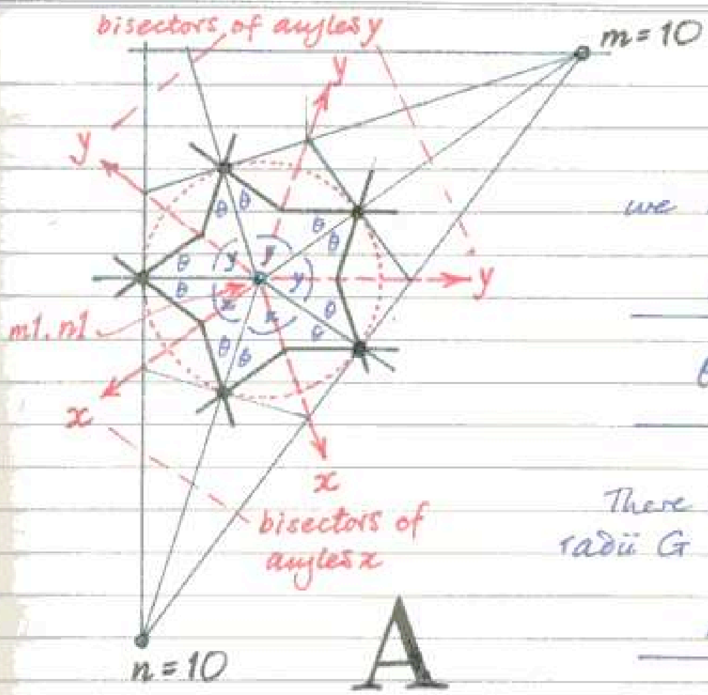
In type III patterns, as in type II, there are peripheral stars the vertices of which lie on the midpoints of the sides of peripheral pentagons, and on the circumference of peripheral circles (the latter determine the properties of the peripheral pentagons). In type III patterns the peripheral stars have their sides parallel in pairs. In the central member of the (3X2) rhomb series, namely (3X2)10,10/III, pairs of sides are parallel to the three bisectors of angles y and two bisectors of angles x (see figure opposite). However, when $m \neq n$ we find, if the standard peripheral circle construction is used, that the sides of the peripheral stars cannot be made parallel to both the y-bisectors and the x-bisectors simultaneously, but that in each case they can be drawn parallel to either the y-bisectors or the x-bisectors. The first case will be termed m-parallel, the second n-parallel. The central 10,10 can be made both m- and n-parallel simultaneously. When $m \neq n$, the remaining, non-parallel sides of the peripheral stars are either convergent or divergent.

Note that the terms m- or n-parallel in type II refer to the rosettes, whereas in type III they refer to the peripheral stars.

When $m = n$, angles x and y are equal, but when $m \neq n$, then $x \neq y$. Values of these angles are given in the lower table on p. 202; these remain constant for each specific rhomb in types I, II and III.

For m-parallel type III rhombs angle θ (see top left diagram on p. 216) is equal to $\frac{1}{2}y$ throughout the peripheral star. For n-parallel type III rhombs $\theta = \frac{1}{2}x$ throughout. Angle θ is labelled θ_m or θ_n respectively, in these two cases.

Since types II and III are special cases of the same infinite series, radii S, A, B, E and F are identical in both types, and indeed for every variety in the whole series. Radii G and H are variable throughout the infinite series though each is dependent on the value of θ. Both of these radii, G & H, also vary according to whether



A

When $m = n = 10$ then $\theta = \frac{1}{2}x = \frac{1}{2}y = 36°$

However, when $m \neq n$, $x \neq y$, and we must distinguish two cases:—

m - parallel	n - parallel
$\theta_m = \frac{1}{2}y$	$\theta_n = \frac{1}{2}x$

— see p. 218 for values.

There are similarly two cases each for radii G and H in type III rhombs:—

m - parallel	n - parallel
$G_m = \dfrac{E \sin \theta_m}{\sin \alpha}$	$G_n = \dfrac{E \sin \theta_n}{\sin \beta}$
$H_m = \dfrac{F \sin \theta_m}{\sin \gamma}$	$H_n = \dfrac{F \sin \theta_n}{\sin \delta}$

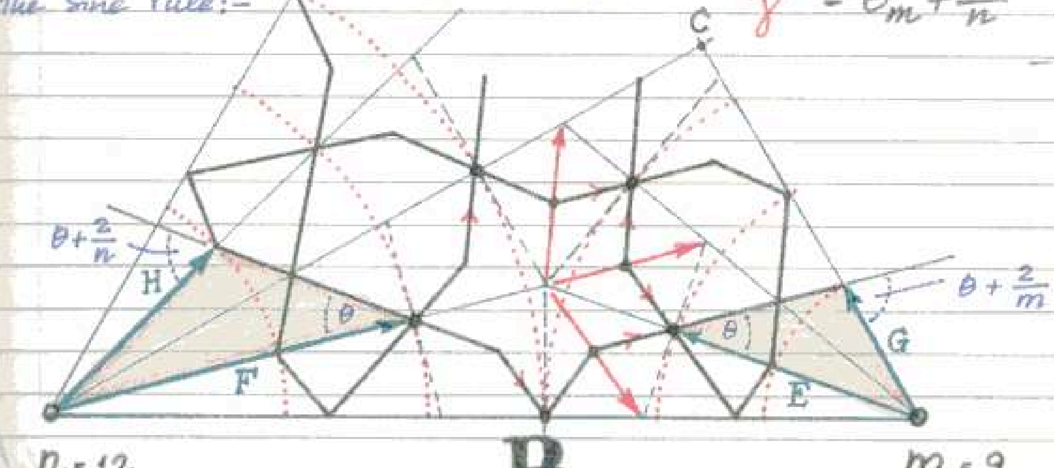
Formulae for G and H can be read off from the diagram below in the coloured triangles, using the sine rule:—

where

$\alpha = \theta_m + \frac{2}{m}$ $\quad \beta = \theta_n + \frac{2}{m}$

$\gamma = \theta_m + \frac{2}{n}$ $\quad \delta = \theta_n + \frac{2}{n}$

— see p. 218 for values



B

The example shown here is 9,12/III m-parallel

(3x2) Type III Standard Construction

Wed 20 Feb '85

The m- or n-parallel case is chosen, when m ≠ n. Formulae for the calculation of the G and H radii for type III rhombs are given on p. 216, and their derivation is easily seen by reference to fig. B on p. 216.

Angle θ_m in the m-parallel type III rhomb is equal to $\frac{1}{2}a$ in type I rhombs (see table on p. 74), so the outer stars of the type III m-parallel rosettas are identical to the stars of type I. Only the m-parallel outer stars of type III will come up to form a type I pattern since only in this variety is θ_m equal to the mean of $2/m$ and $2/n$.

Summary of (3x2) Type III features:—

	$m < n$	$m = n$	$m > n$
m-parallel	n-divergent	n-parallel also	n-convergent
n-parallel	m-convergent	m-parallel also	m-divergent

In type III the m- & n-outer cells have $\boxed{\text{3 or 4 points}}$ lying on circles (not including their "shoulder" points) as follows:—

		$m < n$	$m = n = 10$*	$m > n$
m-parallel	m-cells →	4 on circle	4 on circle	4 on circle
	n-cells	3 on circle 1 OUTSIDE	4 on circle	3 on circle 1 INSIDE
n-parallel	m-cells	3 on circle 1 INSIDE	4 on circle	3 on circle 1 OUTSIDE
	n-cells →	4 on circle	4 on circle	4 on circle

* When m=n=10 pattern is simultaneously m- & n-parallel.

Wed 20 Feb 1985

Calculated Values for Radii & Angles of Type III rhombs (Standard Constr.ⁿ)

including G & H radii for Type II rhombs.

		7,28	8,16	9,12	10,10	12,8	14,7	18,6	30,5		
	s	0.0913	0.1344	0.1543	0.1625	0.1627	0.1549	0.1351	0.0918	s	
m	A	0.1896	0.3244	0.4240	0.5	0.6072	0.6784	0.7660	0.8736	A	m
m	E	0.1191	0.2168	0.2969	0.3633	0.4659	0.5410	0.6428	0.7866	E	m
n	B	0.8104	0.6756	0.5760	0.5	0.3928	0.3216	0.2340	0.1264	B	n
n	F	0.7242	0.5544	0.4420	0.3633	0.2625	0.2020	0.1351	0.0644	F	n
Type II	G	0.0530	0.0898	0.1173	0.1388	0.2247	0.3003	0.4195	0.6222	G	Type II
Type II	H	0.5040	0.3001	0.1973	0.1388	0.1087	0.0899	0.0675	0.0398	H	Type II
Type III	G_m	0.0638	0.1228	0.1763	0.2245	0.3070	0.3744	0.4771	0.6506	G_m	III
Type III	H_m	0.5449	0.3708	0.2797	"	0.1612	0.1277	0.0882	0.0472	H_m	III
Type III	G_n	0.0795	0.1378	0.1851	"	0.2886	0.3400	0.4195	0.5675	G_n	III
Type III	H_n	0.5917	0.3982	0.2912	"	0.1487	0.1081	0.0675	0.0296	H_n	III
	θ_m	32° 8′.57	33° 45′	35°	36°	37° 30′	38° 34′.29	40°	42°	θ_m	
	θ_n	41° 47′.14	39° 15′	37° 30′	36°	33° 45′	32° 8′.57	30°	27°	θ_n	
	α	83° 34′.29	78° 45′	75°	72°	67° 30′	64° 17′.14	60°	54°	α	
	β	93° 12′.86	84° 15′	77° 30′	72°	63° 45′	57° 51′.43	50°	39°	β	
	γ	45°	56° 10′	65°	72°	82° 30′	90° 25′.71	100° (80°)	114° (66°)	γ	
	δ	54° 38′.57	61° 45′	67° 30′	72°	78° 45′	83° 34′.29	90°	99° (81°)	δ	

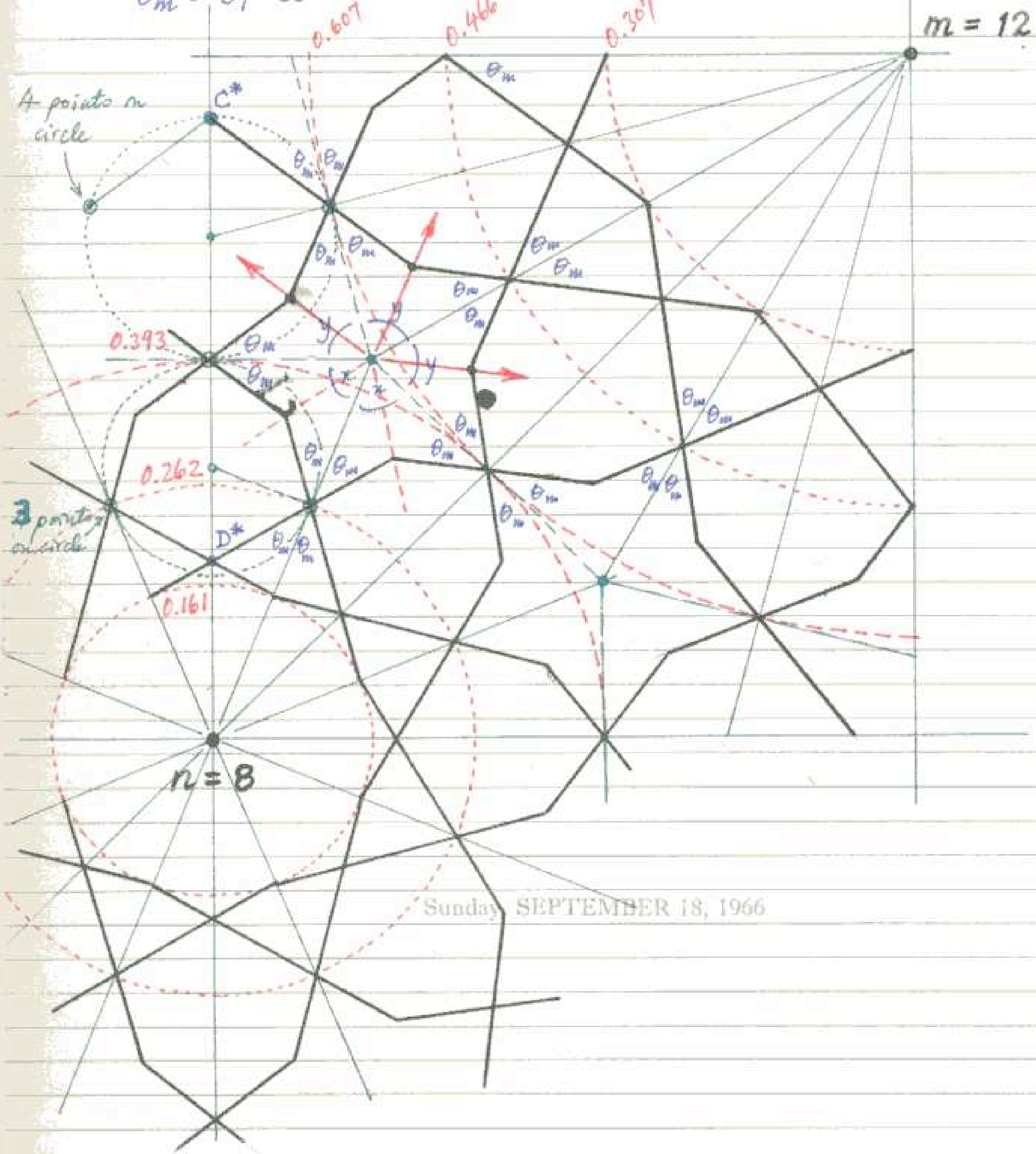
219 (3x2) Type III / 12, 8
STANDARD CONSTRUCTION

Thu 21 Feb 1985

"m - parallel"

Saturday, SEPTEMBER 17, 1966

$\theta_m = 37° 30'$



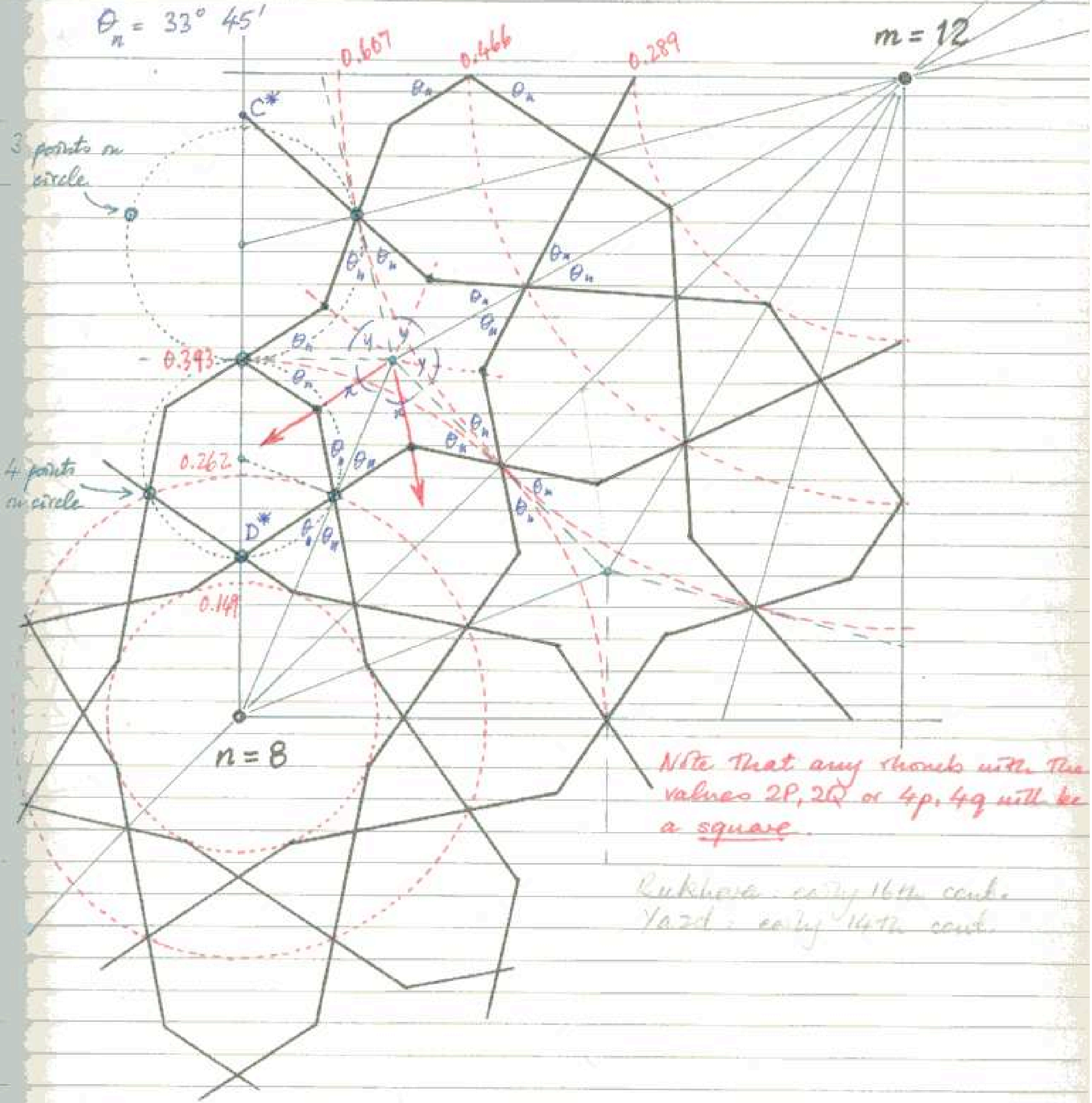
Sunday, SEPTEMBER 18, 1966

* points C, D see p. 225

(3x2)12,8/III

STANDARD CONSTRUCTION

"n-parallel"



Note that any rhomb with the values 2P, 2Q or 4p, 4q will be a square.

Rukhara: early 14th cent.
Yazd: early 14th cent.

*points C, D see p. 225

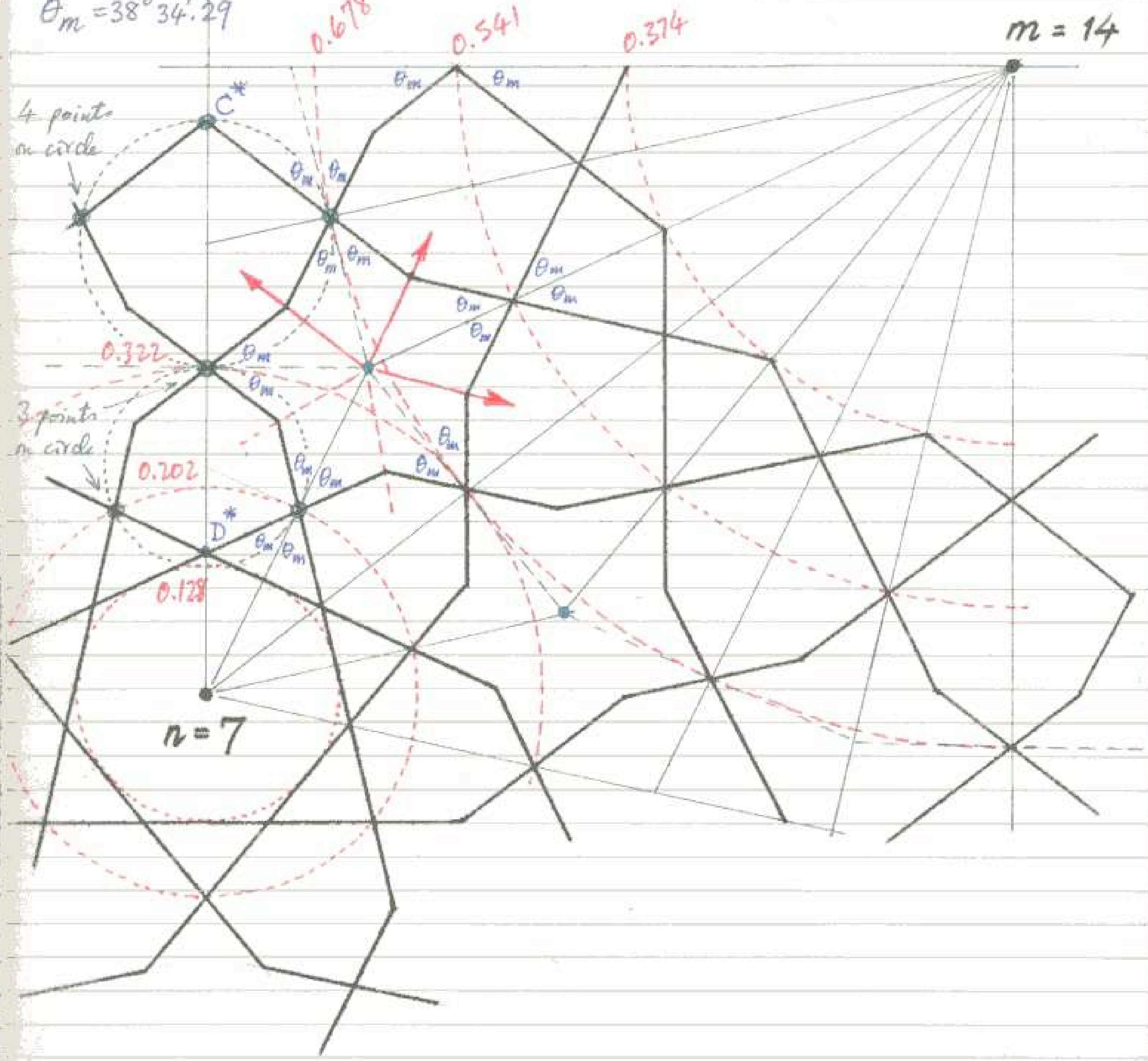
221 $(3\times2)14,7/\text{III}$

STANDARD CONSTRUCTION

Thu 21 Feb 1985

"m – parallel"

$\theta_m = 38°\ 34'.29$



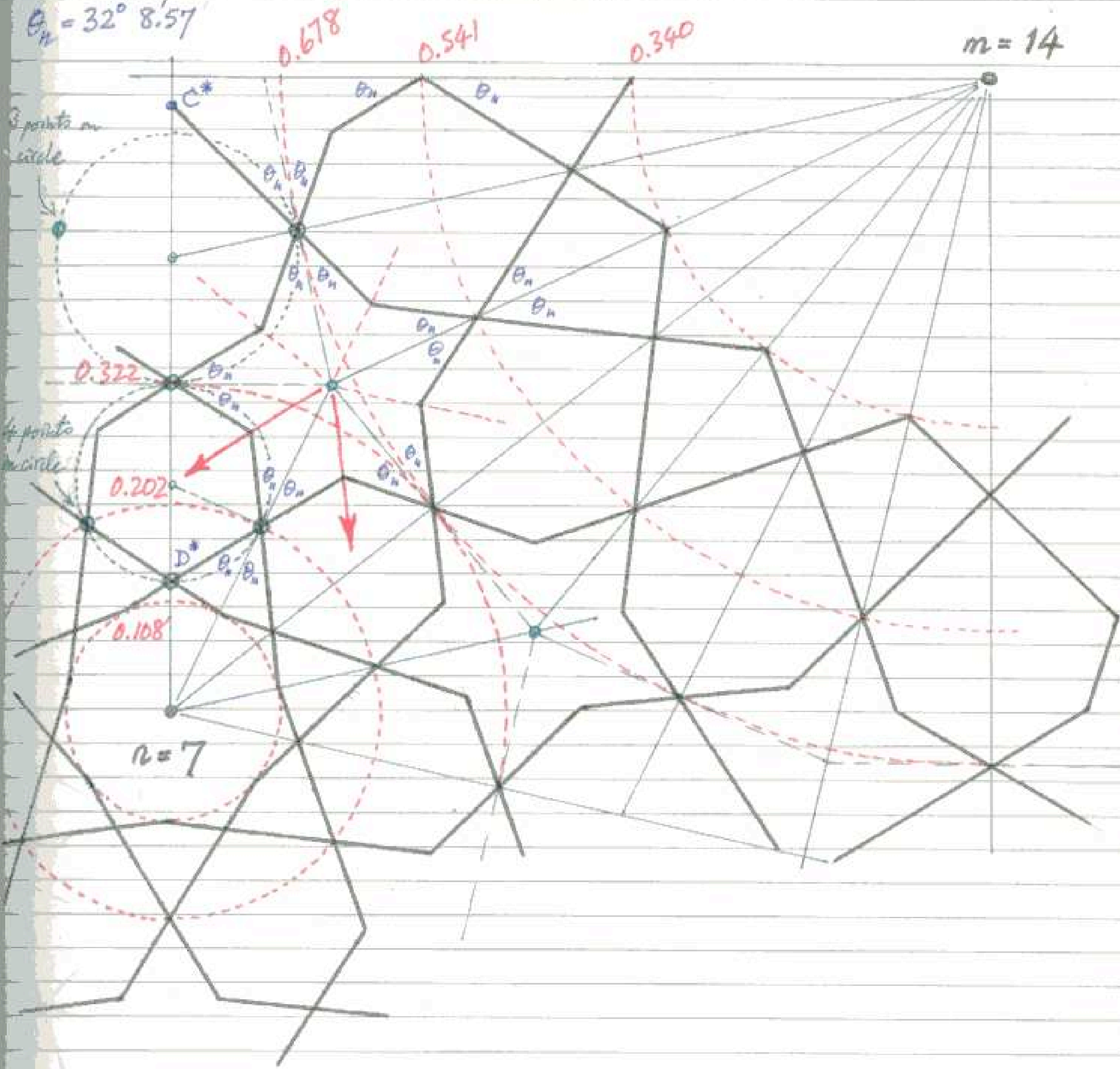
* points C, D see p. 225

STANDARD CONSTRUCTION

"n-parallel"

Wednesday, SEPTEMBER 21, 1966

$\theta_n = 32° \, 8.57'$



* points C, D see p. 225

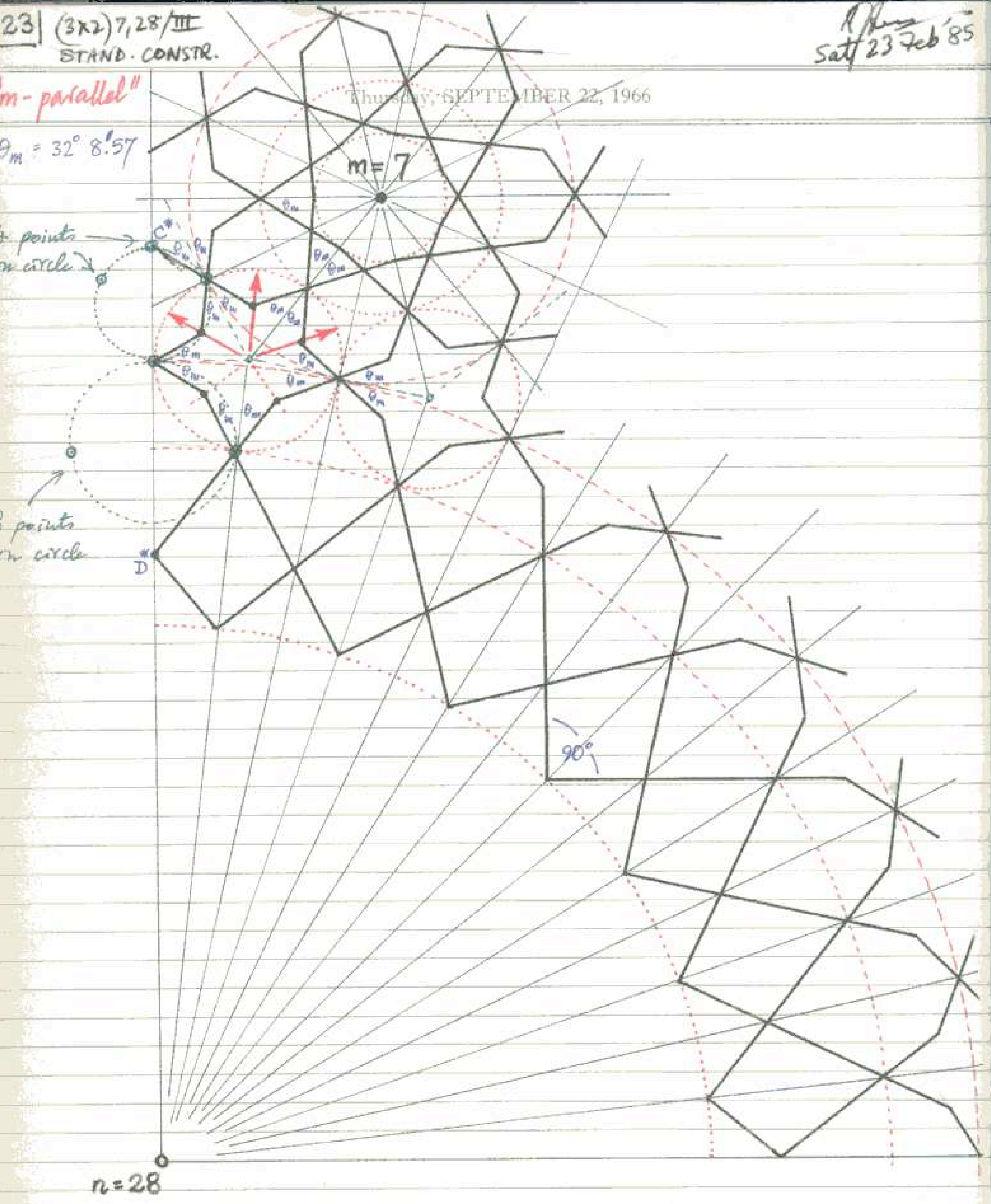
23| (3x2)7,28/III
STAND. CONSTR.

Sat 23 Feb '85

Thursday, SEPTEMBER 22, 1966

"m-parallel"

$\theta_m = 32° 8'.57$

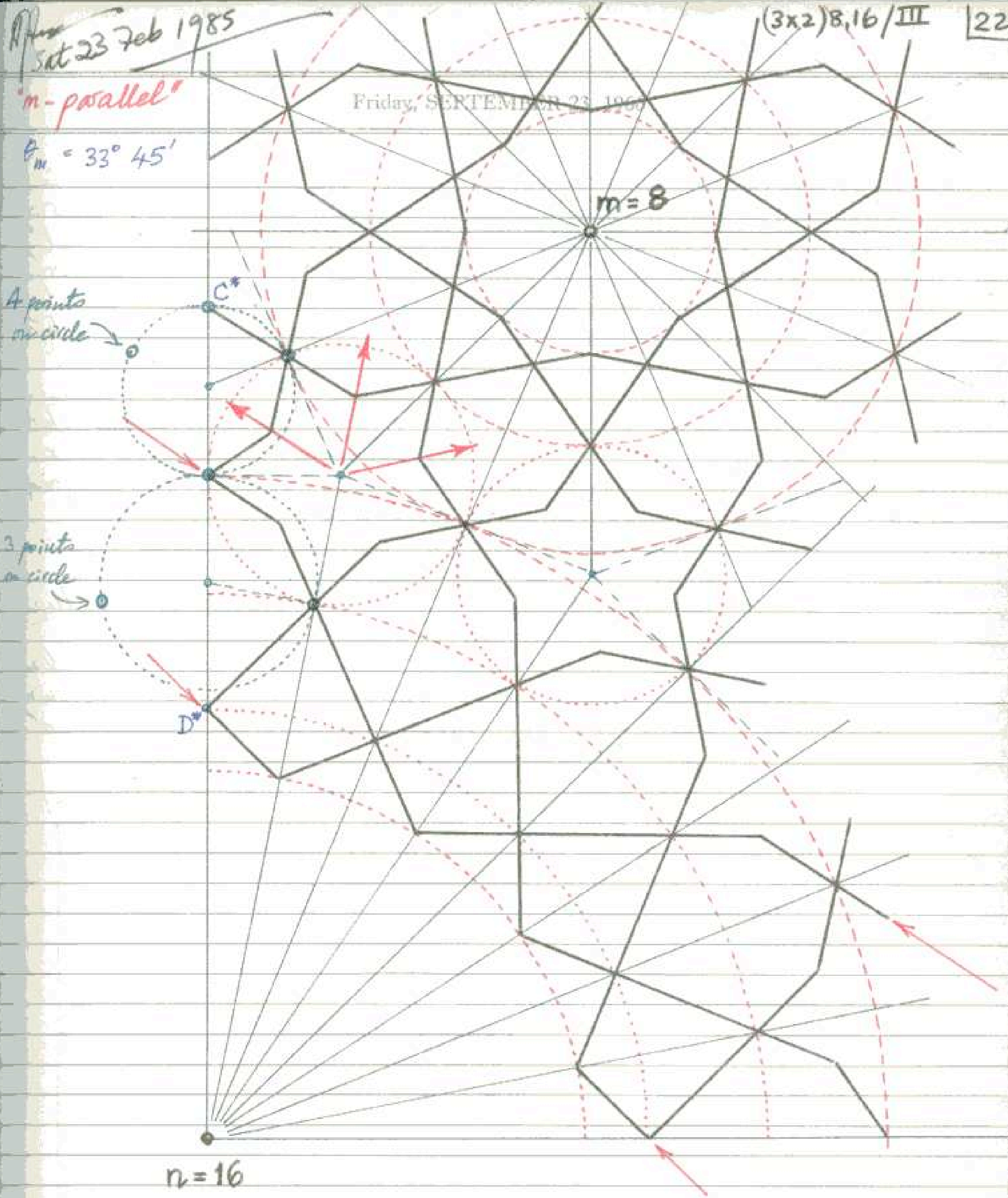


*points C, D see p. 225

Sat 23 Feb 1985

"m-parallel"

$\theta_m = 33° 45'$



* points C, D see p. 225

(3x2) Type III Patterns

RPh Sat 23 Feb 1985

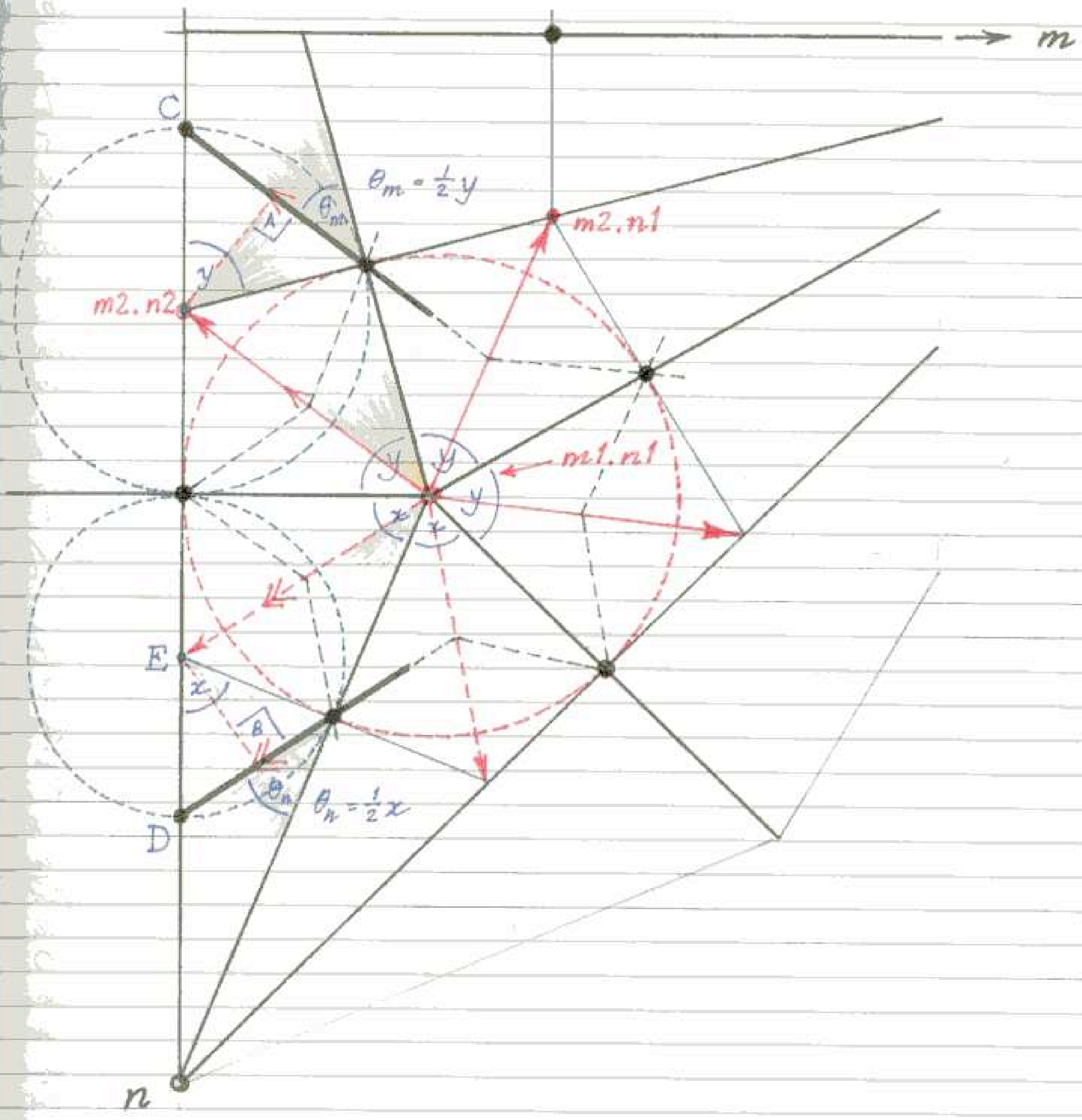
The defining characteristic of type III patterns in (3x2) rhombs is that the sides of the peripheral stars are parallel either to the y-bisectors (= "m-parallel") or to the x-bisectors ("n-parallel") at intersection m1.n1. In the case of m = n = 10 the peripheral stars can be drawn with their sides parallel to both x and y-bisectors, simultaneously, but this is not so when m ≠ n. Thus angle θ, half the angle of intersection of the sides of the peripheral stars at their vertices, is equal either to ½y or to ½x. When $\theta = \frac{1}{2}y$ then angle A is a right angle (see diagram on p. 226, opposite) and point C lies on the green circle centred on point m2.n2. When $\theta = \frac{1}{2}x$ then angle B is a right angle and point D lies on the lower green circle centred on E (the intersection of the x-bisector with radius n2).

When either of points C or D lies on its circle the other point may lie either outside or inside its own circle, and this depends on whether m is greater or lesser than n, as summarized below.

		m < n	m = n = 10	m > n
m-parallel	point C	ON circle	ON circle	ON circle
	point D	OUTSIDE circle	ON circle	INSIDE circle
n-parallel	point C	INSIDE circle	ON circle	OUTSIDE circle
	point D	ON circle	ON circle	ON circle

Sat 23 Feb 1985

(3x2) Type III
STANDARD CONSTRUCTION.



THE GENERAL TYPE I CONSTRUCTION

Wed 27 Feb 1985

Generalization of Pattern Types beyond the (3x2) = [6x4] rhomb series

In previous attempts to classify (3x2) rhomb pattern types at a more fundamental level (see pp. 83, 84 in this notebook) types I, II and III (revised numbering) were grouped together as a fundamental "Group A" (together with their various derivatives, such as types IV, V and VI), on the basis of sharing peripheral elements centred on intersection m1.n1 and defined by outer and re-entrant points of the boundaries of the two kinds of star motif.

However, there is a distinct difference in the formation of the peripheral elements of, on the one hand type I constructions, and on the other, types II & III constructions. In the "standard" construction we have adopted for types II and III, the peripheral stars are formed within "peripheral circles", which determine the radii of the outer stars of both m- and n-rosettes (see p. 199 et seq.). In the case of type I patterns the middle and inner radii of the m- and n-stars are not determined by the radius of the peripheral circle, but by the fact that their sides must be continuous across the gap between vertices B and C in the diagram on p. 228, opposite. The peripheral pentagons of type I patterns are not completely circle-inscribed in the general case, although three vertices, B, C and A in the diagram, always lie on the peripheral circle, i.e. the in-circle of triangle MNE, which is the 2nd collateral triangle as previously defined. The (3x2) rhomb series constitutes a special case: here point F additionally always lies on the peripheral circle, but not point G, except when $m = n = 10$. The latter forms a unique case, where the peripheral pentagons are completely regular and therefore circle-inscribed. In this same series, when $m > n$ point G lies inside the peripheral circle whereas when $m < n$ point G lies outside the peripheral circle. (It is important to notice that in the diagram opposite M and N are arbitrary labels, and only attain the significance usually attributed to them in these notes when the line MN is the side of a $(p \times q) = [P \times Q]$ rhombus.)

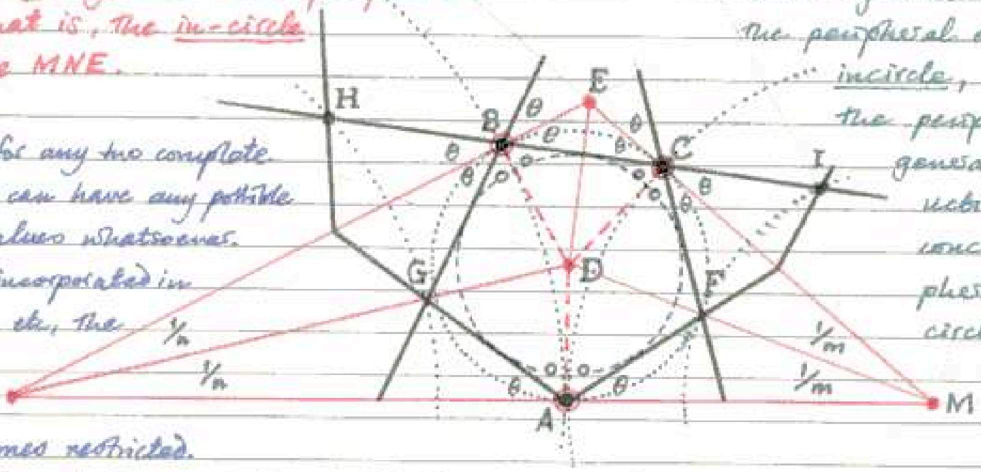
Wed 27 Feb 1985

Definition and Proof.

Points ● always lie on the peripheral circle, that is, the in-circle of triangle MNE.

For the generalized Type I construction the peripheral element has a single incircle, which is not so for the peripheral stars of the generalized Type II/III construction. This incircle is concentric with the peripheral circles, i.e. the incircle of triangle MNE.

Note that for any two complete stars M, N can have any possible integral values whatsoever, but when incorporated in a rhombus etc, the number of possible values becomes restricted.



Two stars, centred on M, N with M and N points respectively, and sharing an outer point A such that M, A and N are collinear points. Points A, B are two outer points of the N-star, repeated regularly round point N at angles of $2/n$, so that AN = BN and so on for all remaining radii. Similarly points A, C are two outer points of the M-star, repeated regularly round point M at angles of $2/m$, so that AM = CM. The angles at the vertices of both M- and N-star are equal to 2θ, that is, opposite angles at all crossovers are equal. The slope of the lines through the vertices is determined by the requirement that points H, B, C and I are collinear, which is the defining characteristic of Type I patterns in general. Or, more simply by joining vertices B and C and extending this line at each end.

In ΔMNE, D is the intersection of the bisectors of angles M and N; it is therefore obvious that DE is the bisector of angle E.

Join DA, DB and DC - - - - .

Triangles BND, AND are congruent (2 sides and included angle) therefore BD = AD;
Triangles CMD, AMD are congruent (2 sides and included angle) therefore CD = AD;
therefore BD = AD = CD

Triangle BCD is therefore isosceles, and ∠CBD = ∠BCD. But because of congruence of triangles BND, AND, ∠CBD = ∠FAD. Similarly, ∠BCD = ∠GAD. Therefore all angles marked ° are equal, and DA, DB and DC are perpendicular to sides MN, EN and EM respectively, in triangle MNE.

It follows that the relative radii of two stars linked by a type I construction are identical to the relative inradii of two regular polygons sharing an edge.

Note that the above construction and proof are valid whether or not M, N are integral. For example, the number of cases when point F (near M) also lies on the peripheral circle is infinite, though all lie on the curve for the (3×2) rhomb series.

GENERALIZED TYPE I CONSTRUCTION

Wed 27 Feb 1985

There is a simple relationship by means of which one can tell whether points F & G (p. 228 and figs. A, B opposite) lie on, inside or outside the peripheral circle, that is, the in-circle of the <u>second collateral triangle</u>

On p. 67 we noted that angle $\theta = \frac{1}{m} + \frac{1}{n}$ in <u>all</u> type I constructions.
From the same source we note the angle $\phi = \frac{1}{2} - \theta - \frac{1}{m}$
$= \frac{1}{2} - \frac{2}{m} - \frac{1}{n}$.

Angle ϕ is equal to, smaller than, or greater than θ. Clearly, when $\theta = \phi$, point F lies <u>on</u> the peripheral circle, as in the (3×2) rhomb series. In this case we have

$$\frac{1}{2} - \frac{2}{m} - \frac{1}{n} = \frac{1}{m} + \frac{1}{n}, \quad \text{that is, } \frac{3}{m} + \frac{2}{n} = \frac{1}{2}. \quad \text{.........1}$$

When $\phi < \theta$ point F is <u>inside</u> the peripheral circle. In this case we have

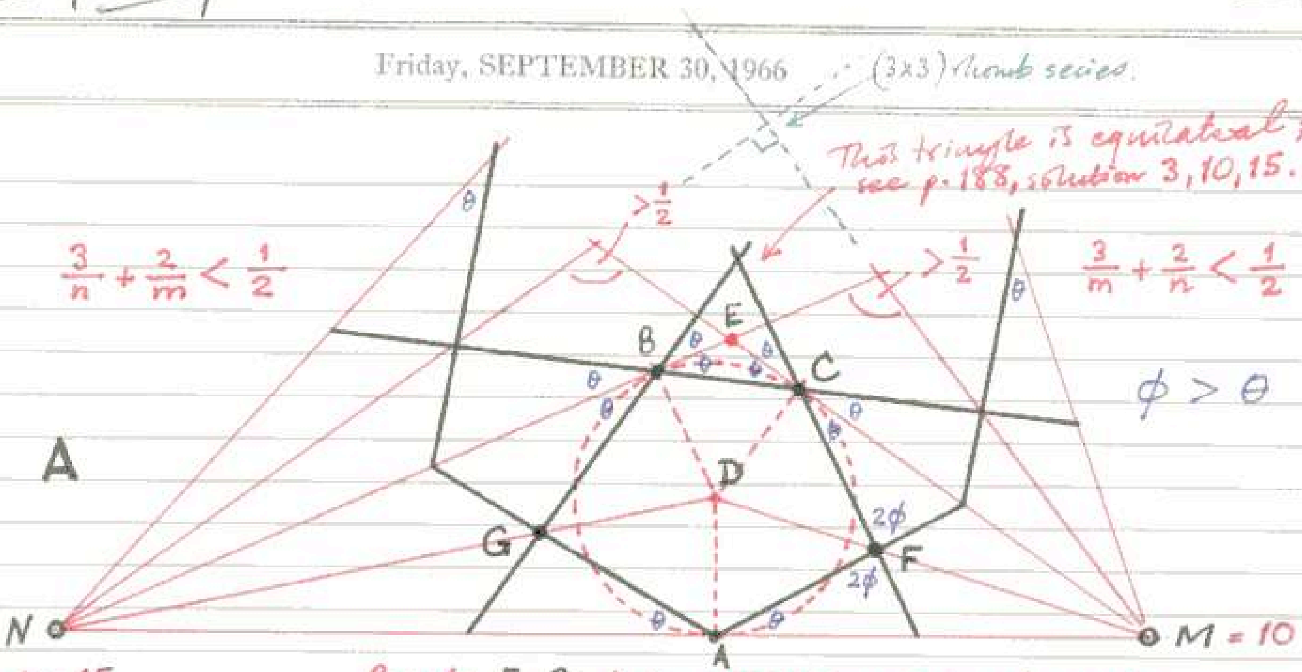
$$\frac{1}{2} - \frac{2}{m} - \frac{1}{n} < \frac{1}{m} + \frac{1}{n}, \quad \text{that is, } \frac{3}{m} + \frac{2}{n} > \frac{1}{2} \quad \text{(see fig. B)} \quad \text{.........2}$$

When $\phi > \theta$ point F lies <u>outside</u> the peripheral circle. In this case we have

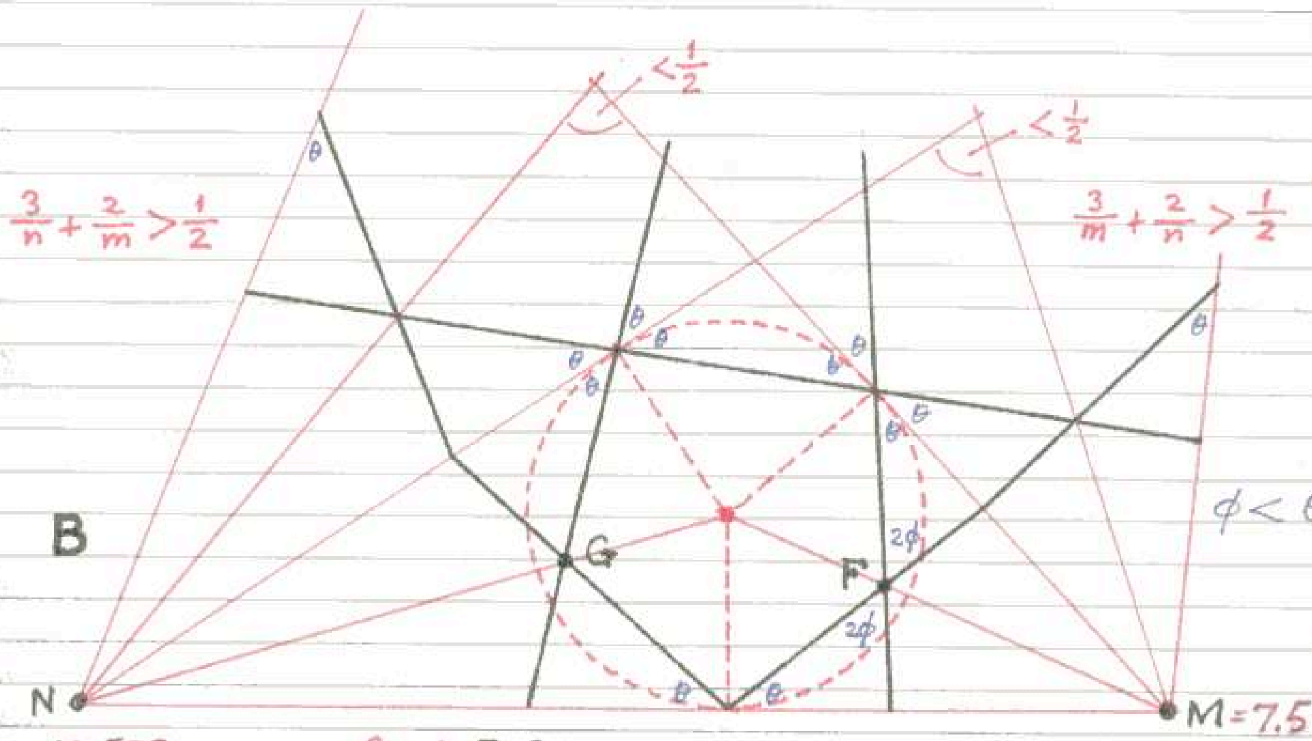
$$\frac{1}{2} - \frac{2}{m} - \frac{1}{n} > \frac{1}{m} + \frac{1}{n}, \quad \text{that is, } \frac{3}{m} + \frac{2}{n} < \frac{1}{2} \quad \text{(see fig. A)} \quad \text{.........3}$$

Similar arguments apply to point G, after reversing m and n in the three given expressions.
Obviously expression (1) above corresponds to the relation between <u>m</u> and <u>n</u> in the (3×2) rhomb series, where point F always lies on the peripheral circle. When $3/n + 2/m = 1/2$ also, <u>m</u> is equal to <u>n</u> and we have the central pair of values in the series, namely (3×2) 10, 10 / I. Thus when expression (1) holds the pair of values <u>m</u>, <u>n</u> must lie on the (3×2) curve, whether or not the values are integral. For the distribution of these properties among all possible pairs of integral values see the diagram on p. 232. →

(3x3) rhomb series



Points F, G both OUTSIDE peripheral circle.

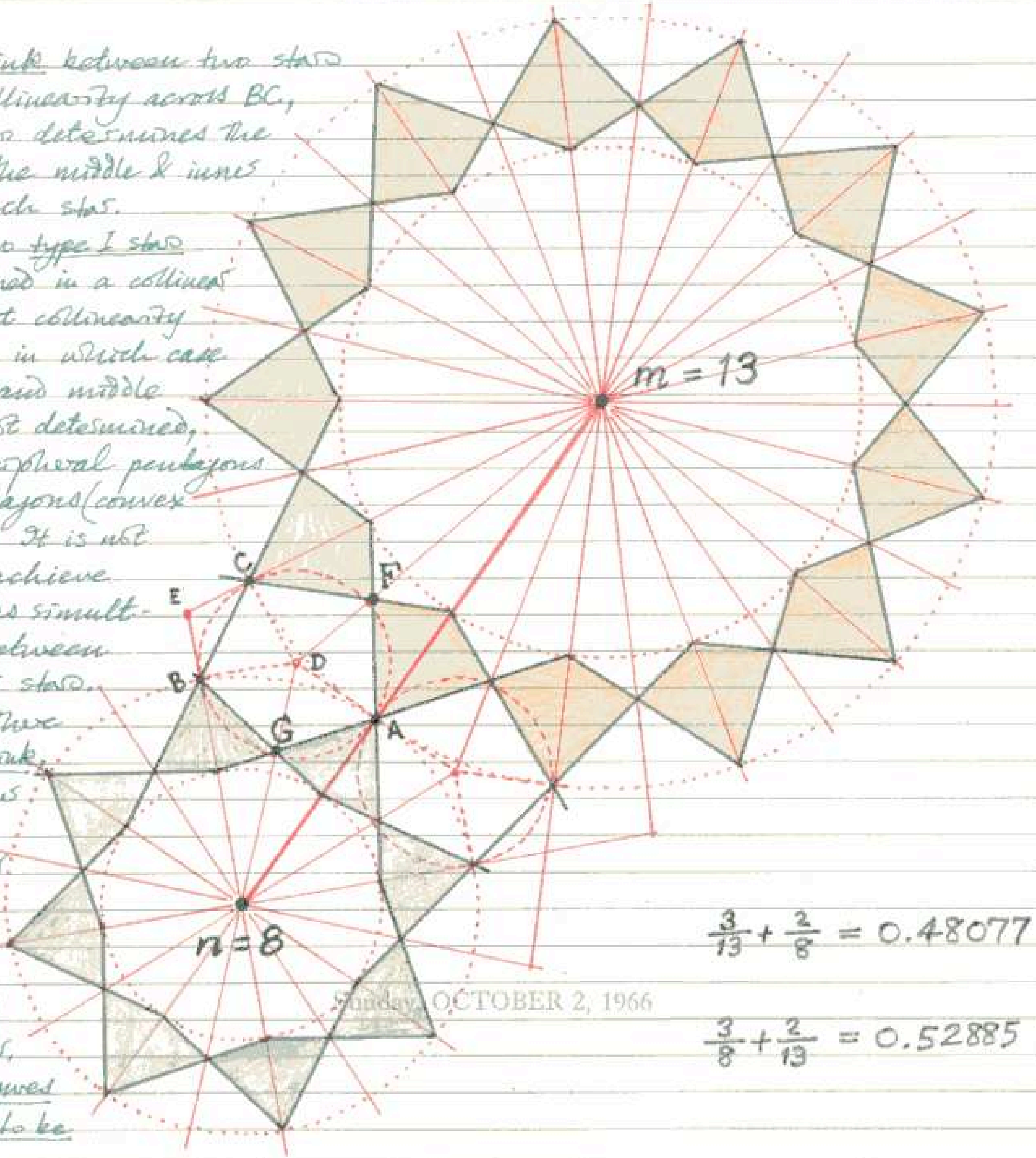


Points F, G both INSIDE peripheral circle.

PROPERTIES OF TYPE I CONSTRUCTIONS FOR VARYING STAR-NUMBERS

Thu 28 Feb 1985

A type I link between two stars requires collinearity across BC, which thus determines the lengths of the middle & inner radii of each star. However, two type I stars may be joined in a collinear link without collinearity across BC, in which case the inner and middle radii are not determined, and the peripheral pentagons become hexagons (convex or concave). It is not possible to achieve type I links simultaneously between different stars. Even when there is no type I link, once the inner and middle radii of one star are chosen this decides the lengths of the radii of the other star, if vertex figures are required to be rectangles.



$$\frac{3}{13}+\frac{2}{8} = 0.48077$$

$$\frac{3}{8}+\frac{2}{13} = 0.52885$$

One of the smallest cases in which F is outside, and G is inside the peripheral circle. This also illustrates the fact that this (and other) constructions can be achieved along a collinear link between any pair of stars, odd or even, whether or not they can form an integral rhombus. The above pair can form a non-exact $(3\times2)\overline{13,8}$ or $(3\times2)\overline{8,13}$, but no exact rhomb.

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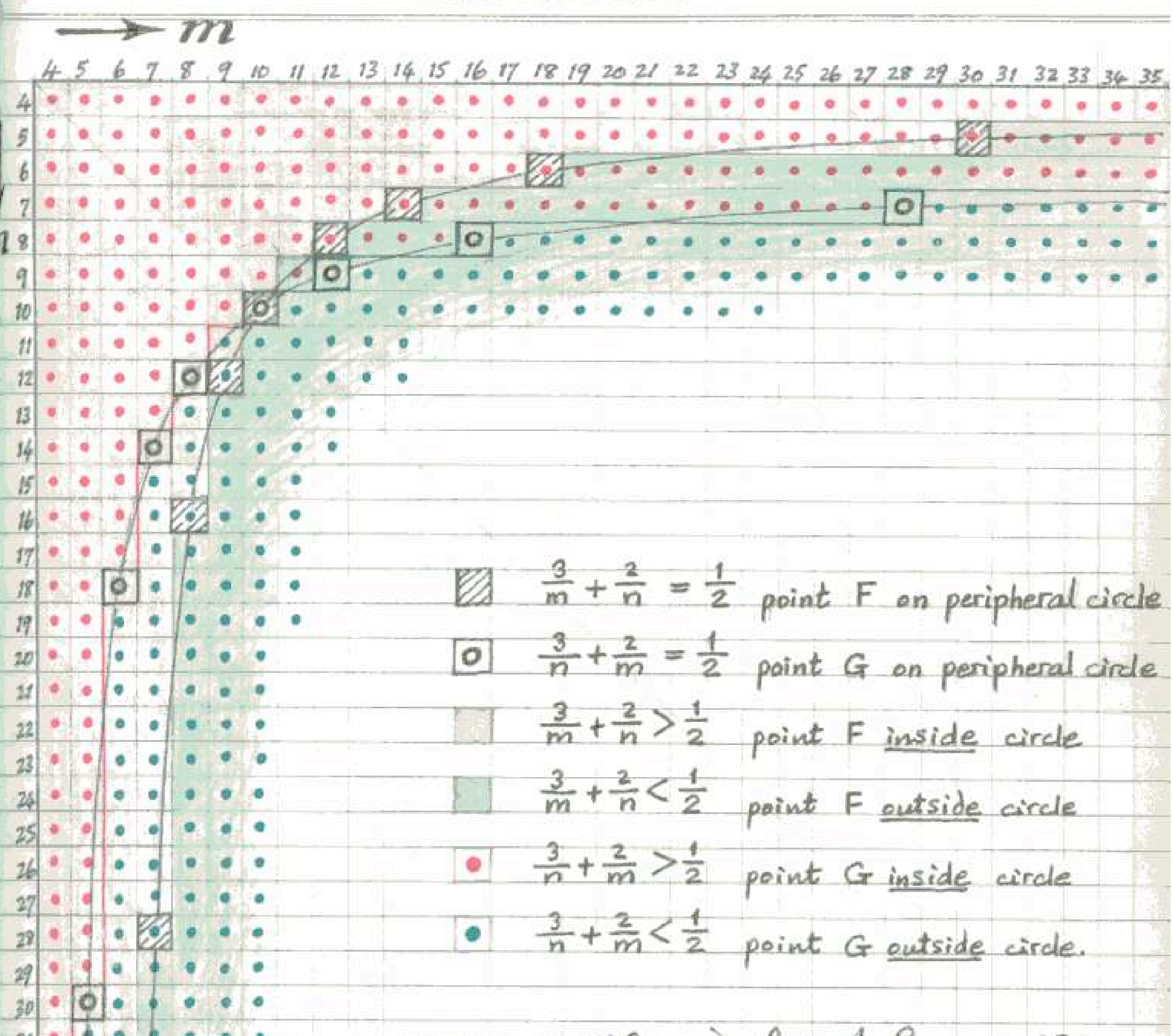

— see p. 229 and figs. A, B on p. 230 for the location of points F and G. For non-integral values the curves shown roughly indicate the boundaries between different zones of the diagram.

There is a narrow range of pairs of values for which one of points F & G lies inside and the other lies outside the peripheral circle, simultaneously. These are the pairs marked ▣ or ▣.

Pf ... Fri 1 March 1985

GENERALITIES IN A SCIENTIFIC STUDY OF ISLAMIC STAR PATTERNS

It is well known that the muslim craftsmen and artisans responsible for the development and execution of the geometrical star patterns left behind no general treatise laying down rules to be followed in their design or construction, nor did they attempt to define criteria by means of which the correctness of any individual pattern might be judged. But examination of a large body of this ornament suggests that a number of rules for construction were indeed followed, whether passed on by word of mouth (or merely by example) or even perhaps unconsciously adopted.

It has been the usual practice for centuries, at least for master craftsmen to gather together collections of working drawings of geometrical patterns for their own use, with indications of the main construction lines. A number of such collections are available for study in various modern museums, but these are no older than the sixteenth century, whereas the origin of the true star patterns goes back at least to the tenth century, about three hundred years after the birth of Islam. In most of the extant working drawings the constructions are geometrically simple, involving circles centred on the intersections of various pairs of radii emanating from the centres of the main star motifs. Many constructions are more or less specific to a single pattern, although some can clearly be generalized or adapted to include stars with different numbers of points. However, the craftsman's ability to generalize his pattern constructions in this way varied widely, and frequently depended on a deeper understanding of the underlying geometry of the construction than many craftsmen in fact possessed.

A scientific and mathematical study of Islamic star patterns, if systematically carried out, requires the precise formulation of rules for construction if one is to define generalities underlying series of related patterns. From observation of large number of authentic patterns we can discern certain consistently applied constructional procedures, or arbitrarily applied conventions which, if universally adopted enable us to develop "standard", general constructions to which we can then apply precise mathematical analyses. It must be remembered, however, that every constructional rule or procedure we may agree on will be disregarded in some authentic ornament, whether through ignorance on the part of the artisans responsible, or because aesthetic considerations (or other geometric priorities) dictated that the pattern should be drawn otherwise. Nevertheless, one can detect in all Islamic geometrical ornament a trend toward greater symmetry at all levels within each pattern, so in adopting rules which will allow our mathematical analyses greatest power and application we naturally choose or devise constructional rules which will maximize symmetry.

The most general rule clearly concerns the regularity of the motifs themselves:—

1. All n-pointed star motifs are finite and regularly formed, i.e. they possess either dihedral (D_n) or cyclical (C_n) point symmetry, or they are drawn as regularly as possible.

A geometrical analysis becomes easier if we restrict our research to rectilinear, interlacing patterns and in fact this type is by far the most common of all Islamic geometrical ornament. If we regard most Islamic geometric patterns as edge-to-edge plane tilings (not all of them are) in which three or more cells or tiles meet at each vertex, then the interlacing patterns are characterized by the fact that every vertex is 4-way, i.e. four cells and four edges meet at every vertex. Furthermore, alternate or opposite angles are equal at each vertex, that

Apply Fri 1 March 1985

is, the vertex figure at every vertex is a rectangle. This means that each vertex of the tiling becomes an intersection or crossover of a pair of straight lines. Only in the true interlacing patterns can the pattern lines be represented as interlacing bands or ribbons which weave alternately over and under one another. This is summarized in a second general rule:—

2. The vertex-figure at every vertex or crossover is a rectangle.

At the lowest level of complexity each star motif remains in contact with its nearest neighbours, that is, each nearest pair of motifs shares a common vertex. Furthermore, each pair of motifs forms a collinear link, i.e. their centres and the shared vertex lie on a single straight line. However, in some patterns the star motifs, although forming a collinear link, do not share a common vertex (consider for example, the Rpl(3×2)10,10/VIIII pattern at the bottom of page 50). The essence of a collinear link is that one radius (~~or~~ axis of symmetry) in both star motifs should lie on a single straight line.

3. At the simplest level of complexity each star motif of a neighbouring pair is orientated relative to its neighbour by the formation of a collinear link between them. — Other kinds of links are defined on p. 157 et seq..

The fact that authentic star patterns do not invariably follow rules such as those just indicated leads to the recognition of a fundamental dichotomy in the study of Islamic geometric ornament. On the one hand we can study them on a purely theoretical, idealized level, applying various rules to their construction — rules which are to some extent arbitrary, although based on close observation of general trends in the construction of authentic

In "non-exact" patterns rules (1) and (3) cannot both hold simultaneously.

examples. On the other hand we can limit our study solely to genuine patterns and their constructions as they occur in many different techniques in Islamic art. The drawback of the second approach as a sole means of study is that frequently a single pattern will have many varying constructions, although all its variations may well be topologically identical. We are thus often reduced to analysing patterns individually when there is little or no evidence of an awareness of general constructions applicable to different patterns of the same geometrical type.

Clearly the approach of the original artisans and pattern designers was to a large extent pragmatical rather than theoretical, and did not involve the application of any deep understanding of the underlying geometry of the patterns. In spite of claims that the earliest development of Islamic geometrical patterns arose through the intervention of professional mathematicians (or even that the artisans themselves acquired a knowledge of mathematics), it seems likely that in many examples of authentic Islamic ornament we are witnessing the application of what might be termed "folk-geometry", that is, practical geometrical techniques and constructions developed slowly through trial and error experimentation within a basic framework of simple motifs and schemes for their symmetrical repetition on a plane surface. As an example let us take the first rule, mentioned on p. 234, where the additional directive "or they are drawn as regularly as possible" was appended with authentic geometric patterns specially in mind. In the case of many, perhaps the majority of authentic patterns the motifs can be regarded as having perfect symmetry, in a theoretical sense, beyond the small practical

independently of

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errors inherent in any process of drawing them with less than perfect instruments. There are some patterns, however, in which perfect symmetry, in an absolute mathematical sense, of all motifs simultaneously is impossible, if we assume that all lines are strictly collinear. For example, the centres of three different star motifs may form a triangle, yet the sectors of the three motifs forming the internal angles of the triangle do not sum exactly to 180°. The error may be so small as to be undetectable by the eye, and may even be unnoticeable when subjected to practical measurement with the usual geometrical instruments. It is probable that the muslim craftsman made no mental distinction between exact and inherently approximate constructions of this kind. He was probably not aware that the three interior angles of any triangles sum to 180° and so was not in a position to notice such an inherent error, unless it were so large that no amount of careful drawing could produce a correctly constructed pattern. Had any such unavoidable error been pointed out to him, he would probably have dismissed the observation as irrelevant and outside the immediate aims of his pattern making.

The artisan was aware that most of his patterns could be constructed satisfactorily and as exactly as his instruments and techniques allowed, but he was almost certainly unaware that many of these patterns fitted together perfectly in an absolute, mathematical sense. He would have been incapable of making the theoretical generalizations which we are continually trying to formulate in the pages of this notebook (in fact most modern authors writing on Islamic patterns also seem either to lack this ability or to be unaware of the possibility of making such generalizations). Consequently the artisan would not have appreciated the fact that a type I relationship, such as that illustrated on page 231, is inherently impossible between three different stars simultaneously. For him the proof of the feasibility of any pattern lay in its practical realization as a satisfactory and aesthetically pleasing construction. He was adept at masking any unavoidable errors by freehand adjustment of the lengths or slopes of lines within a pattern, as he was also adept at masking errors of layout, as when the repeat distances of a pattern differed slightly at each side of a doorway and did not quite match up across the top. Human errors of this kind are no more frequent than we might expect from reasonably skilled workmen thoroughly versed in the techniques of their tasks, but who are nevertheless occasionally distracted from the complexities of their pattern making by the joking and banter of their fellows. The modern muslim craftsman has his transistor radio as an added distraction, but his idle chatter probably differs little in content from that of his counterpart of a few centuries ago.

Even though muslim craftsmen may not have made a distinction between exact and inherently non-exact constructions, such a distinction becomes necessary if we are to pursue a systematic mathematical analysis and synthesis of Islamic star patterns. Occasionally the two divisions, exact and non-exact, can be studied together. For example, the enumeration of (p×q) rhomb sizes (pp. 11-14)*. Here, all exact solutions lie on the positive arms of various hyperbolas, but all possible non-exact solutions may also be discovered on the same graphs as pairs of values lying close to the curves on which lie the exact values – the nearer to the curves, the more nearly exact is any approximate solution.

* semi-symmetrical rhombs are dealt with on pp. 92-92; integral polygons from p. 88 onwards.

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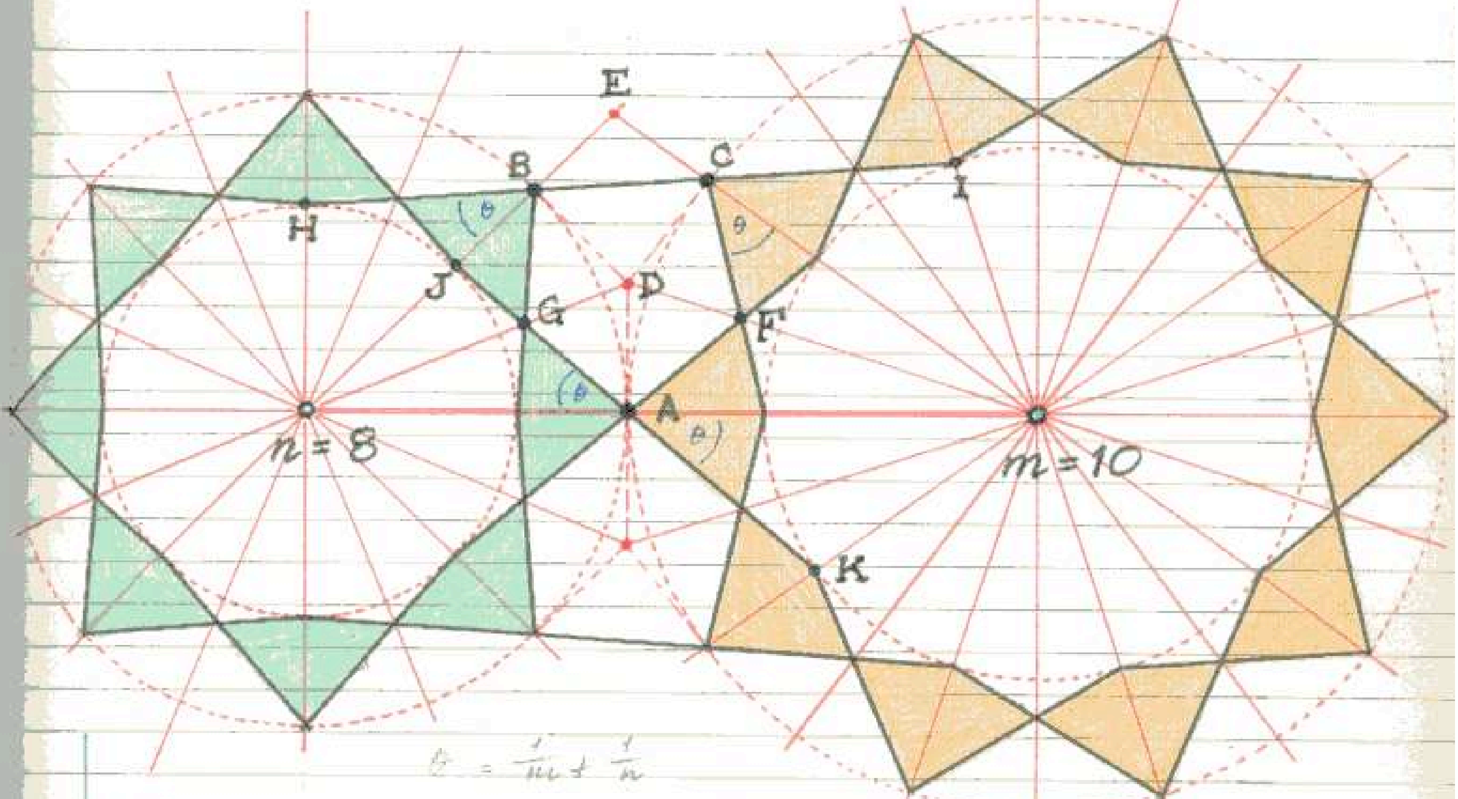
In other cases non-exact solutions are found more easily if specific methods are devised to search for them. For example, in the case of non-exact patterns the special methods I have discovered in recent years have revealed large numbers of exquisitely beautiful patterns, many of which are illustrated in my various files and notebooks. The discovery of means for studying rhombic tilings has greatly helped the realization of many different kinds of star patterns, and has also led me to the discovery of a unique set of rhombic tilings, to which I have given the name "axially centred". In every zone of such special tessellations, the centres of every rhomb in the zone lies on a single straight line – the zone axis. This restriction gives rise to 34 distinct tilings, in none of which can there be more than four different kinds of rhomb.

A synthetic study of Islamic star patterns may be pursued as a special branch of plane geometry, starting from its own axioms.* Examples of the latter are the basic rules 1–3 already mentioned. These are not entirely arbitrary, since they are specifically designed to increase the overall symmetry of the constructions based upon them. But like all useful axioms, further truths may be deduced from them, and a consistent body of geometrical theorems can be built up. For example, from rules 1 and 2 (pp 234, 235) we can immediately deduce that in any pattern in which all stars are in contact with their nearest neighbours throughout the pattern, however many kinds of stars there are, the angles formed by the sides of the stars through their outer points must be a constant size throughout the pattern (see for example the illustrations on pp. 184 – where 4-, 6- and 12-stars lie on the vertices of a [2×2×2] triangle – and 190).

Many people have mistakenly assumed that the apparently perfect symmetry and regularity of

* "definitions" would perhaps be a better word.

→ p241



A *type I link* is defined as a collinear link between two type I stars*, regularly formed according to rules 1 and 2 (pp. 234, 235), in which there is simultaneously collinearity of points H, B, C and I, and of points J, G, A and K. It is found that the radii of any given pair of stars joined by means of a type I link are in a specific ratio unique to that pair (see p. 228) defined by the location of the median point A (see p. 27) which is the point where a perpendicular from D (the 1st collateral intersection m1 : n1 – see p. 25) intersects the collinear radii forming the line MAN. For further generalities on the geometry of type I links see pp. 227–232.

Since the slope of the line HI (on which lie B, C) is unique for a given pair of star-numbers m, n it is obvious that a third star, different from m and n could not form a type I link simultaneously with m and n nor even with one of them but not the other.

The value of angle θ is unique for each m, n pair.

* sharing a single outer point.

Mon 4 March 1985

Islamic patterns is evidence for the possession of an extensive knowledge of mathematics, or at least of geometry, on the part of the skilled artisans responsible for this decoration. In the first place the regularity is often only apparent; closer inspection shows many irregularities and inconsistencies, indicating a great deal of uncertainty in the minds of the original craftsmen as to exactly how many of the patterns should be constructed. Secondly, those patterns which do appear to have been constructed with perfectly regular proportions are mainly those which are easier to construct anyway, especially those in which there occurs only one kind of star, superimposed on a grid of squares or of equilateral triangles. Patterns on this kind of basis have clearly caused the craftsmen few problems, and virtually every suitable combination of stars centred on the diads, triads, tetrads and hexads of these groups will be found to occur somewhere in Islamic geometric art. However, the introduction of two or more different kinds of stars simultaneously within the same pattern can create difficulties, especially if more than one kind of star must relate in a similar geometrical manner to shared peripheral elements. The artist may ignore such relationships, or he may not have noticed them, but if he is aware of them he will also need to possess the additional special knowledge of whichever constructional methods will show this relationships to best advantage. At its simplest this will require an ability to construct the correct ratio between the lengths of the radii of two neighbouring stars. Sometimes this ratio is not very critical, as in the illustration at the top of p. 242, opposite (some rosette relationships with peripheral stars are fairly tolerant to slight errors), but in other cases absolute precision is necessary, as in the type I look (p. 231 and 240.).

As already mentioned, it has been claimed

→ p. 243

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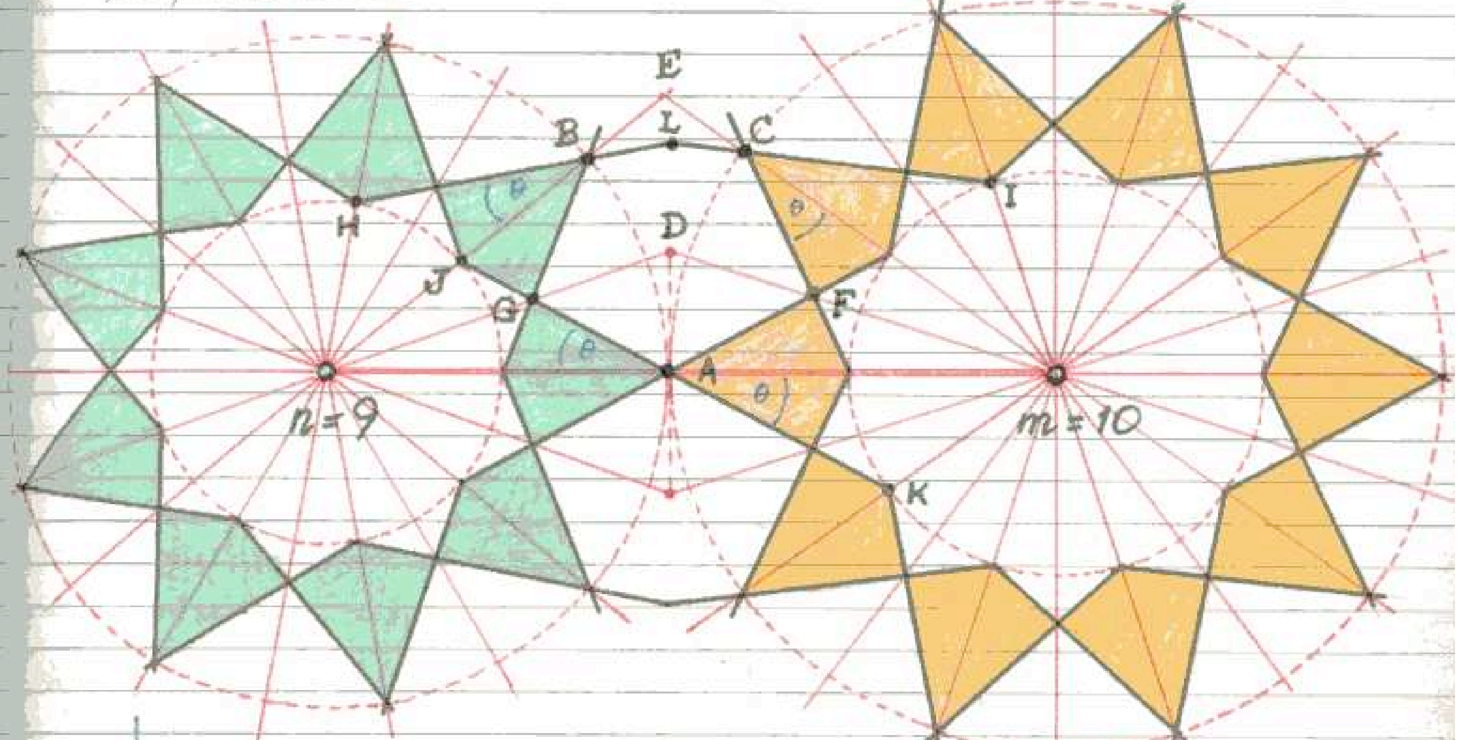
COLLINEAR LINK WITH TYPE I STARS

In this construction points A, B & C still on a peripheral circle.



Two type I stars may of course form a collinear link without necessarily forming a <u>type I link</u>. In the diagram above the sides HB and IC are not collinear but meet outside the star at a point L at an angle other than 180°. However, we still maintain rule 2 (p. 235), so that angle θ is identical for each star, although no longer absolutely fixed as it is in a type I link (see p. 240). This type of construction normally arises because one or other of the stars is constructed strictly as a star-polygon that is, the slope of its sides is determined by joining a pair of non-adjacent vertices (although in a strict mathematical sense this may result in a compound star rather than a star polygon proper). The pattern of 8-stars shown on p. 176, D shows an example of this variety. A strict <u>type I link</u> of this same pattern is the familiar and ubiquitous "star & cross" pattern (see, for example, p. 168, A).

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that there is some documentary evidence* for the intervention of professional mathematicians in the instigation and design of the earliest star patterns, in the 10th century. This is not impossible, but the evidence of the authentic patterns themselves shows that if any strictly mathematical intervention occurred at all it must have been sporadic and minimal. Certainly there does not appear to have developed a continuing tradition of collaboration between mathematicians and craftsmen in the design or execution of star patterns; throughout most, if not indeed all, of the history of Islamic star patterns. The craftsmen thus seem to have worked on their own without outside interference, on the design and construction of their geometrical patterns.
* see pp 171 – 175

NOTE

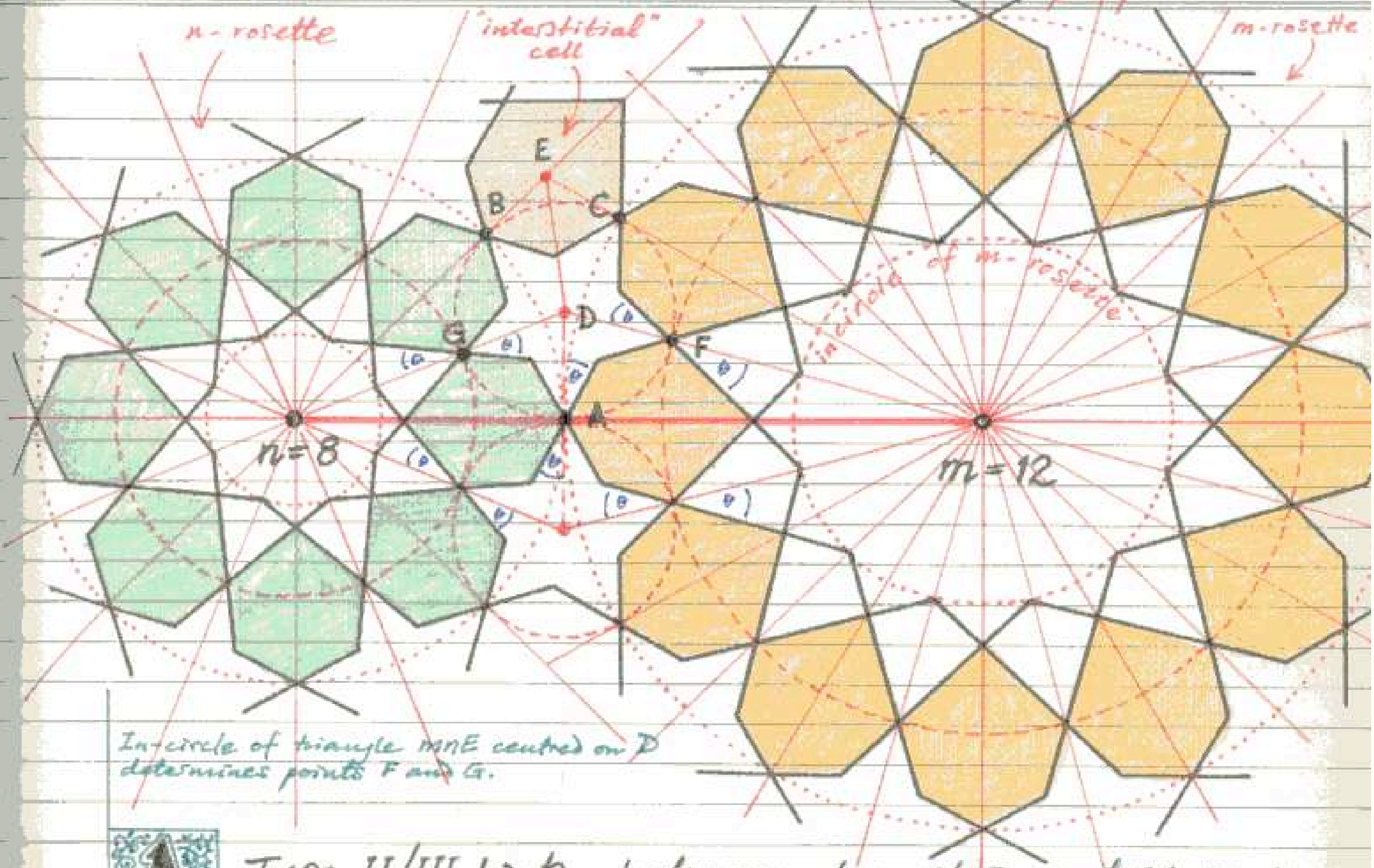
Wed 6 March 1985

Specific "Types" of Links** The designation of these link types (pp. 240, 244) derives from the named "types" of pattern realizations in the (3x2) rhomb series (see p. 83), but the link types have a general application to a collinear link between any two star-centres independently of their occurrence in a (p x q) rhomb series or other integral polygon series (see p. 88 et seq.). These link types are designed to form a standard series of constructions applicable to collinear links between integral numbered star-centres (but they are clearly applicable to non-integrally numbered "star-centres" also). As such they are intended to produce the highest degree of symmetry attainable between the elements of each construction, whether rosettes, stars and their individual cells, peripheral elements, and "interstitial" elements, along with rules for the preservation of specific desired features in each construction. ** See p. 269 et seq. and figs 272, 274.

⇒ Many of the terms used so far are unsatisfactory and should be regarded as provisional only, even though the concepts they describe are probably valid and useful.

= general realization, neither rosettes parallel sided nor peripheral stars.

Monday, OCTOBER 17, 1966



In-circle of triangle mnE centred on D determines points F and G.

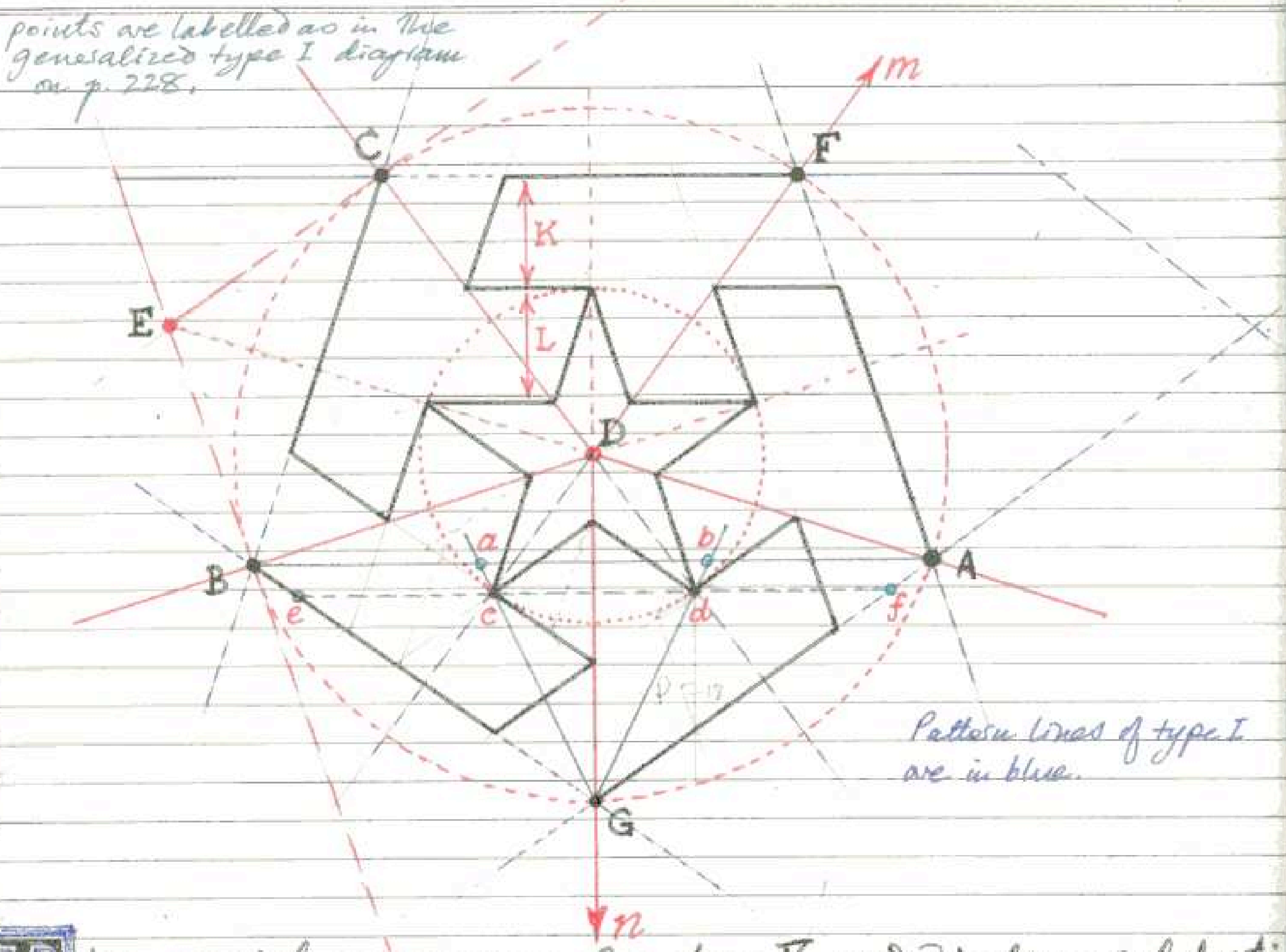
A Type II/III Link between two star-centres m, n may be defined as possessing rosette motifs sharing a common outer point on their collinear link, and with their outer stars determined by the radius of the peripheral circles common to both rosettes. The peripheral circles are in fact the incircles of the 2nd collateral triangles mnE. Angle θ is constant throughout any one construction, but choice of the actual value of θ is arbitrary in the general construction. The radius of the incircles of each rosette depends on the value chosen for θ, but the interstitial cell centred on E is always congruent to the outer cells of the m-rosette, when $3/m + 2/n = 1/2$, and this requirement gives rise to the 8 pairs of integral values in the (3×2) - $[6\times4]$ rhomb series. Thus the pattern in a type II/III link is not unique, but the relationship between pattern elements is rigidly determined.

245 TYPE V — elaboration of Type I.

(3x2) rhomb series $m = n = 10$

Wed 6 March 1985

Tuesday, OCTOBER 18, 1966

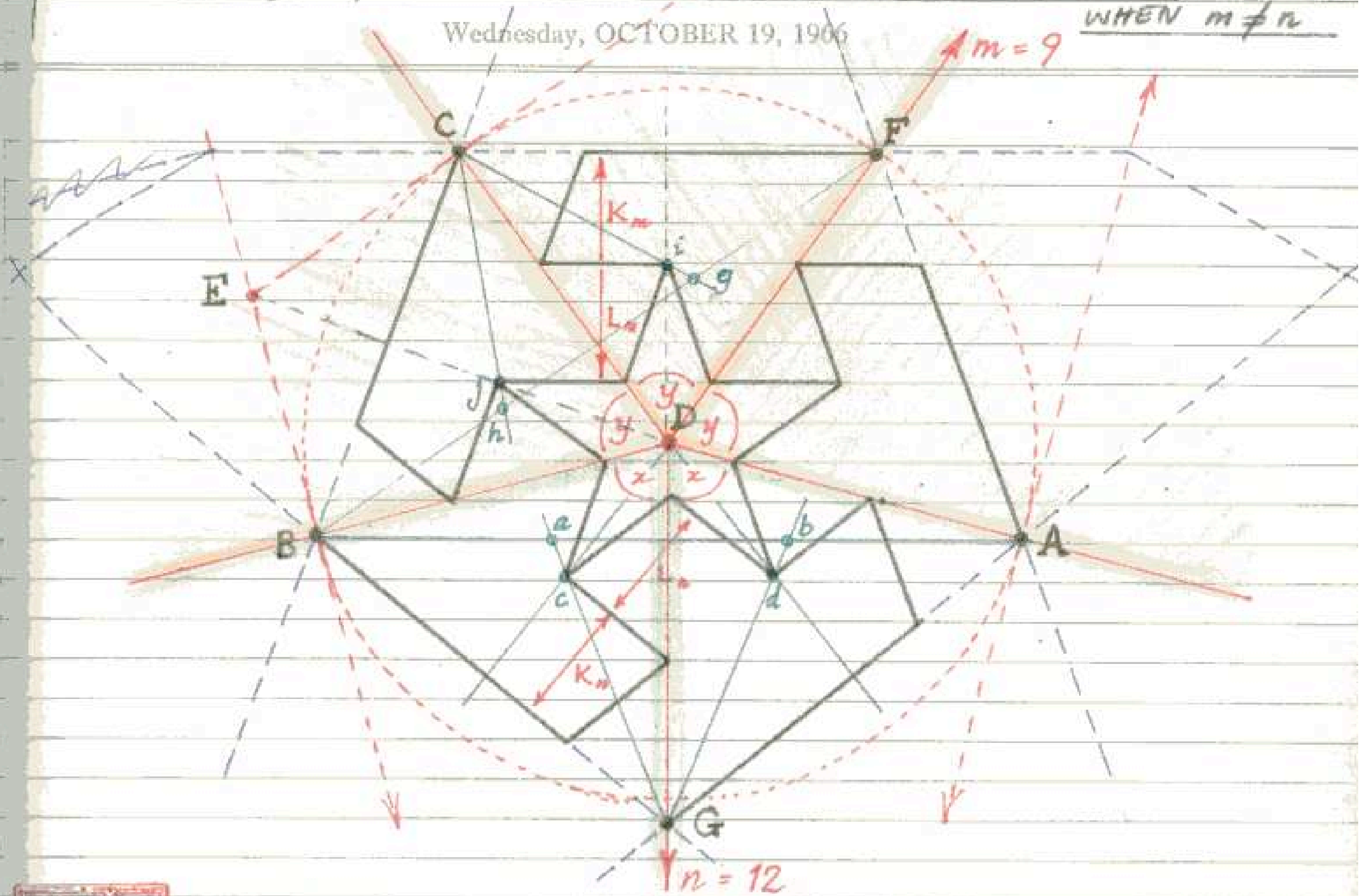


The special appearance of a type V modification or elaboration of (3x2)10,10/II (see p.56) derives from the special way in which the peripheral pentagons are filled. In the above diagram, AGBCF is one such peripheral pentagon. Widths K and L must be equal, which determines the circumradius of the small central 5-star. The construction which ensures that K = L has been briefly noted on p.42 previously. Points c, d are located such that ec = cd = df on line ef. Thus, on any line parallel to ef, whose outer extremities lie on GB and GA draw two points dividing the line into 3 equal parts. In the diagram pentagonal diagonal AB is chosen and divided into 3 equal parts to get points a and b. Join Ga and Gb. Points c and d are obtained from the intersections of Ga and Gb with the bisectors of angles BDG and ADG respectively. As already noted on p.42 however this cannot produce a satisfactory result in the interstitial region.

Wed 6 March 1985

TYPE I PERIPHERAL PENTAGON INFILLING FOR (3x2) RHOMBS WHEN $m \neq n$

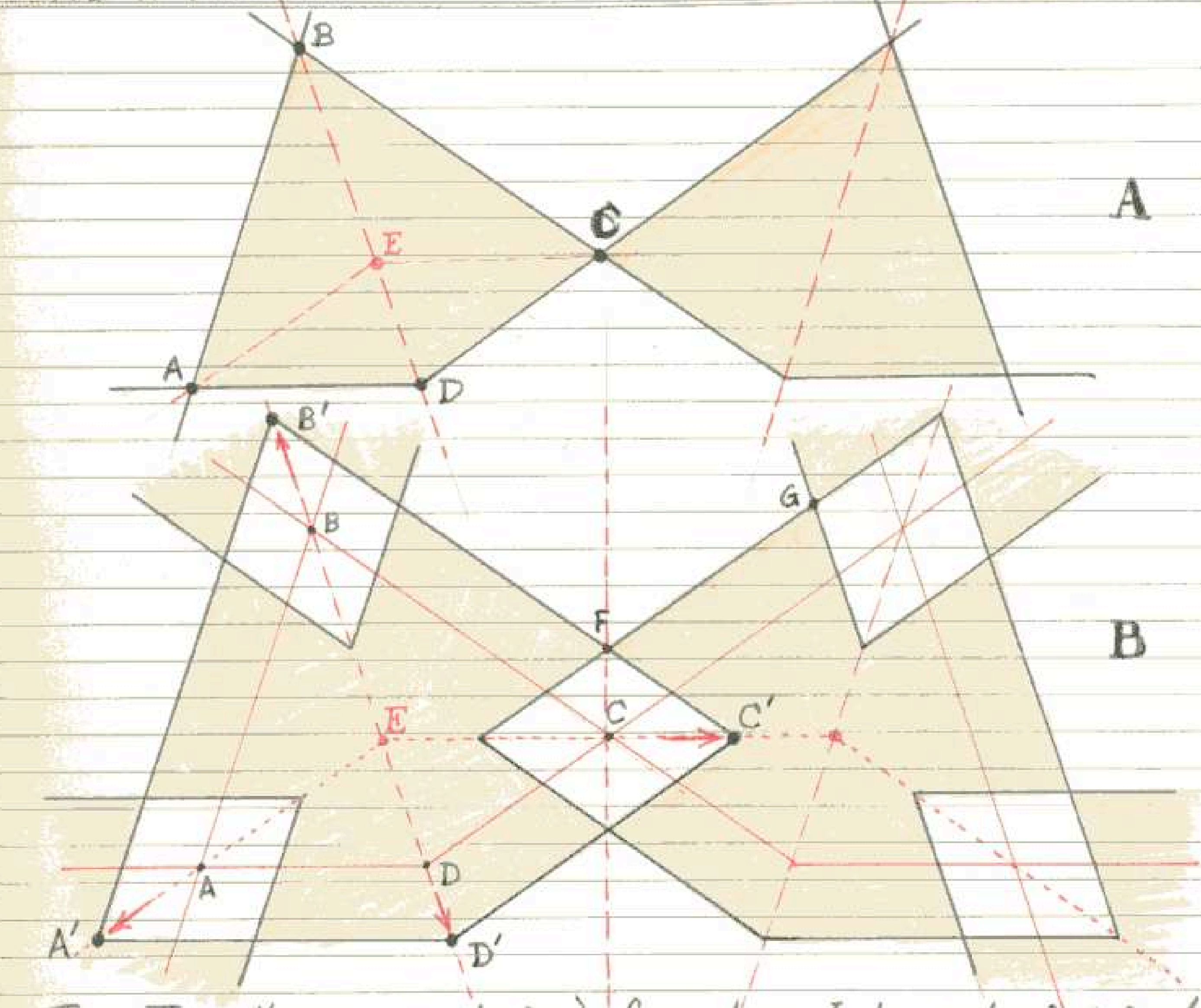
Wednesday, OCTOBER 19, 1966



When $m \neq n$ in the (3x2) rhomb series, the construction on p. 245 cannot be carried out consistently throughout the area of the peripheral pentagons of the type I pattern. Consequently a number of compromises become necessary. If we start the construction in the m-zone (orange) so that $K_m = L_m$ with the central 5-star on a single circle then $K_n \neq L_n$, and vice versa. Clearly we cannot preserve all the characteristic features of the regular pentagonal construction, and there is little justification in trying to devise a mathematically exact solution which will give the best result. It is better to resort to freehand drawing to try and mimic the regular case as far as possible. The same conclusion applies also to the type IV variety (see pp. 42, 56)

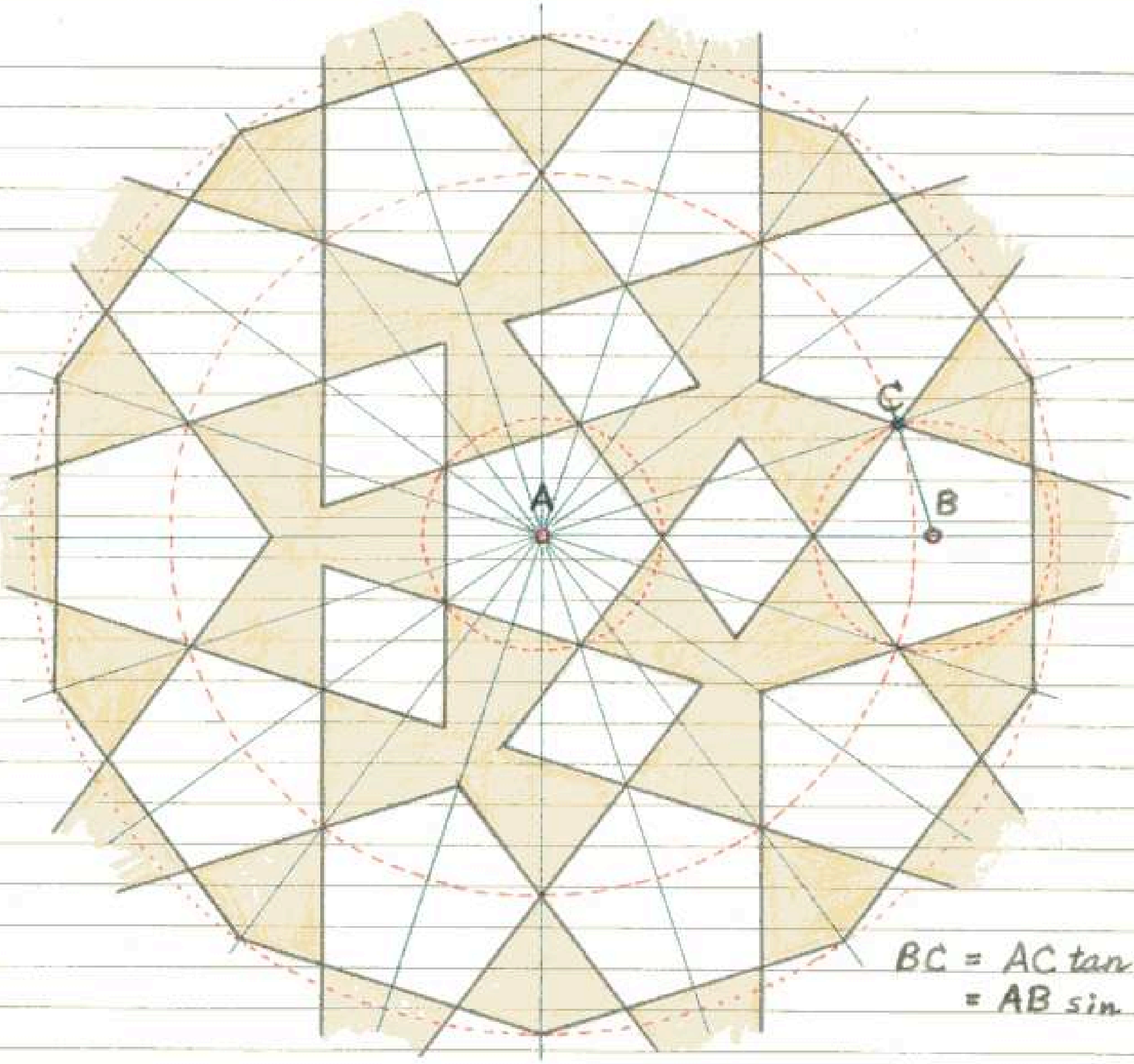
After Thu 7 March 1985

FORMAL DERIVATION OF TYPE VI PATTERNS:



Type VI patterns are derived from type I by enlarging each kite shaped area ABCD (fig. A above) about its centre E until adjacent kites overlap in small rhombs, fig. B, above. Initially the degree of overlap is arbitrary, but certain decagonal patterns allow the degree of overlap to be precisely fixed (fig. on p. 248), that is, the ratio of the side of each small rhomb to the length of the sides of the pentagons in the pattern (i.e. length FG in fig. B, above).

Since the short side of the expanded kite is no longer collinear with the long side of the adjacent kite (although in decagonal patterns the two sides are still parallel) we can



$BC = AC \tan 18^\circ$
$= AB \sin 18^\circ$

make use of this fact in type VI derivatives when $m \neq n$ in the (3×2) rhomb series. The non-regular pentagons in these cases can be adjusted by altering the slopes of the overlapping kites which border them, until they become as nearly regular as possible (indeed this can be done in type VI pattern in other rhomb series). Of course, the small rhombs are no longer rhomb-shaped, but the distorted shapes of these is not so obvious as would be that of the unadjusted non-regular pentagons. Bourgoin (1879) refers to this process as "ajustement pentagonal".

In any true interlacing pattern, that is a pattern

TYPE VI

"AJUSTEMENT PENTAGONAL" (x+y) Thu 7 March 1985

COLLINEARITY OF D-A'-A
· non authentic solution

possessing everywhere 4-way vertices and rectangular vertex figures (p. 235), all pattern areas can be labelled in one of two "modes", for example + or −, black or white, etc. It is always possible to carry out the process of enlarging and overlapping the pattern areas belonging to one mode or the other, and in a general sense, this may be regarded as a "type VI" process. In our original type VI the alternative process would have been to enlarge and overlap the peripheral pentagons of a type I pattern, rather than the outer cells, or "kites" of the type I star. In decagonal patterns this alternative "type VI" does not seem to occur in authentic Islamic star patterns, but such alternatives occur in other styles of star ornament.

Pentagonal adjustment of a type I pattern will entail the least distortion the more nearly regular are the original peripheral pentagons. That is angles a, b and c (figs A–C opposite) must be nearly equal. If the m- and n-stars are drawn regularly then points G' and F' (in fig. B) must still lie on radii ND and MD respectively. It is not so important that points B' and C' remain on lines BD and CD* although they should not lie too far from those lines, so as to preserve the impression of a collinear link between adjacent peripheral pentagons, whether they are regular or not. It must be remembered that if point B' for example is moved off line BD by altering the slope of line G'D' then the line G'A' will have to be altered by the same amount to preserve the regularity of the peripheral lines around the n-motif. If the line B'C' is altered however no such difficulty is encountered. Pentagonal adjustment in most cases is more a question of aesthetic judgement than of strict mathematics, and it is therefore not surprising to discover that muslim craftsmen excelled at producing well balanced patterns of this type. They are particularly numerous in Cairo and the Middle East.

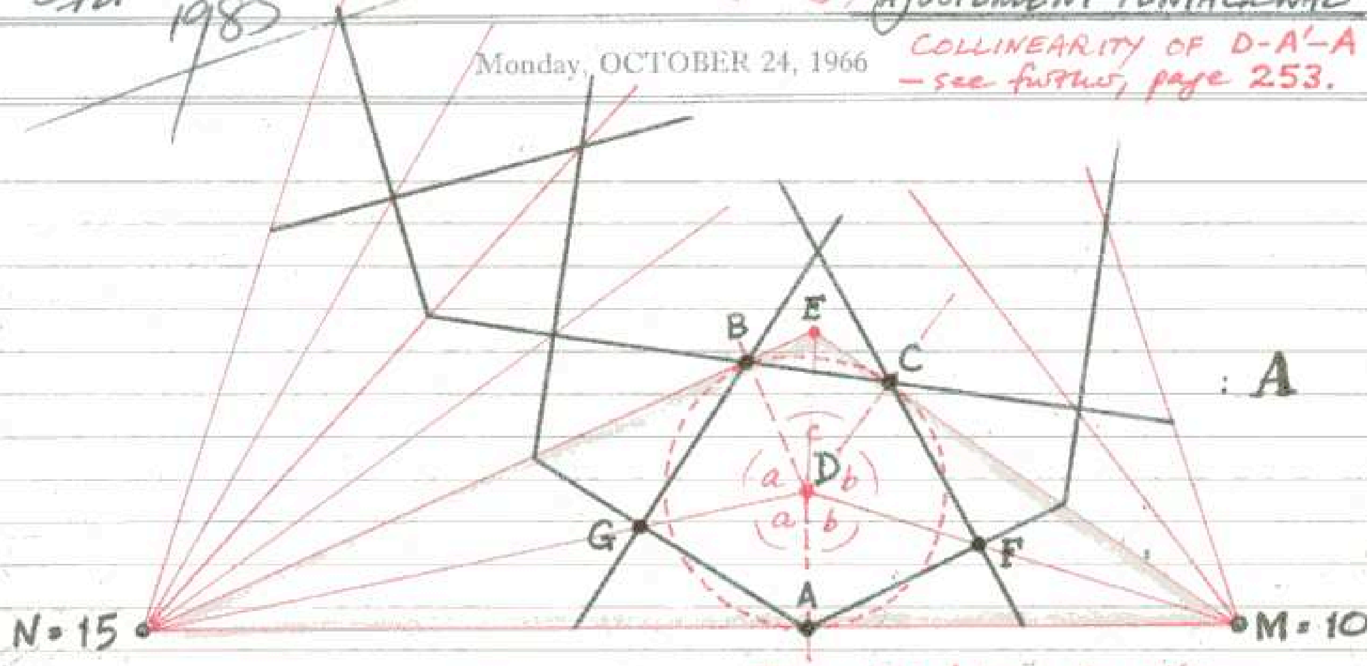
Sunday, OCTOBER 23, 1966

* or that point A' should lie on AD.

continues → p. 253

COLLINEARITY OF D-A'-A

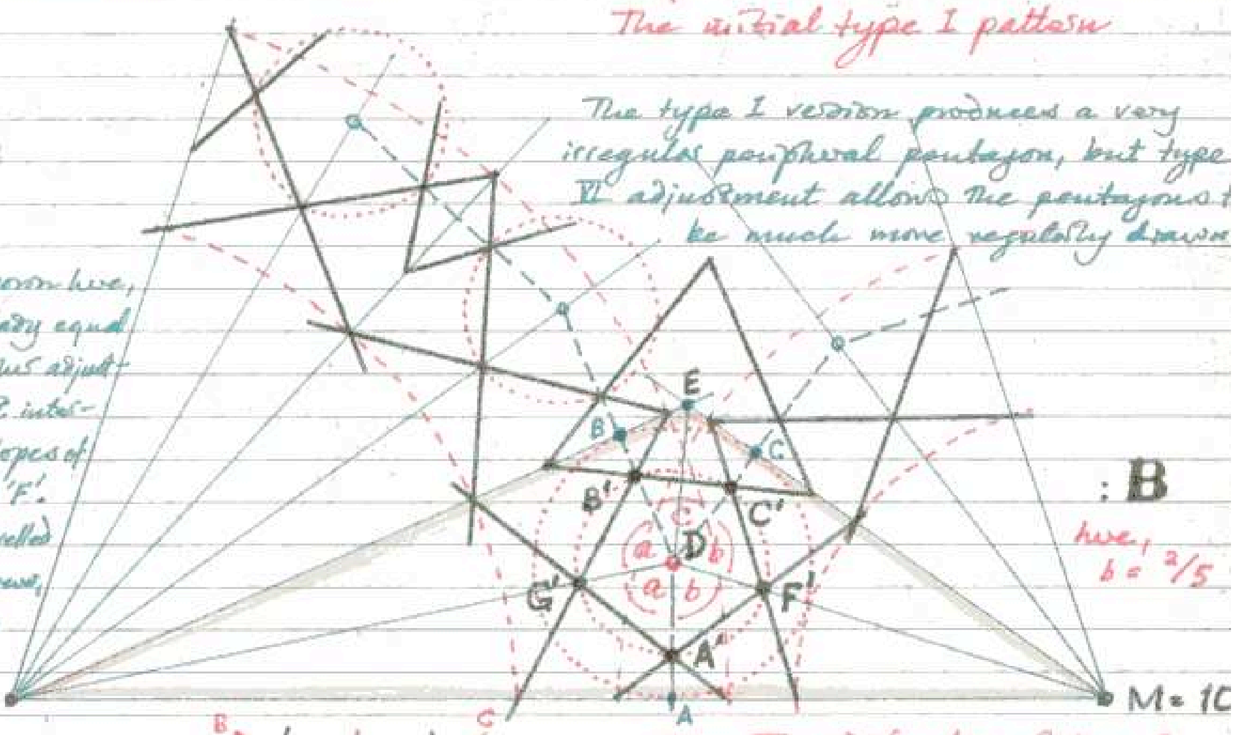
— see further, page 253.



The initial type I pattern

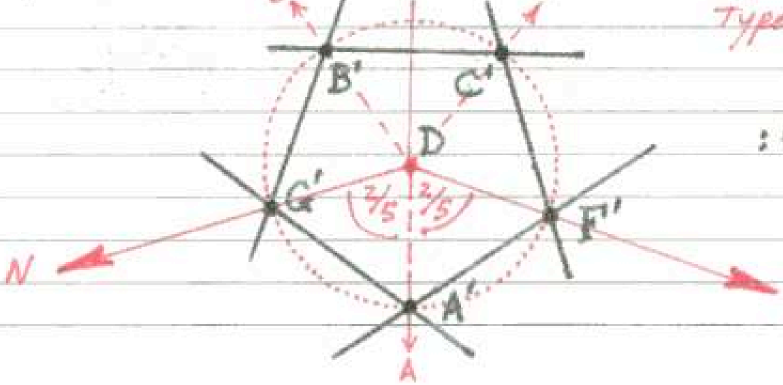
The type I version produces a very irregular peripheral pentagon, but type VI adjustment allows the pentagons to be much more regularly drawn.

In the case shown here, since b is already equal to 2/5, any further adjustment should not interfere with the slopes of lines C'F' and A'F'. B'C' could be swivelled about point C', however, to bring B' slightly nearer centre N.



here, b = 2/5

Type VI modification of type I = "ajustement pentagonal"



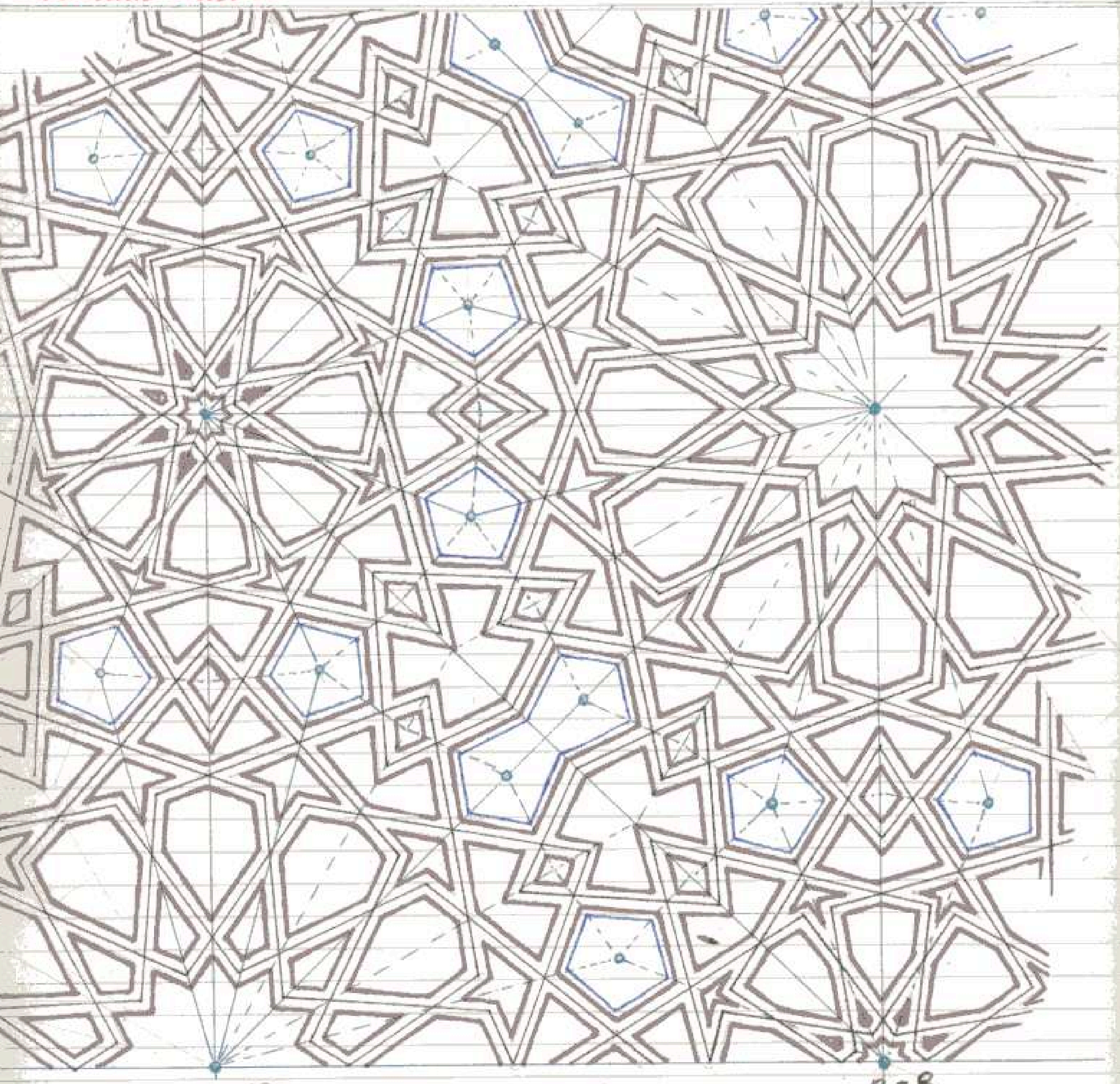
:C Regular pentagon.

$a = b = c = 2/5$

251 TYPE VI
"AJUSTEMENT PENTAGONAL" $\alpha = \frac{1}{2}(x+y)$ Fri 8 March 1985

Non-collinearity of D-A'-A
= authentic solution

Tuesday, OCTOBER 25, 1966



m = 12 n = 8

$\alpha = 71^\circ.25$

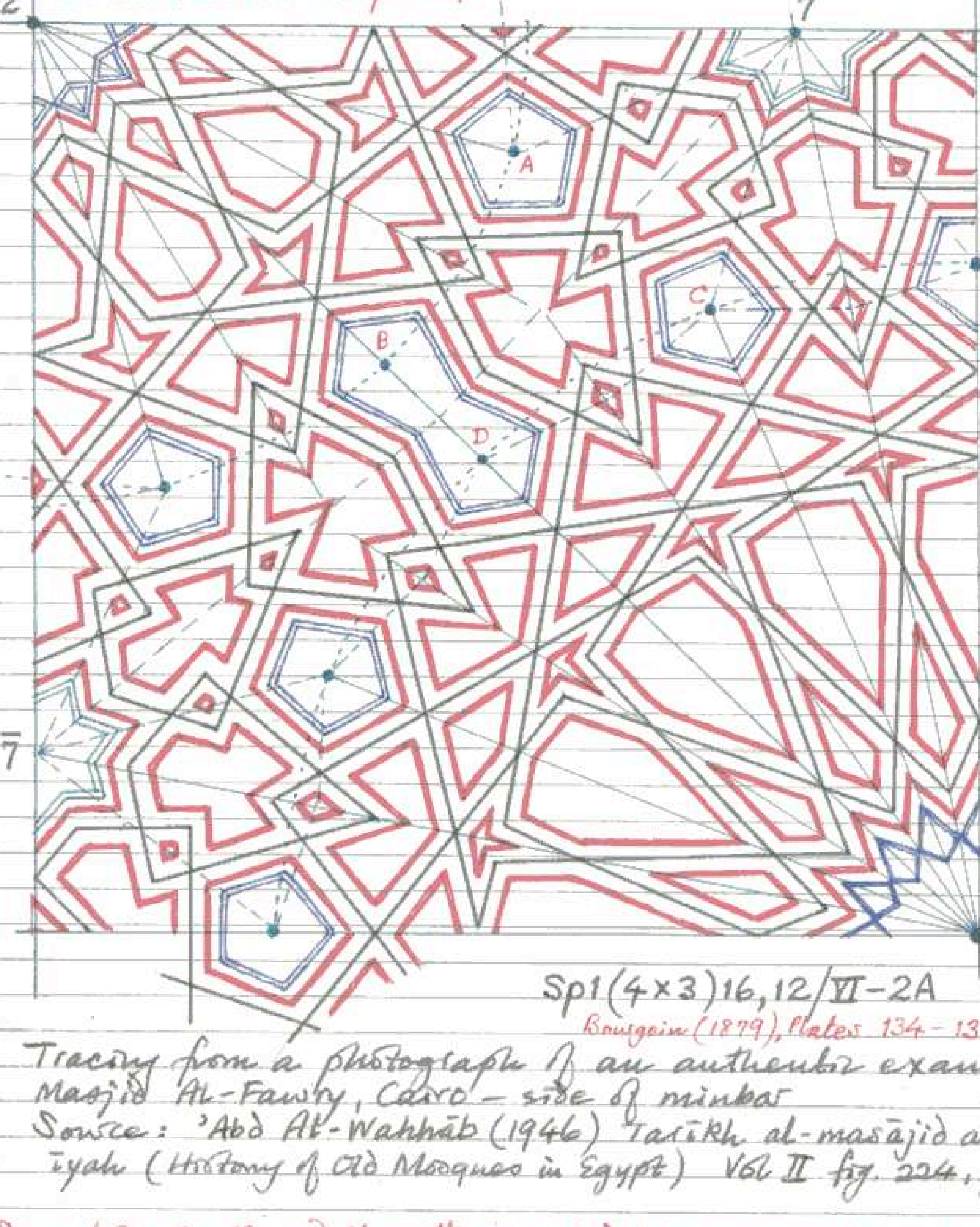
Sp1(3x2)12,8/VI This is my own construction; Bourgoin (1879) has a version of this pattern (his plate 116) but his drawing is rather clumsily executed, and it is doubtful whether it resembles any authentic example at all closely.

Fri 8 March 1985 $\alpha = \frac{1}{2}(x+y)$. TYPE VI

"AJUSTEMENT PENTAGONAL"

cf. pp. 296, 297 Wednesday, OCTOBER 26, 1966 NON-COLLINEARITY of D-A'-A = authentic solution.

For construction see p. 284



N.B. Distance from A-C is equal to A-B, but is slightly less than C-D

Sp1(4×3)16,12/VI-2A

Bourgoin (1879), Plates 134-135.

Tracing from a photograph of an authentic example, Masjid Al-Fawry, Cairo – side of minbar

Source: 'Abd Al-Wahhāb (1946) Tarīkh al-masājid al-Athar-iyah (History of Old Mosques in Egypt) Vol II fig. 224, p. 145.

Since both the 12- and 16-rosette are linked to a non-regular, "interstitial" 7-rosette, the value of α in each case depends on the manner of formation of this 7-rosette

TYPE VI

"AJUSTEMENT PENTAGONAL" AUTHENTIC METHODS ~~After~~ Sat 9 March 1985.

NON-COLLINEARITY OF D-A'-A Thursday, OCTOBER 27, 1966
: authentic solution.

$\alpha = \frac{1}{2}(x+y)$

Although the preliminary solution described on pp 249-250 preserves collinear links between adjacent peripheral pentagons this is considered a less important feature in authentic type VI patterns. In authentic patterns the primary aim seems to be to draw the peripheral pentagons as regularly as possible, when $m \neq n \neq 10$. This is achieved by bisecting angle MDN (see fig. ~~of~~ p. 254); this creates four equal angles $\underline{\alpha}$ at point D, after symmetrical repetition round centres M and N. General expression for the normal constituents of angle MDN, namely $\underline{x}$ and y, are given on page 67 (where they are labelled $\bar{i}$ and $\bar{j}$) and for (3×2) rhomb values on p. 204. Specific values for (3×2) rhombs are given at the bottom of page 202.

Given the four angles α the vertices of the peripheral pentagon are determined by a circumscribing circle centred on point D. The radius of this circle is arbitrary, but should be between about 0.6 to 0.75 of the distance DA. However, although the starred rosettes which are tangent to the peripheral pentagons do not have a completely determined radius, the range of lengths permitted is limited by the range just recommended for the size of the circumcircles of the peripheral pentagons. On the other hand, the angles between line segments within the rosettes are determined, as are the relative lengths of the radii of their outer star and incircles, if we follow the standard rosette construction, based on peripheral circles, as described on pp. 199, 200 et seq.. The drawback of this method is that the smaller of two rosettes always has obtuse terminal segments* (see the 8-rosette on page 251), resulting automatically in a divergent sided rosette according to the standard construction. In many authentic examples there seems to have been a desire to avoid divergent sided rosettes with obtuse terminal segments, in favour of reflex rosettes tangent to the type VI peripheral pentagons (see the 7-rosettes on pp. 252 and 254). There are thus two alternative possible realizations of type VI patterns

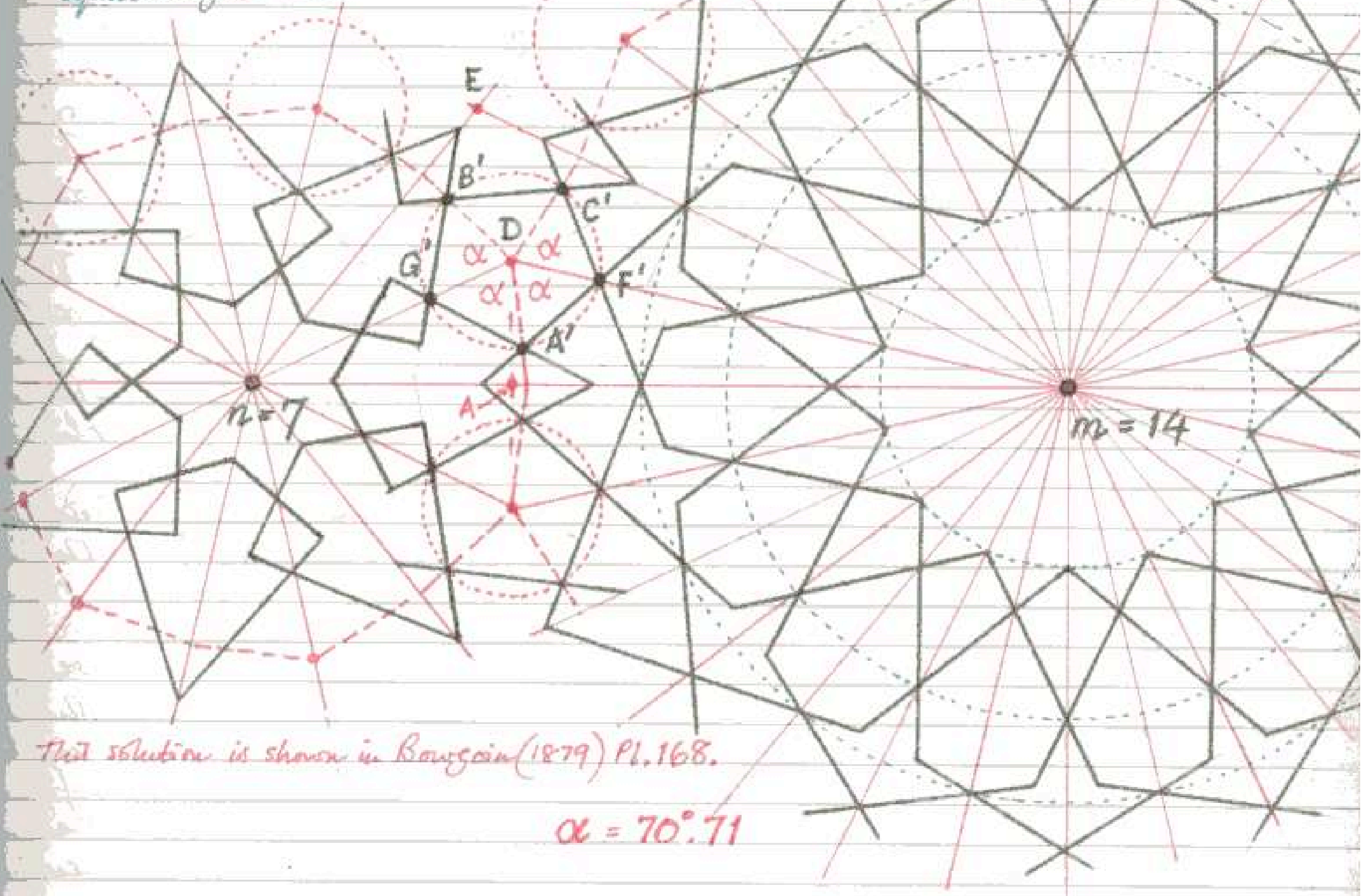
* This is true only of lower (p×q) series, including (3×2) – see diagram on p. 258.

Sat 9 March 1985

$\alpha = \frac{1}{2}(x+y)$
$= \frac{1}{2} - \frac{1}{2m} - \frac{1}{2n}$

TYPE VI
"AJUSTEMENT PENTAGONAL" 254
NON-COLLINEARITY OF D-A'-A
The usual authentic solution.

Angle MDN is bisected to produce equal angles α.



This solution is shown in Bourgoin (1879) Pl. 168.

$\alpha = 70°.71$

255 | TYPE VI PATTERNS
STANDARD CONSTRUCTION.

Wed 13 March 1985

Saturday, OCTOBER 29, 1966

In what we shall now regard as the "standard" type VI construction the five vertices of the type VI peripheral pentagons are determined by the formation of four equal angles α round point D, the 1st collateral intersection (see pp. 25-26). Since the peripheral pentagons are inscribed within a small circle of arbitrary radius, its radii enclose four congruent isosceles triangles with apical angle α and basal angles ψ. The main motifs used in type VI patterns are either starred type II rosettes or reflex rosettes (figs. A, B on p. 256 opposite). In the case of starred type II rosettes the angle at the starred vertex is 2ψ, and this value is fixed and unique for each given pair of star-motifs m, n.

For a starred type II rosette with parallel sides and collinear terminal segments (a P/C-rosette) with n vertices, the angle at each vertex is equal to $S = \frac{n-4}{n}$. If the rosette is of the "standard" peripheral circle construction, we can say that if $2\psi > S$ then the rosette will have divergent sides and obtuse terminal segments (a D/O-rosette); such a rosette is shown in fig. A opposite. If, on the other hand $2\psi < S$ then the rosette will have convergent sides with acute terminal segments (a C/A-rosette, as in the 14-rosette on p. 254). Since the value of 2ψ depends on the value of α which in turn depends on the values for m and n, we can thus determine beforehand the kinds of type II rosettes which will result from the use of a given pair of centres m, n.

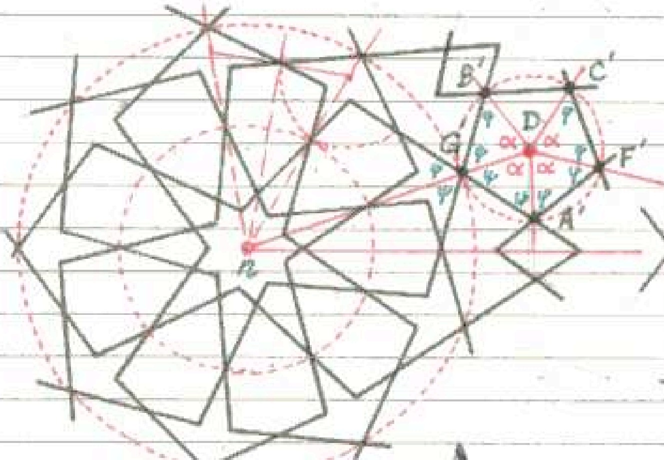
Sunday, OCTOBER 30, 1966

$$\alpha = \tfrac{1}{2} - \tfrac{1}{2m} - \tfrac{1}{2n} \qquad 2\psi = \tfrac{1}{2} + \tfrac{1}{2m} + \tfrac{1}{2n}$$

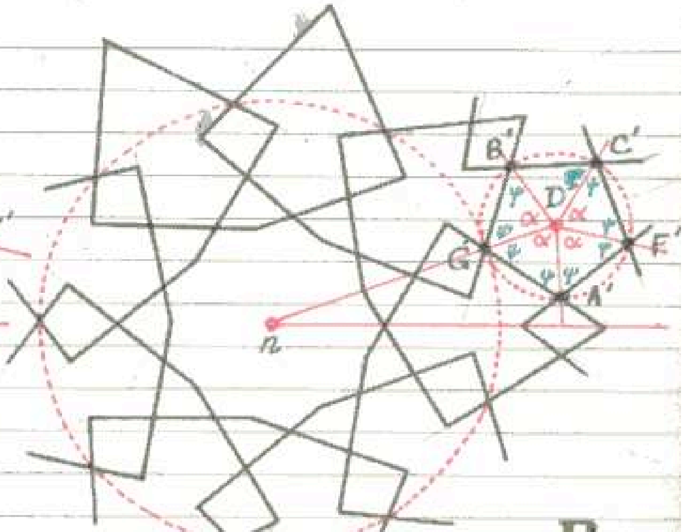
In this form, the equations allow a quick check for every pair of values m, n to determine the types of rosettes which result. As will be seen from the table on p. 258 the lowest numbers give two D/O-rosettes, two higher numbers give two C/A rosettes, while a mixed pair gives one D/O- and one C/A rosette (the D/O being the smaller).

NON-COLLINEARITY of D-A'-A

• authentic solution.



A

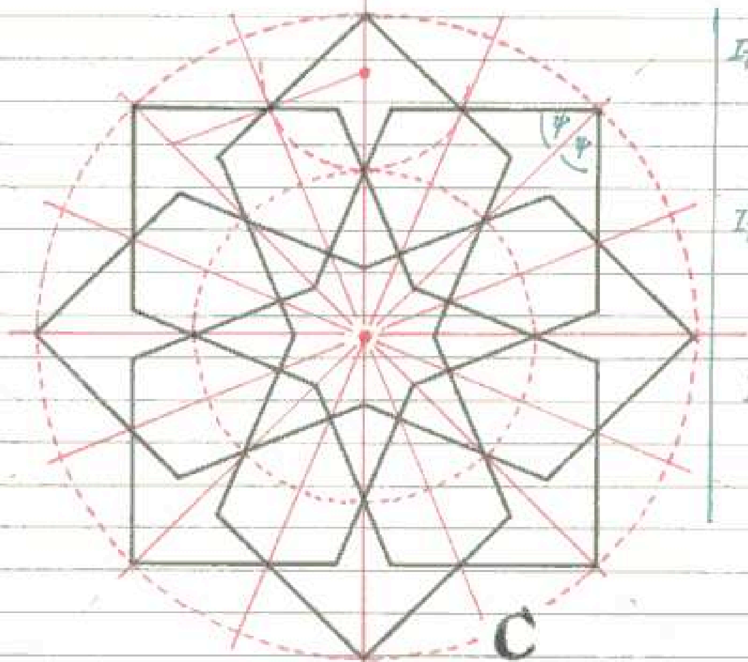


B

Formation of starred type II rosette within circle nG'. This is a starred D/O-rosette.

Formation of reflex rosette within circle nG'.

A given pair m, n determines the value of $\alpha = \frac{1}{2}(x+y) = \frac{1}{2} - \frac{1}{2m} - \frac{1}{2n}$. Angle ψ is thus fixed for each m, n pair, and therefore the type of rosette, whether P/C, C/A or D/O, is completely determined.



C

If $2\psi = \frac{n-4}{n}$ rosette is parallel-sided with collinear terminal segments (P/C - rosette)

If $2\psi > \frac{n-4}{n}$ rosette is divergent sided with obtuse terminal segments (D/O - rosette)

If $2\psi < \frac{n-4}{n}$ rosette is convergent sided with acute terminal segments (C/A - rosette).

A starred type II 8-rosette, with parallel sides and collinear terminal segments (P/C rosette). $2\psi = \frac{n-4}{n}$

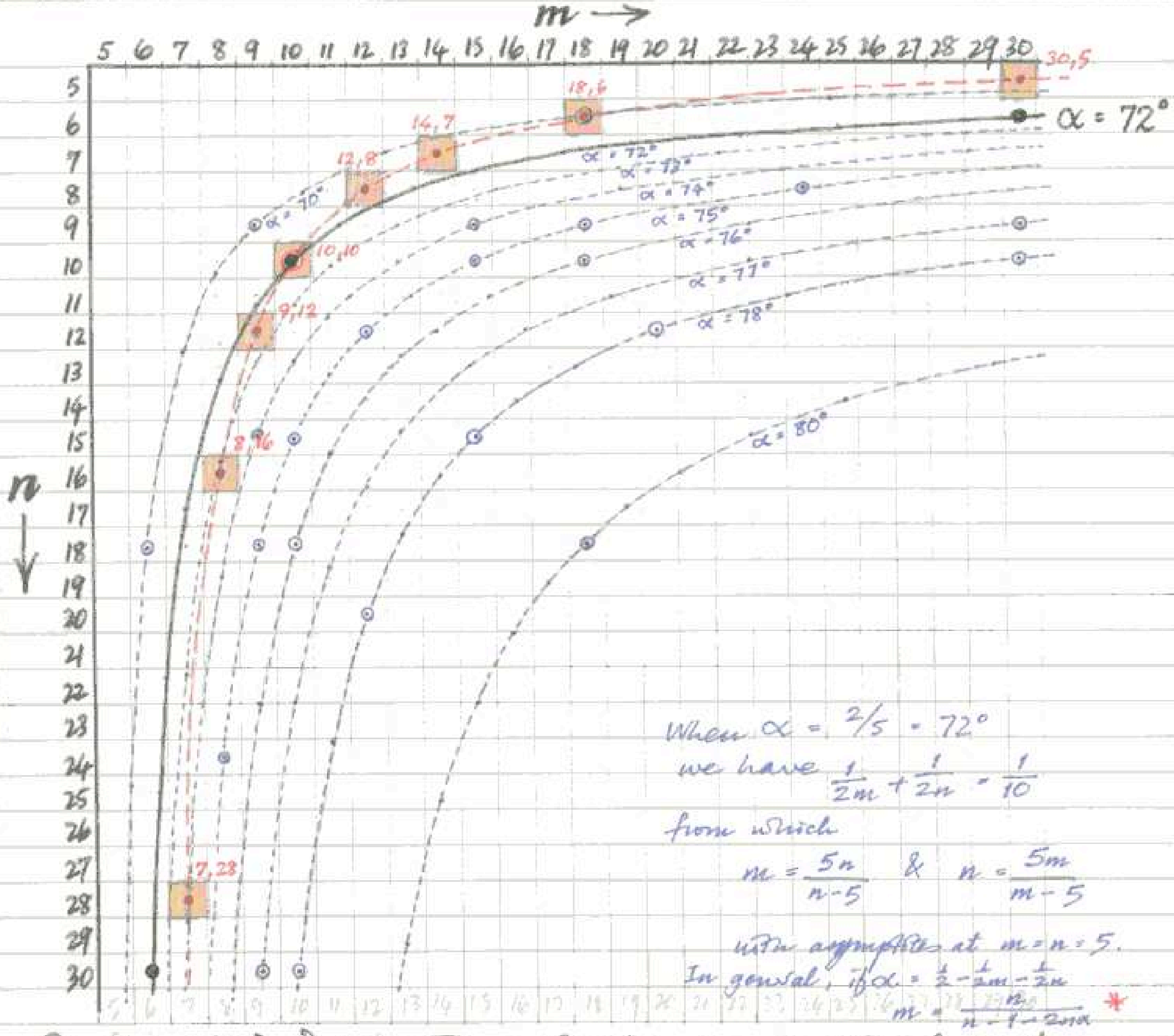
257 | TYPE VI PATTERNS

STANDARD CONSTRUCTION = α-CONSTRUCTION

Wed 13 March 1985

Curve for $\alpha = 2/5 = 72°$

Tuesday, NOVEMBER 1, 1966



When $\alpha = 2/5 = 72°$

we have $\frac{1}{2m} + \frac{1}{2n} = \frac{1}{10}$

from which

$m = \frac{5n}{n-5}$ & $n = \frac{5m}{m-5}$

with asymptotes at $m = n = 5$.

In general, if $\alpha = \frac{1}{2} - \frac{1}{2m} - \frac{1}{2n}$

$m = \frac{n}{n-1-2n\alpha}$ *

In the standard type VI construction the peripheral pentagons will be closest to a regular pentagon the closer angle α is to $72° = 2/5$. The graph above shows the curve for which $\alpha = 72°$ (which has only 3 integral points) and superimposed on this the curve of values for the (3×2) rhomb series. This is one of the closest (p×q) curve and thus gives particularly accurate values for the peripheral pentagons of type VI patterns. As will be noticed the approximate rhomb $\overline{(3\times2)}$ 11, 9 (both ways) also gives accurate type VI results.

* for purposes of calculation α is expressed as a fraction of 180°, as usual.

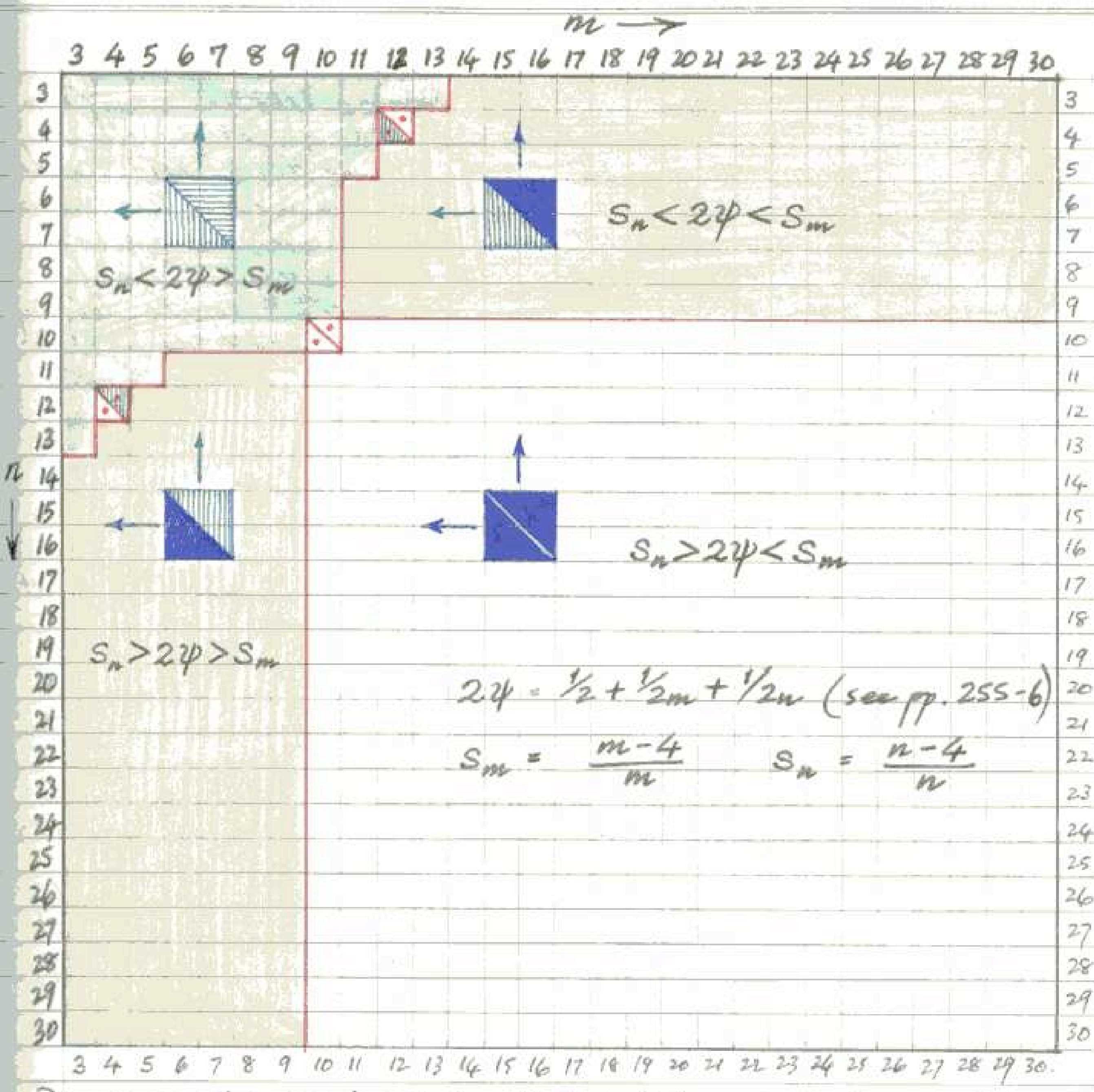


Diagram showing types of rosettes determined for each pair m, n of star-numbers in type VI patterns, assuming the α-construction outlined on pp. 253-256. Rosette types are

- = P/C, parallel-sided with collinear terminal segments
- = C/A, convergent-sided with acute terminal segments
- = D/O, divergent-sided with obtuse terminal segments.

Cases in which star number is less than 5 are of theoretical interest only. Thus, for practical purposes the only case with P/C-rosettes is the pair 10,10.

TYPE VI PATTERNS

SHAPES OF PERIPHERAL PENTAGONS.

Wed 13 March 1985

In the standard construction (or "α-construction") for type VI patterns the precise shape of the peripheral pentagons, and their closeness to a regular pentagon, is dependent on the exact value of the angle α (see fig. on p. 254). This angle varies according to the values m, n for a given pair of star motifs, and is equal to $1/2 - 1/2m - 1/2n$, that is, $\frac{1}{2}(x+y)$ - see for example, fig. on p. 201. Clearly the most regular looking peripheral pentagons will be those for which α is close to $72° = 2/5$. But acceptable results are obtainable within the range of about 70° – 75° as may be seen from the drawings given on p. 260, opposite. For values outside this range it is possible to adjust the pentagons still more, but at the expense of the perfect regularity of one or both of the star-motifs concerned. Examples of this further "ajustement pentagonal" are shown in a number of authentic Islamic patterns.

Curves for varying values of α are given in the diagram on p. 257, and used in conjunction with such diagrams as those on p. 260, an approximate idea of the shape of the peripheral pentagons can be obtained, appropriate to any pair of star motifs with values m, n.

It is interesting that there are only two pairs of values which will give a perfectly regular peripheral pentagon: 10,10 and 6,30 (see diagram 257, the $\alpha = 72°$ curve).

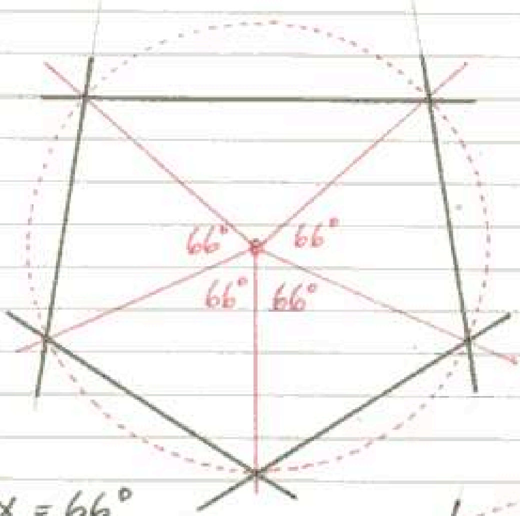
Wed 13 March 1985

SHAPES OF PERIPHERAL PENTAGONS for VARYING α

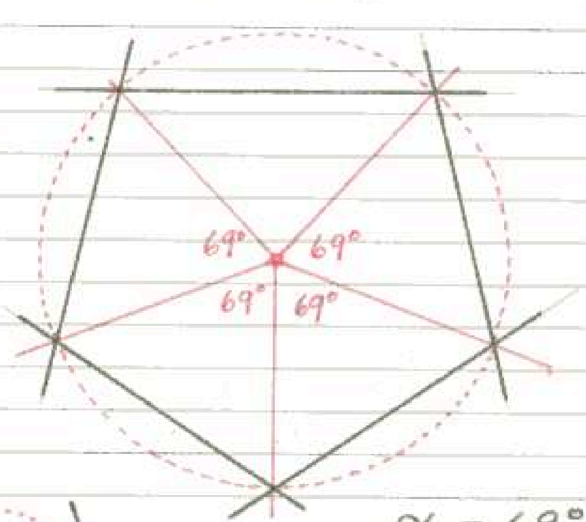
Friday, NOVEMBER 4, 1966

α - CONSTRUCTION.

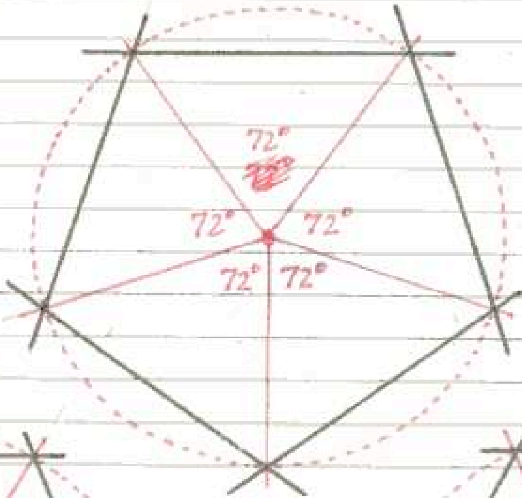
$$\alpha = \frac{1}{2} - \frac{1}{2m} - \frac{1}{2n}$$



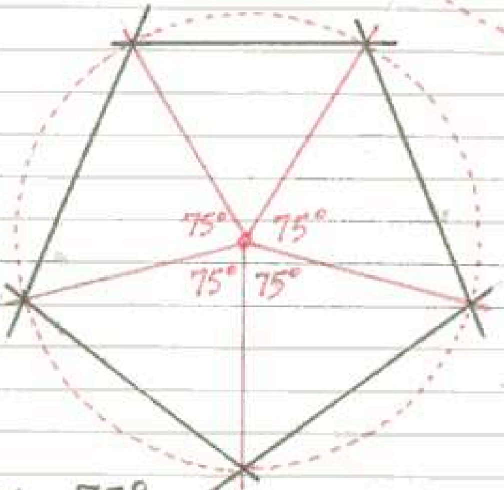
$\alpha = 66°$



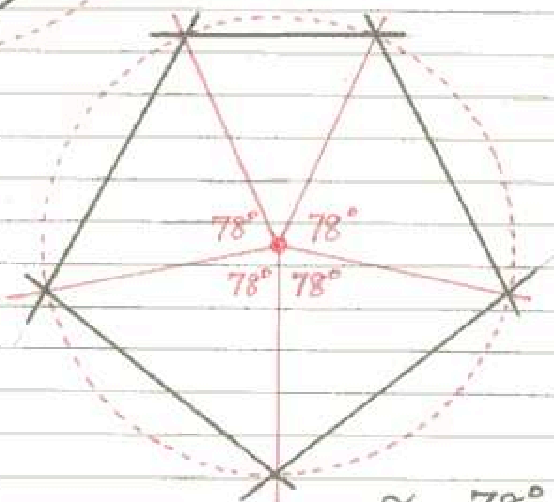
$\alpha = 69°$



$\alpha = 72°$ (regular)



$\alpha = 75°$



$\alpha = 78°$

261 TYPE VI PATTERNS
STANDARD, α-CONSTRUCTION

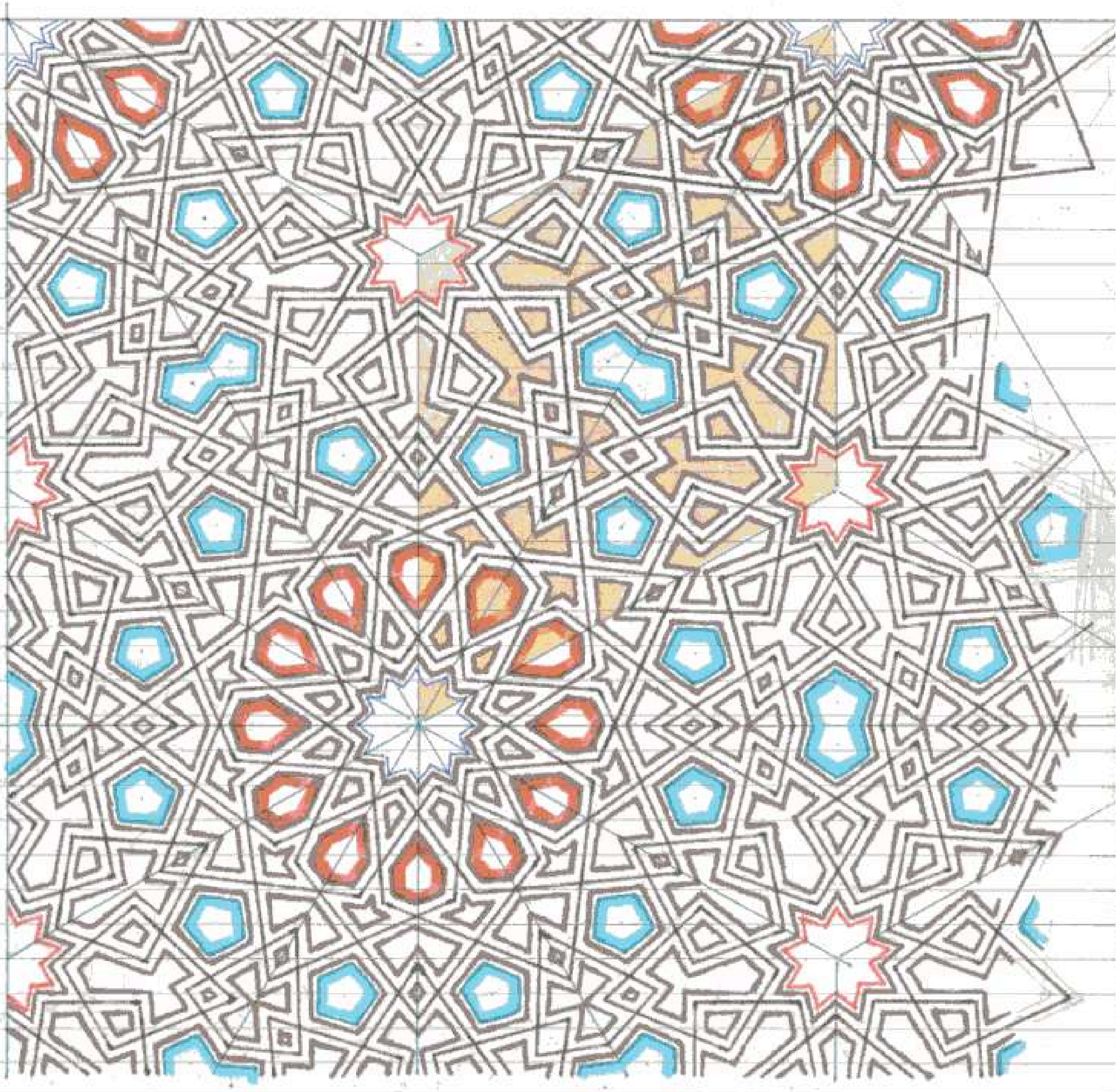
Thu 14 March 1985

Saturday, NOVEMBER 5, 1966

(3x2) 8,16/VI

Incorporating the rhomb (3x2)8,16/VI (part of one such rhomb is coloured orange). The 8-stars are on the vertices of t{4,4}, the 16-stars centred on the octagons. α = 73°.125, so the pair 8,16 gives quite accurate peripheral pentagons; cf diagram 257. If type II rosettes are drawn the 8 is D/O, the 16 is C/A (see diagram 258). This occurs in Cairo. Bourgoin (1879) gives it on his plate 152.

On 12s alone this is $\boxed{H1(4\times2)\,12,12/VI-2B}$



H1(3×2) 9, 12/VI (one rhomb is outlined in orange).

α = 72.°5 so peripheral pentagons are fairly regular, in fact these are the most regular pentagons of the (3×2) rhomb series, apart from 10, 10.

Occurs in Cairo; e.g. minbar in mosque of Emir Zeyn ed-Din Abu Bakr b. Mazhar el Ansari (1479) [Burlington Mag. 35: 243, 1919]. See also Bourgoin (1879) Pl. 124 - different proportions

263 | TYPE VI PATTERNS : α- CONSTRUCTION

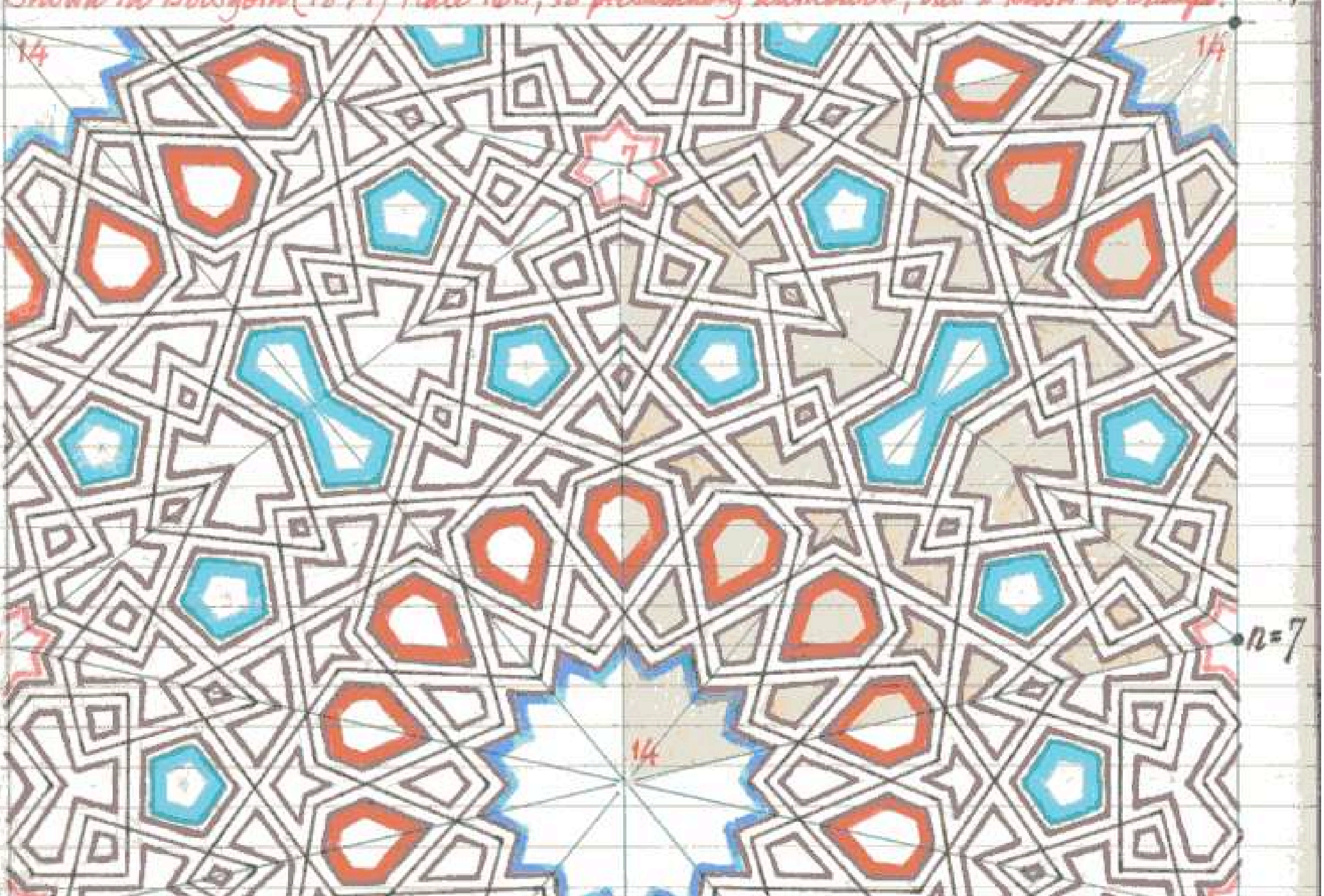
Fri 15 March 1985.

On the 14s alone, this is (4×3)14,14/VI–2A
and this can be adapted to other (4×3) rhombs – see p.252 & 286.

Tuesday, NOVEMBER 8, 1966

H2 AAB 14,7/VI incorporating (3×2)14,7/VI, coloured orange

Shown in Bourgoin (1879) Plate 168, so presumably authentic, but I know no example!



Fri 15 March 1985

STANDARD, or α - CONSTRUCTION

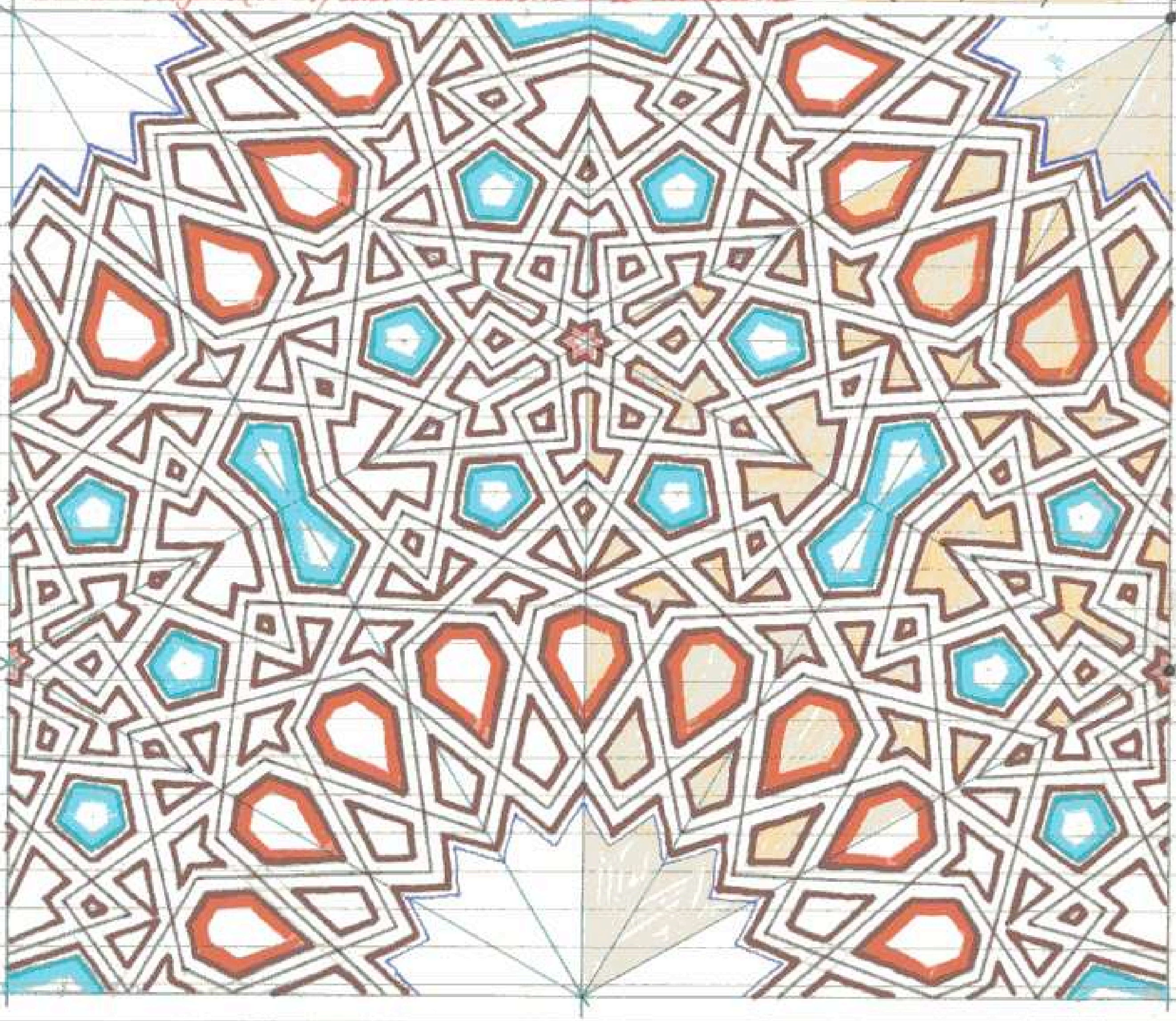
Wednesday, NOVEMBER 9, 1966

On the 18s alone, this is (6×3)18,18/VI–2A.

Not in Bourgoin (1879) and not known to be authentic

H1 (3×2)18,6/VI

m=18



n=

The pattern opposite can be regarded either as an arrangement with two kinds of rhomb – (3×2) 14,7 and (3×1) 7,14 : H2 AAB 14,7/VI or with 1 kind of rhomb – (4×3) 14,14 in which the 7-stars are regarded as interstitial elements. Other patterns on the same (4×3) basis may be constructed, and Sp1 (4×3) 16,12 is shown on p. 252, but here the approximate 7-stars (which are interstitial) act as a common point of coordination for the peripheral pentagons with dissimilar main stars – 16s and 12s – so this is a now a case of multiple peripheral coordination.

α - construction

Fri 15 March 1985

Type VI Patterns: ~~Varieties~~ degrees of Peripheral Pentagon Coordination

~~Varieties~~ Degrees of "coordination" here refers to the number of star-centres with which the peripheral pentagons form simultaneous collinear links by means of their vertices.

1-coordination: 1 radius of each pentagon lined up with a single star-centre, although the opposite edge of the pentagon may be aligned with a rotocentre with a low degree motif (triangle or square) collinear with the first collinear link. The pentagons can be drawn perfectly regularly (see fig. on p. 266 opposite).

2-coordination: pentagons with collinear links simultaneously to two star-centres, which may be regular or non-regular. Angle α constant for any one pattern.

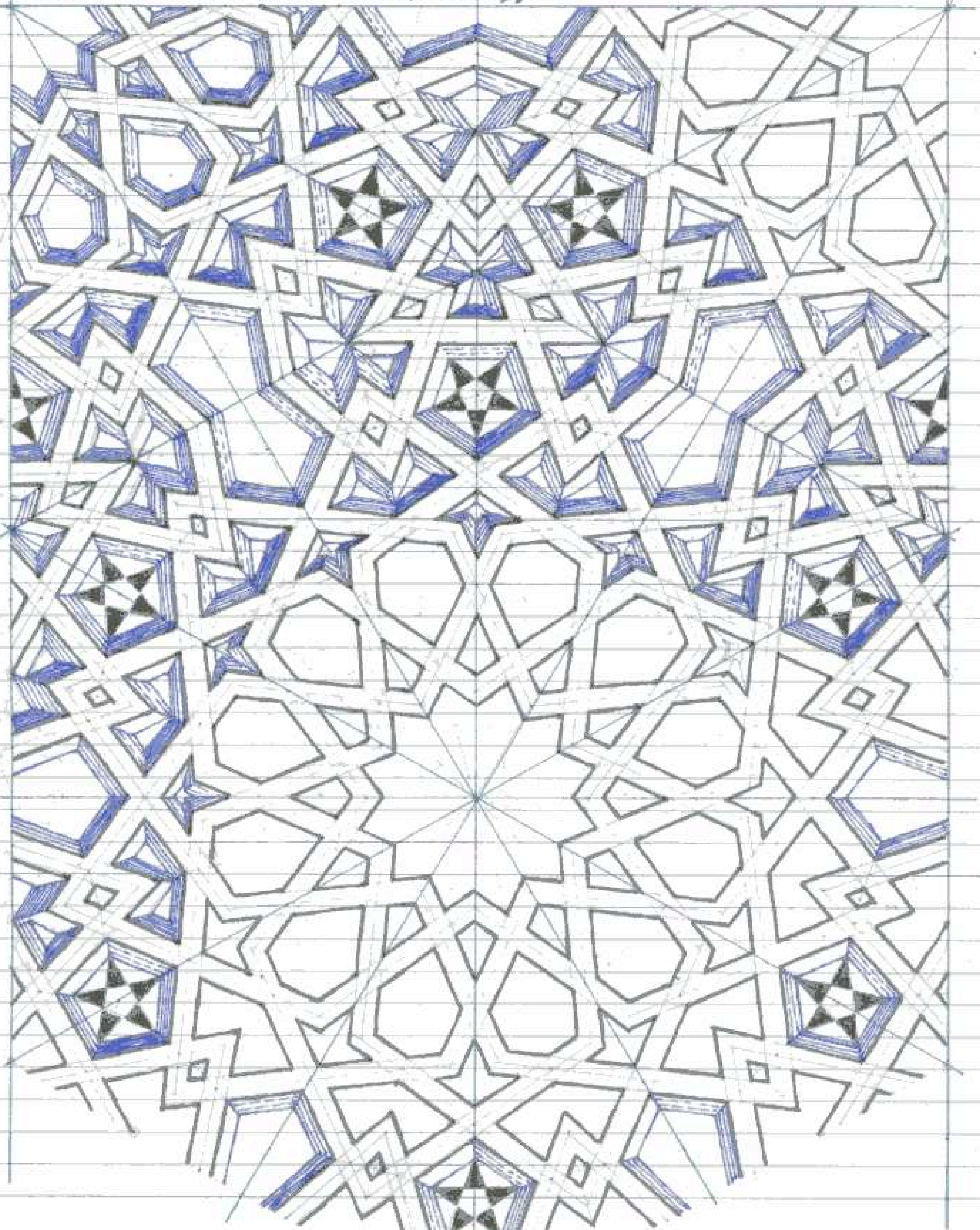
>2-coordination: between more than two star-centres, some of which may be interstitial or secondary stars. However, no one pentagon need be aligned with more than two centres simultaneously. Angle α cannot have a constant value throughout the pattern. An example of "multiple-coordination" is shown in the authentic type VI pattern on p. 252, where the peripheral pentagons surrounding the interstitial 7-stars are coordinated with both the 12-rosettes and the 16-rosettes. The differences between the two values for α are small, however, even if we assume the 7-star is regular. For the 7,12 coordination α = 69°.643, whereas for the 7,16 coordination α = 71°.518

Fig. 266 opposite: Peripheral pentagons are regular pentagons, their ~~vertices~~ centres lying on ~~reg~~ vertices of regular 12-gons and equilateral triangles, which share a common edge length. Circumradii of pentagons is exactly 3/8 of this edge length. The resulting proportions of the pattern match the original so closely that it is almost certain this was the basis on which the latter was laid out.

Fri 15 March 1985

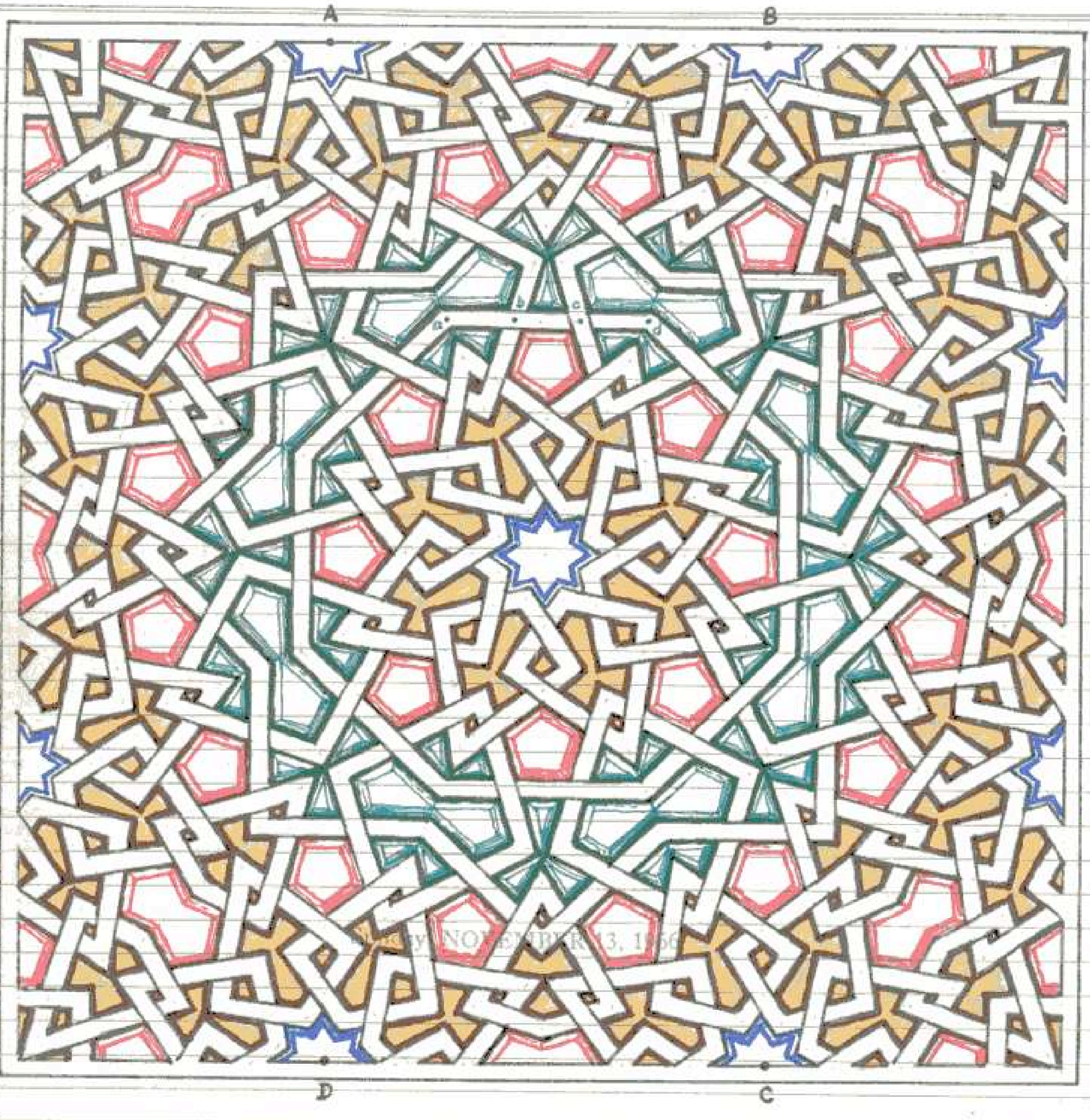
H1(4×2)12,12/VI-2A

Cairo: inlaid door from tomb of Mu'ayyad.



267 | Type VI Patterns
1-coordination

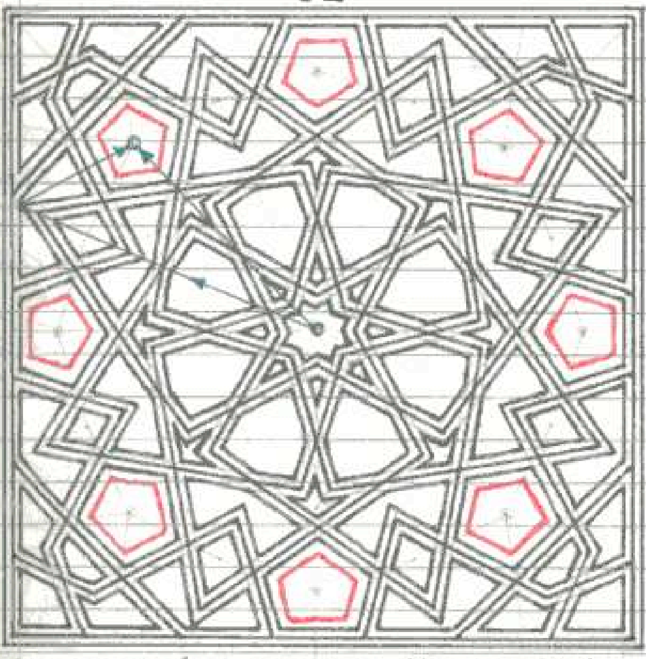
Sat 16 March 1985



This square panel is developed from a rectangular panel on centres ABCD, from the Wekala of Kait Bey (15th cent.), Cairo (S. Lane-Poole, 1886). The pentagons are regular, but although the pattern is related to fig. 268 A the construction is different. On the upper edge of the inner octagon, points a, b, c & d are equidistant. Related to the type VI with 16 & 8, but here the pentagons have $\alpha = 73°.125$ (see p. 261).

Sp1(2x2)8,8/VI-2A

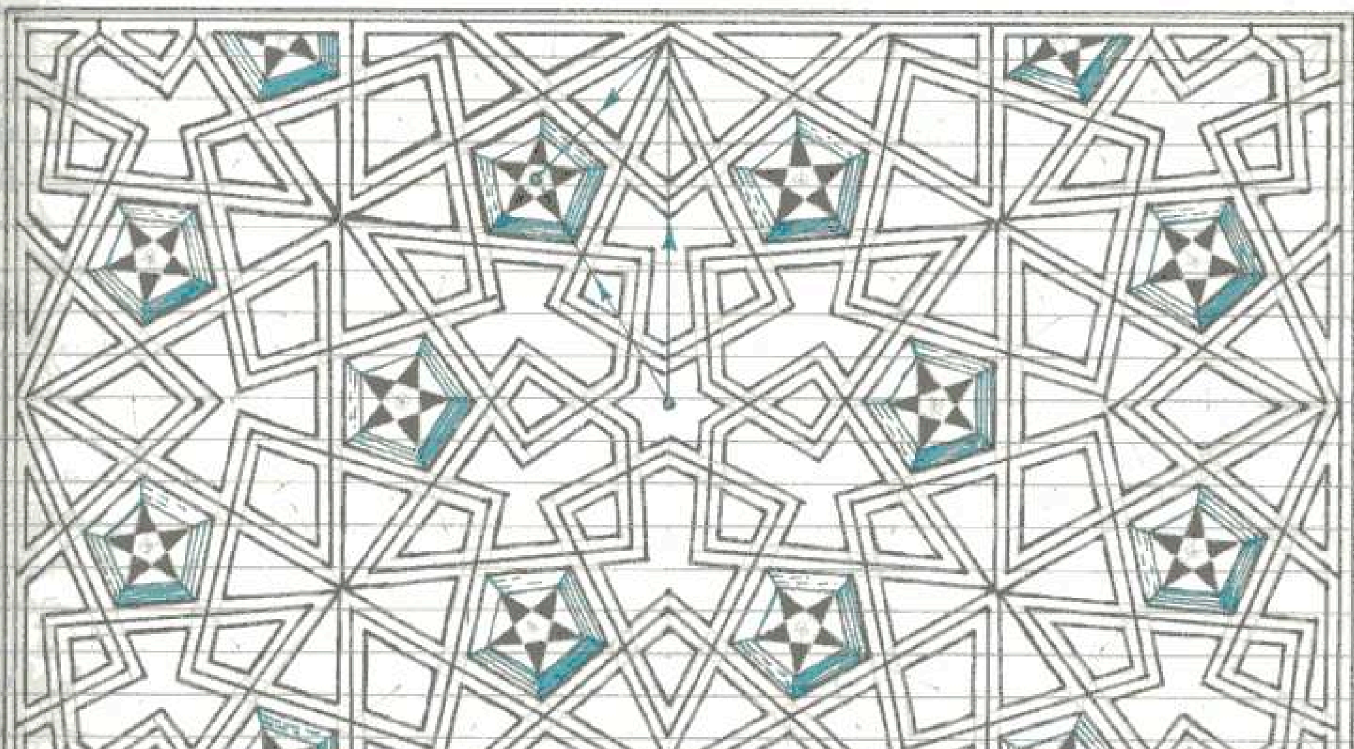
A



Cairo; minbar, mosque ibn Al-Ala'

B

Pentagons non-regular, BAC = 30°



C Pentagons regular, construction corresponding to A above. See also a construction corresponding to B above in Rempel (1961) fig. 102.

(original -)

Mon 18 March 1985.

Tuesday, NOVEMBER 15, 1966

Types and Degrees of Coordination of Peripheral Elements

We shall use the term "peripheral coordination" to refer to the particular geometrical relations between peripheral elements pertaining to one or more star-motifs, including any small adjustments or approximations necessary to preserve collinear links or to make the peripheral elements as regular as possible. As we have just seen in type VI patterns, Bourgoin's (1879) "ajustement pentagonal" is one aspect of peripheral coordinations.

When we have a pair of stars or star-motifs M, N in a conjoint or direct collinear link there is a small space remaining between the shared vertex and the adjacent M- and N-vertices. In this area lies the first collateral intersection m1.n1. This area may be termed the peripheral space, and any subsidiary pattern element P with at least approximately regular P-fold symmetry (P>2) drawn here (and centred on m1.n1) may be termed a peripheral element. The interstitial space in contrast may be defined as the space remaining at the centre of any polygon formed by collinear links (the sides of the polygon) between pairs of adjacent star-motifs (centred on the vertices of the polygon).

Peripheral elements as thus defined are linked, or coordinated with 2 principal star-motifs M and N, and so the peripheral coordination is of degree 2. Different degrees of peripheral coordination have been briefly defined on page 265. In most patterns peripheral coordination is of degree 2 and involves 5-fold elements, which are usually only approximately regular. There is a finite number of cases for each regular peripheral element P with exact P-fold symmetry in which collinear links can be maintained between every possible pair of centres, M, N and P, in an example of 2-degree coordination. When perfect regularity is incompatible with collinear links between all possible pairs of centres, the regularity of the main star-motifs, M and N takes precedence, and either (a) the peripheral element P has only approximately P-fold symmetry, or (b) the links M-P and N-P are only approximately collinear. The first alternative is the more usual.

Regular peripheral elements related to just one star motif have already been investigated on pp 19-20 (see also pp 181-190). Perfectly regular peripheral coordination of degree 2 can only be achieved with a pair of dissimilar principal stars or star-motifs if we relax the requirement that the P-P link must be collinear. As we have seen, this is normally achieved in type VI coordination, using the standard, or α-construction, whether or not the peripheral elements retain perfectly regular P-fold symmetry. Again, the number of solutions is strictly limited for principal star-motifs and peripheral elements which are both regular.

A purely geometrical enumeration of all possible kinds of links between M, N and P centres in degree 2 peripheral coordination would produce many cases which were of little use in pattern formation, so we will limit ourselves to the more useful among them, and in particular to those cases actually employed in authentic Islamic geometrical ornament. Since the peripheral element is either approximately or exactly symmetrical about both radii M1 and N1, and does not overlap the radii collinear with the line joining centres M and N, it is entirely contained within the 2nd collateral triangle.

We shall base a preliminary classification of types of peripheral coordination of degree 2 on

271 PERIPHERAL COORDINATION
DEGREE 2

RAH Tue 19 March 1985.

Those cases in which $M = N$ and M, N and P all possess regular rotational symmetry. The two star-motifs will usually be of the same type, and will join up with their shared peripheral elements in an identical manner (we have seen that this condition is frequently relaxed in type VI patterns, to avoid the use of D/O-rosettes, but even in such cases both rosettes could have been constructed on the same basis, whether C/A, P/C or D/O-rosettes).

Considering first the relation between a pair of adjacent peripheral elements: if a neighbouring pair of peripheral elements shares an edge, or one or two vertices, then they are <u>conjoint</u>, otherwise <u>disjoint</u>. Secondly, consider the relation of the shared peripheral element with its M- and N-motifs: if the peripheral element presents a vertex to its parent motif, then we may refer to this as <u>vertex-coordination</u>. If it presents an edge, or a pair of edges bordering a re-entrant angle to its M- and N-motifs we refer to this as <u>edge-coordination</u>. Finally, in each case the peripheral element may be realized either as a P-gon or as a P-star. Consideration of all possible combinations of these alternatives leads us to recognize 8 possible types of peripheral coordination of degree two, all ~~except one~~ of which occur in authentic Islamic ornament. These are shown in fig. 272 for $P = 5$, $M = N = 10$; and partially for $P = 7$, $M = N = 14$ in fig. 274. In the terminology of pp 19-20 $d = 2$ in the first case, and $d = 3$ in the second case, and we only consider cases in which d has the same value toward both the M and N sides of the peripheral element.

Such a classification of peripheral coordination types is broader and at a more fundamental level than the previously recognized <u>pattern types I-XII</u>, since the latter were based purely on the central values, $M = N = 10$, of the (3×2) rhomb series, which

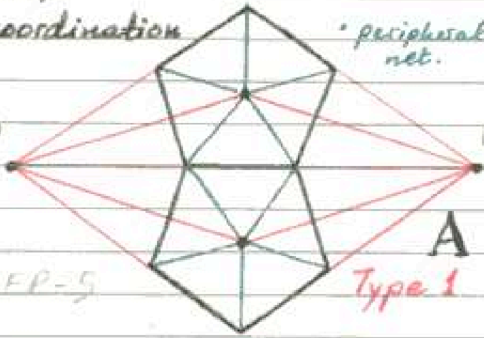
Tue 19 March 1985

N.B. An additional type which might be considered has parallel links between adjacent peripheral 5-stars; this is very rare in authentic patterns.

Friday, NOVEMBER 18, 1966

Conjoint, Edge-coordination 5-gon

M=10



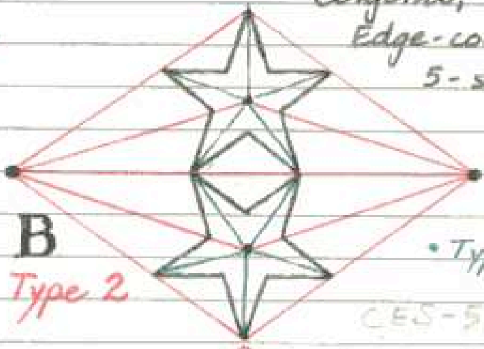
N=10

· peripheral pentagon net.

A Type 1

CEP-5

Conjoint, Edge-coordination 5-star

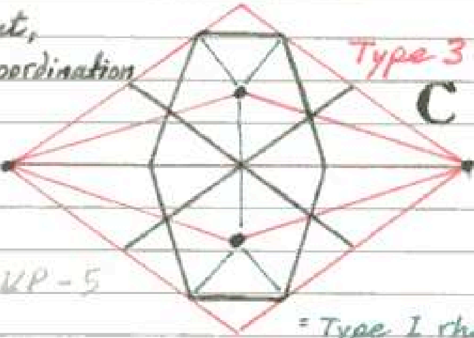


B Type 2

· Types XI, XII rhombs.

CES-5

Conjoint, Vertex-coordination 5-gon

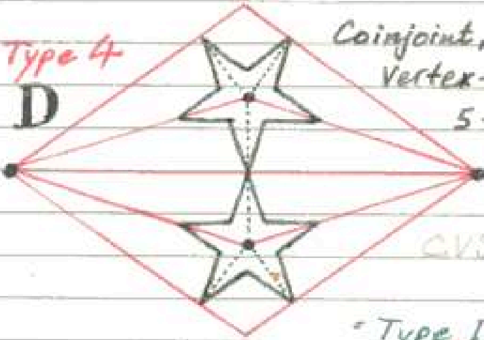


Type 3 C

CVP-5

· Type I rhomb

Coinjoint, Vertex-coordination 5-star



Type 4 D

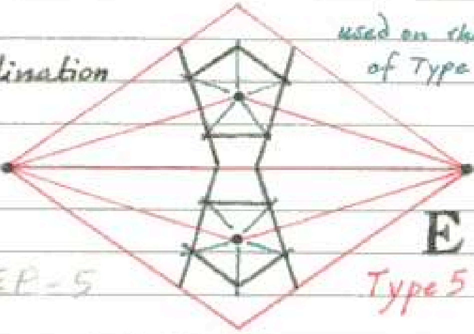
CVS-5

· Type II/III rhomb

↑ CONJOINT

↓ DISJOINT

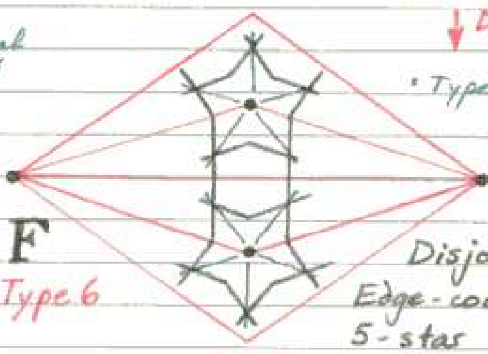
Disjoint, Edge-coordination 5-gon



used on rhomb diagonal of Type VII pattern *

E Type 5

DEP-5



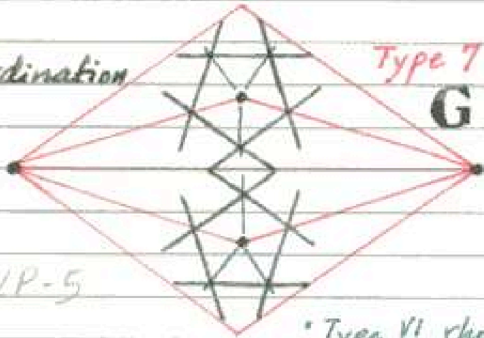
· Type VIII rhomb

DES-5

F Type 6

Disjoint, Edge-coordination 5-star

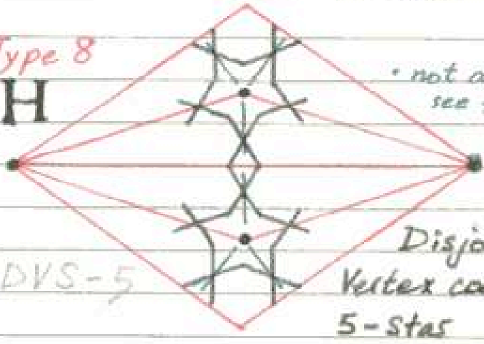
Disjoint, Vertex-coordination 5-gon



Type 7 G

DVP-5

· Type VI rhomb



Type 8 H

· not authentic? see fig 276 and p. 275.

DVS-5

Disjoint, Vertex coordination 5-star

The 8 primary types of peripheral 2-coordination, for P=5 and M=N=10

* a (3×2) rhomb with edge of type 5 is possible, but interstitial pattern is unknown

see fig 275

 Tue 19 March 1985

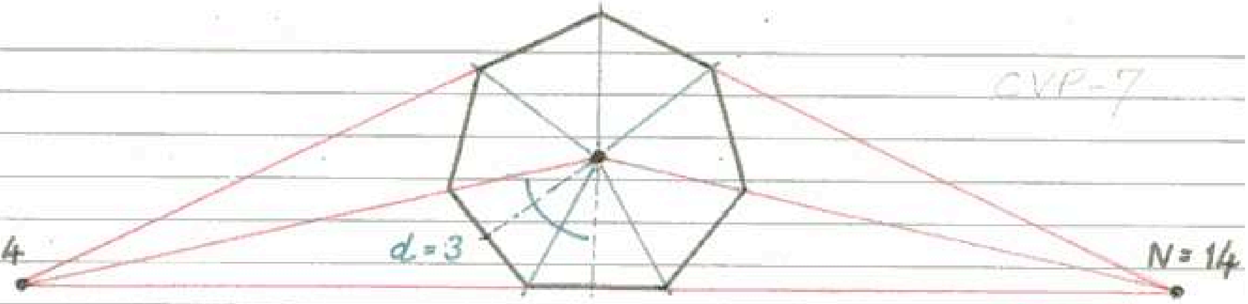
are not always completely adaptable to other rhomb sizes, or even in some cases to extreme members of the (3×2) rhomb series itself. This is primarily because the definition of pattern types involves the nature of the interstitial pattern, in addition to that of the motif type and peripheral elements. Furthermore, a wider viewpoint brings to light many configurations which are not applicable to (3×2)10,10 or other members of the (3×2) rhomb series. The interstitial pattern is quite independent of any particular peripheral coordination, but the amount of space remaining for the interstitial pattern depends on the exact shape of the rhomb or other integral polygon as well as the relative sizes of the constituent star-motifs.

As can be seen from fig. 272 the eight types of peripheral coordination for P=5 are nearly all referable to authentic pattern types in which definite peripheral elements are employed. Type 5 does not occur as a rhomb edge in any of our 12 pattern types, but it is present along the shorter axis of one of the rhombs in type VII; it can also occur along the rhomb edge of many other kinds of patterns. Pattern types IX, X are not represented of course, since these two do not use genuine peripheral elements, although their construction does make use of the first collateral intersection m.t.n.t. It is noteworthy that these two pattern types, like our type VII, makes use of two distinct kinds of motif within one rhomb, which would largely preclude the formation of proper peripheral elements in any case.

A (3×2)10,10 rhomb can be constructed with edges of type 5 peripheral coordination (P=5), but the resulting interstitial pattern is slightly awkward (fig. 275); and this may have deterred the original artisans who were experimenting with new combinations from using this rhomb, since as far as I know it does not occur

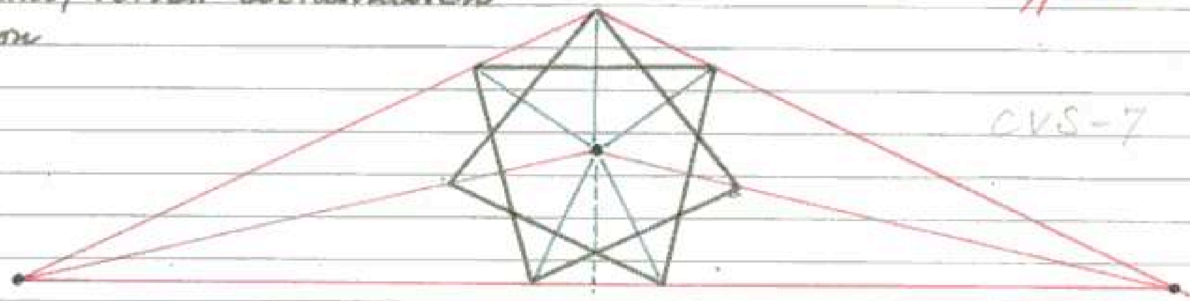
Tue 19 March 1985

Monday, NOVEMBER 21, 1966



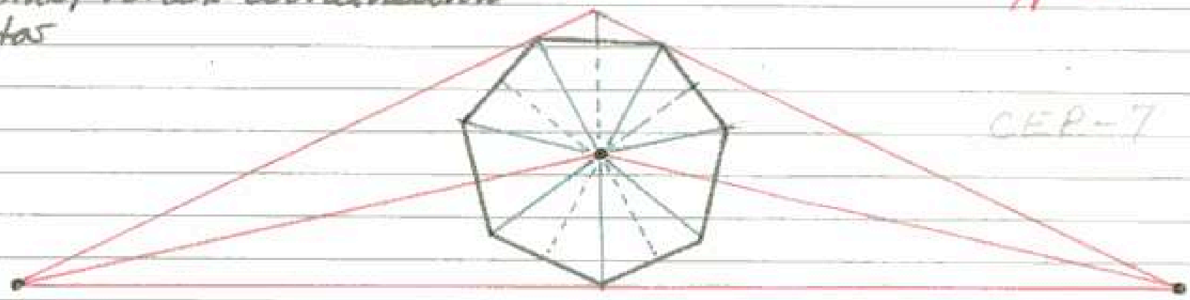
A Conjoint, vertex-coordination 7-gon

Type 1



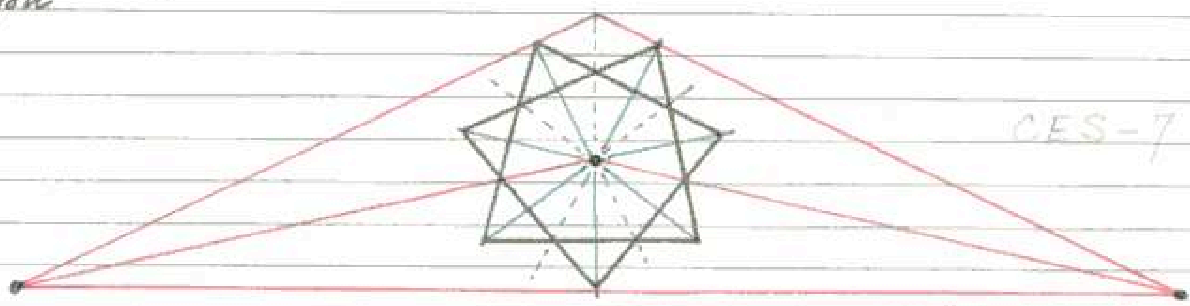
B Conjoint, vertex-coordination 7-star

Type 2



C Conjoint, edge-coordination 7-gon

Type 3



D Conjoint, edge-coordination 7-star

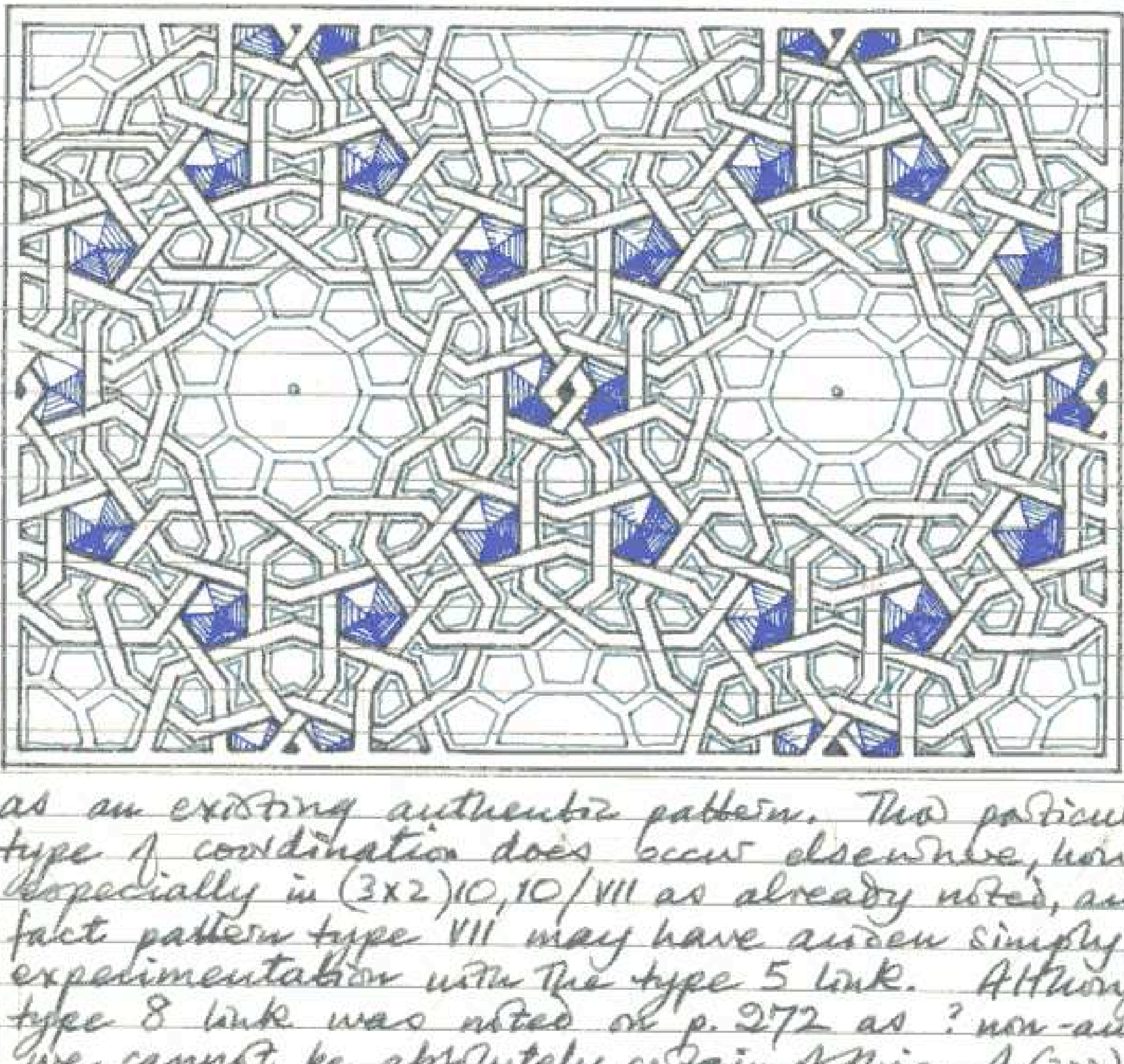
Type 4

Types of peripheral coordination, degree 2, for P = 7

275 | PERIPHERAL COORDINATION DEGREE 2

type 5 for P = 5

Wed 20 March 1985



not authentic, so far as known at present.

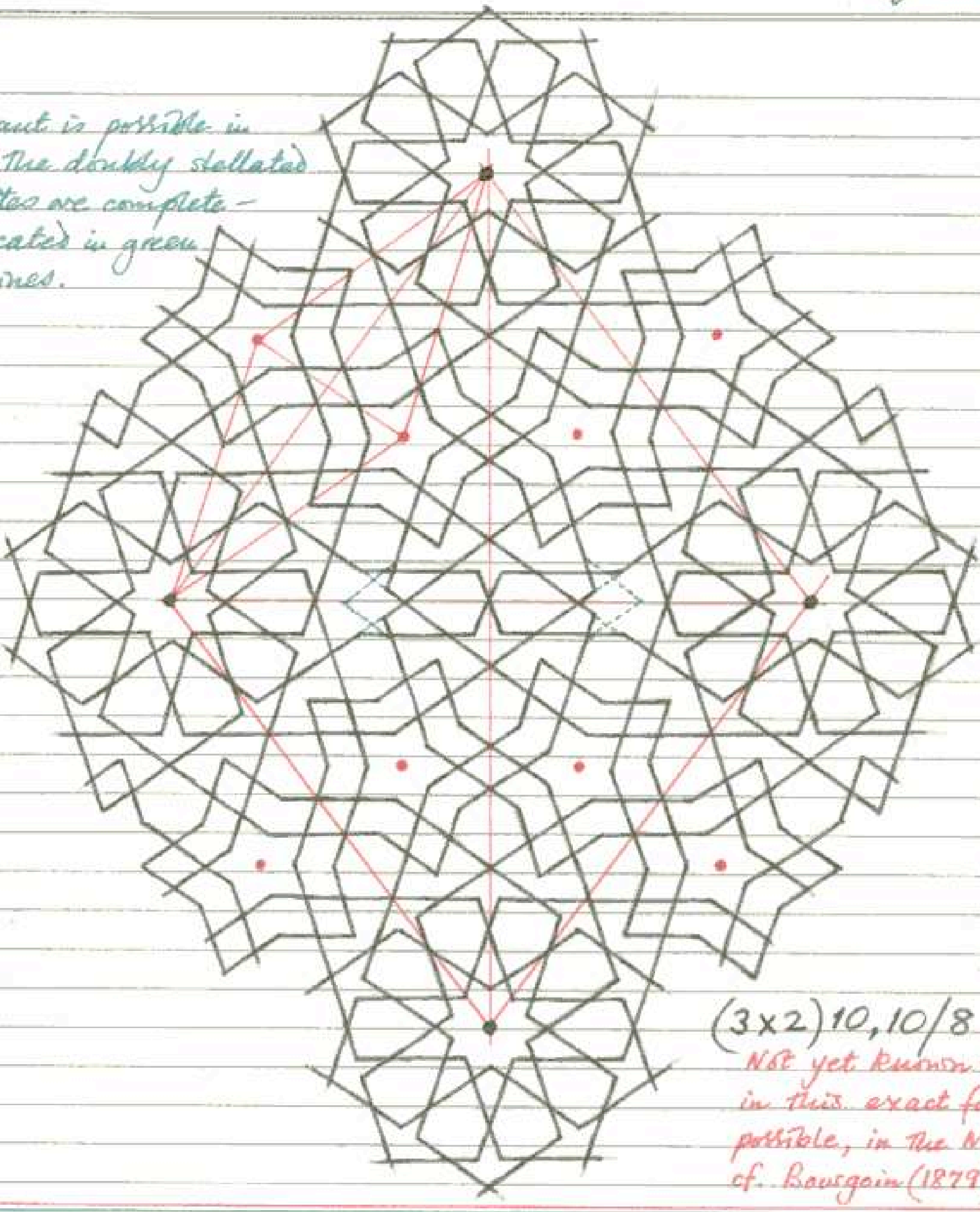
as an existing authentic pattern. This particular type of coordination does occur elsewhere, however, especially in (3x2)10,10/VII as already noted, and in fact pattern type VII may have arisen simply from experimentation with the type 5 link. Although the type 8 link was noted on p. 272 as ? non-authentic we cannot be absolutely certain of this. A (3x2)10,10 rhomb with this link along each edge can be constructed (see fig opposite, on p. 276) with doubly stellated 10-rosettes as the main star-motifs. Bourgoin (1879) gives two patterns (Plates 184, 185) which are rearrangements of elements in this rhomb, so this style of ornament is certainly present in the Middle East, but I don't at present know whether it exists in the form of either the (3x2)10,10/8 rhomb or the type 8 link alone.

The basic concept underlying **coordination** in general is the formation of **collinear links** between adjacent pattern centres having exact or approximate n-fold rotational symmetry ($n \geq 2$), usually pertaining

Wednesday, NOVEMBER 23, 1966

type 8 for P-5

A variant is possible in which the doubly stellated 10-rosettes are complete – as indicated in green dotted lines.



(3x2)10,10/8

Not yet known to be authent in this exact form, but qui possible, in the Middle East. cf. Bourgoin (1879) Plates 184,18

to symmetry group D_n (see p. 181). The essence of peripheral coordination is that any peripheral element should be symmetrical, either exactly or to a close approximation, simultaneously about a radius from each of two main star-motifs.

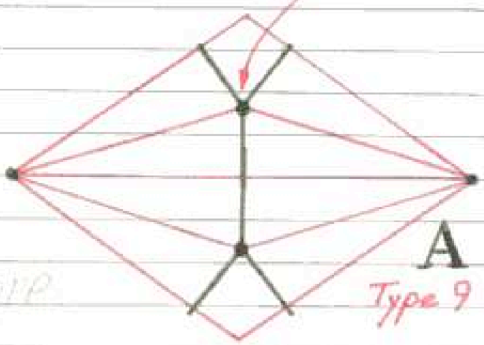
Many star patterns exist which lack any kind of peripheral elements or indeed peripheral coordination.

→

There is a wealth of such patterns, which defy any logical classification beyond one which recognizes the collinear (or other) links between their main star-motifs. Most of them seem to be relatively simple patterns, and it is probably this simplicity which largely precludes the development of peripheral elements. But these relatively simple patterns do not at the present form our main interest.

There are many patterns which, although lacking peripheral elements, do possess a kind of peripheral coordination of degree 2 in which the first collateral intersection m1.n1 (see pp. 25, 26) is used as a point of coordination. Pattern types IX and X are of this type (see p. 52) and it is the m1.n1 intersection which determines the sizes of the interlocked stars and polygons surrounding the main motifs. As previously noticed these present different motifs at the M- and N-centres, and in cases where $m \neq n$ they can usually be freely interchanged, producing quite different effects. The type of coordination shown in fig. 278A opposite produces the familiar net of limiting polygons sharing an edge which intersects the collinear ray at the median point (p. 28). This should rank as a pattern type in its own right since it is so used, either alone, or in combination with type 1 for example, in many authentic patterns.

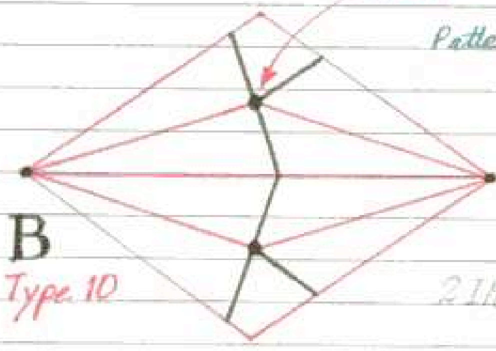
Wed 20 March 1985



201P

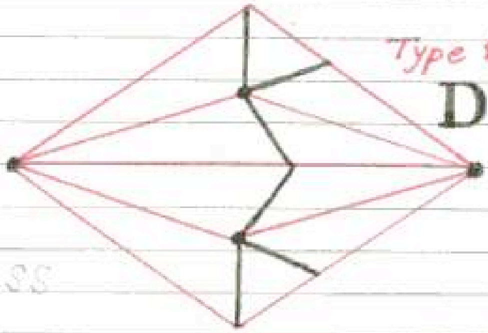
A Type 9

B Type 10



Pattern type IX

21PS



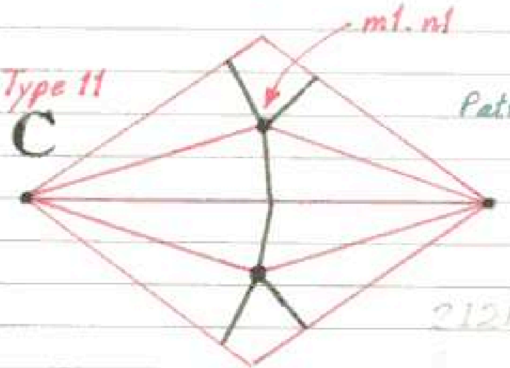
Type 12

D

21SS

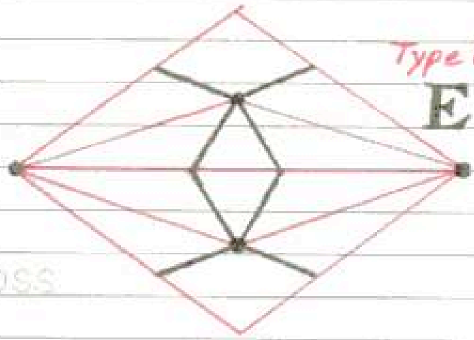
Type 11

C



Pattern type X

212PS



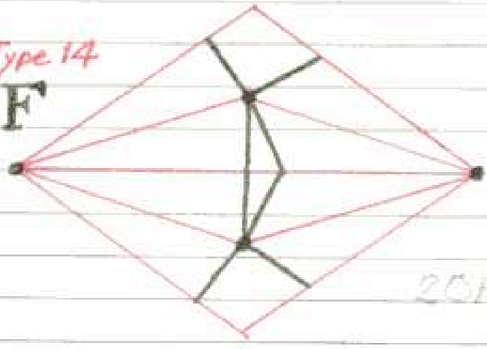
Type 13

E

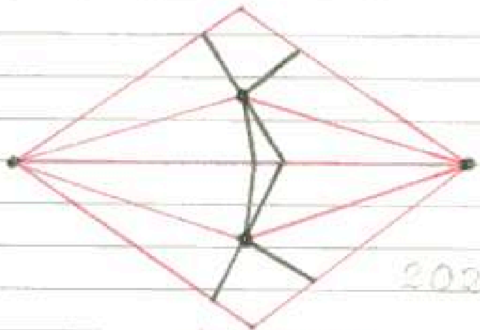
20SS

Type 14

F



20PS



202PS

G Type 15

A. J. Lee Thu 21 March 1985

Saturday, NOVEMBER 26, 1966

Type 17 Links.

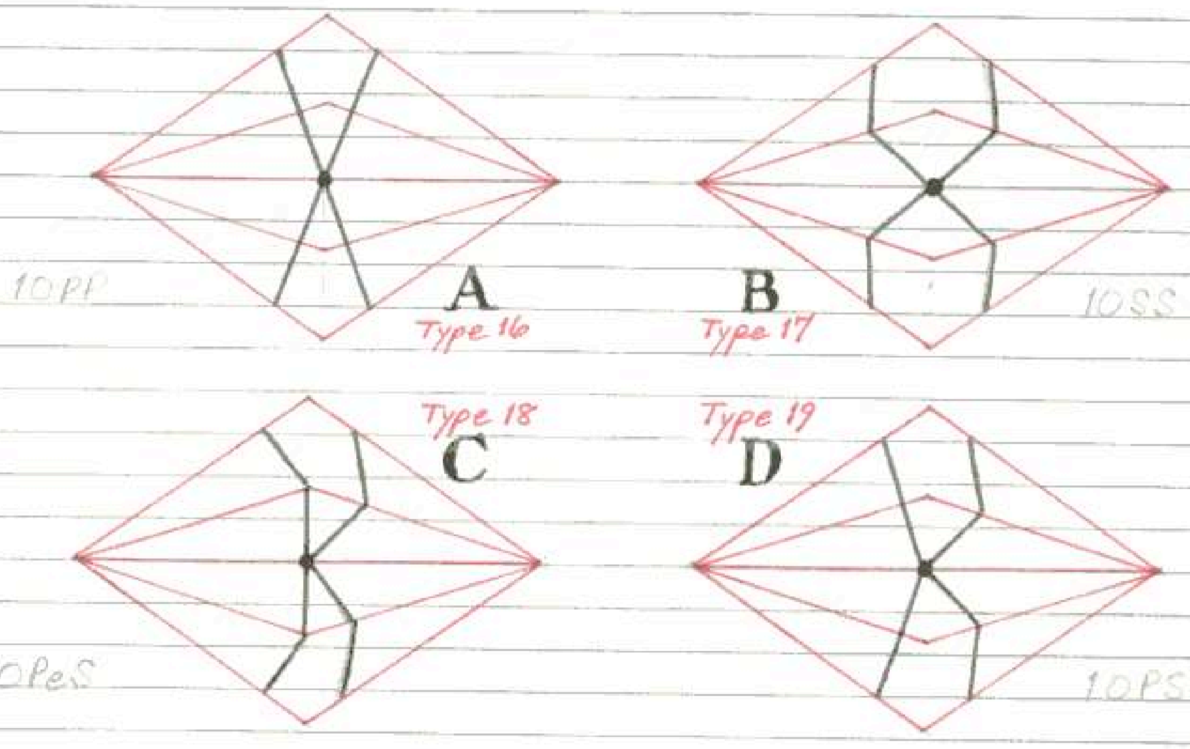
Tessellation of elongate hexagons angles 6/7, 8/7.

Mosque Mesched Makam Ali, nr. Anah, Syria (on Euphrates). N.H. Viollet, 1909.

Sunday, NOVEMBER 27, 1966

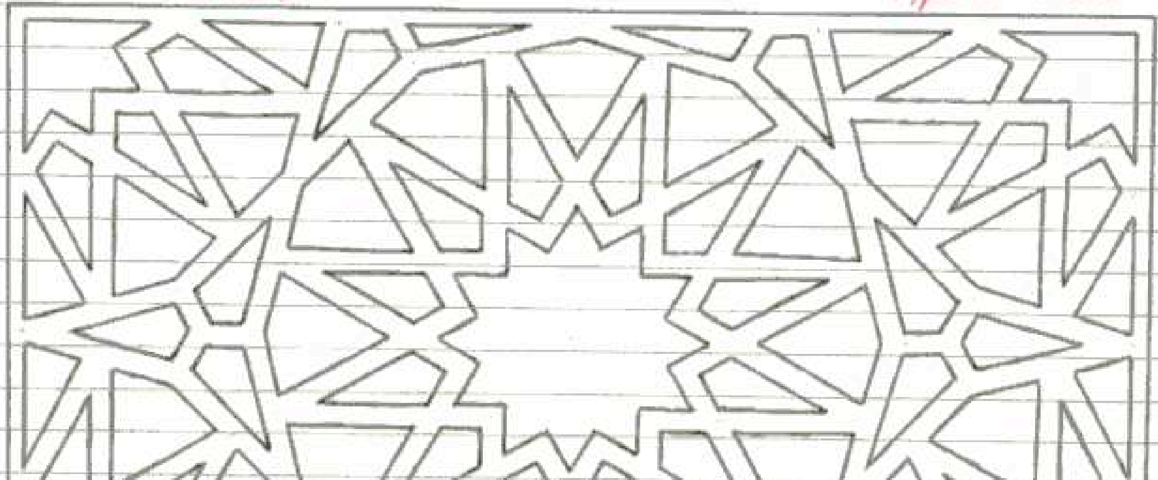
Thu 21 March 1985

Monday, NOVEMBER 28, 1966



Pierced balustrade

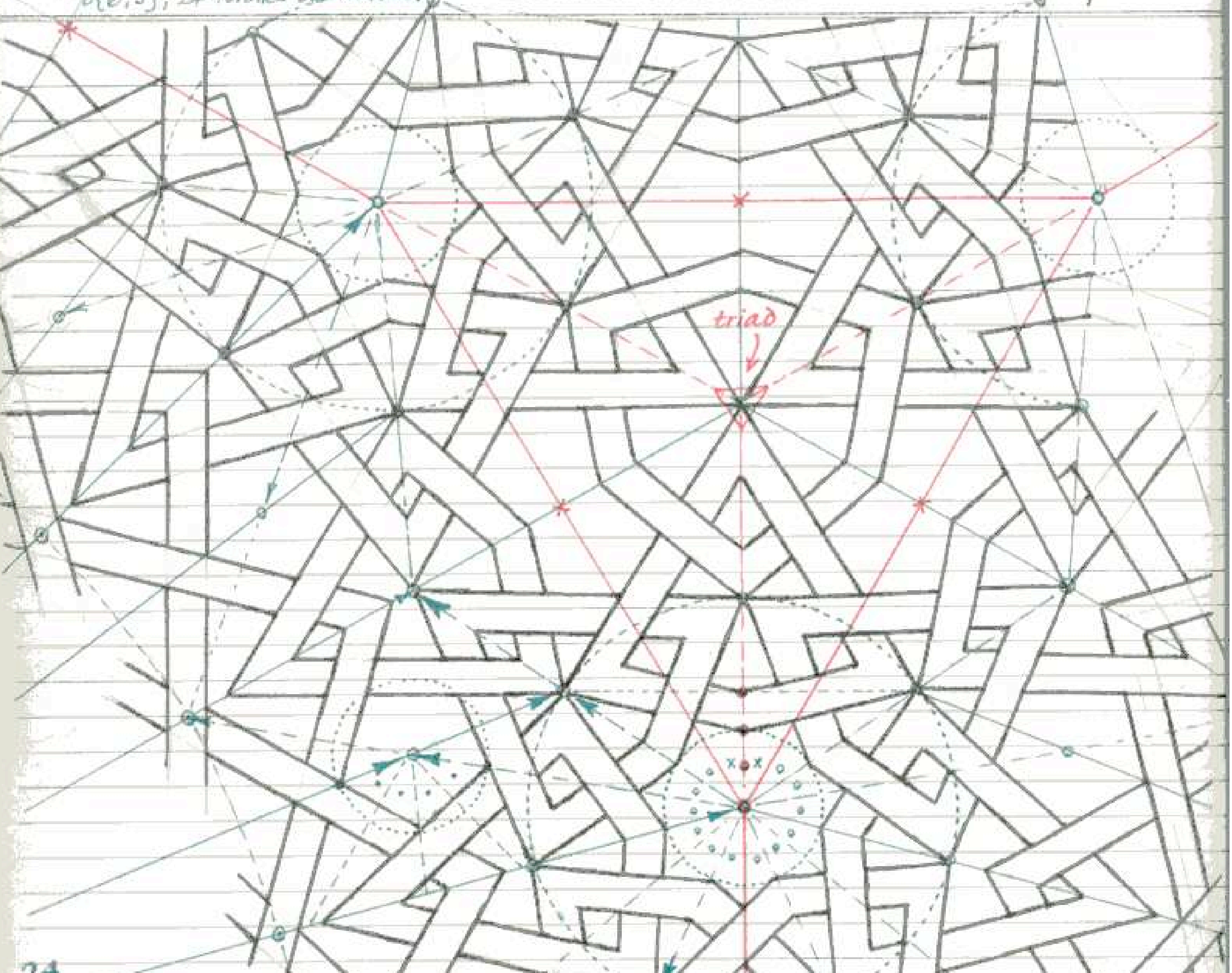
Type 16 links



E H1(4x2)12,12/16 Turkey: Iznik, Mahmud Çelebi Mosque
see K. Otto-Dorn 1941 Das Islamische İznik; Berlin. see Pl. 22.

281 | Turkey: proportions as on Nalindji Turbe (mausoleum), Konya (carved stone) Sat 23 March 1985

Approximate 7-stars on vertices of $t\{6,3\}$; 24-rosettes on hexads.



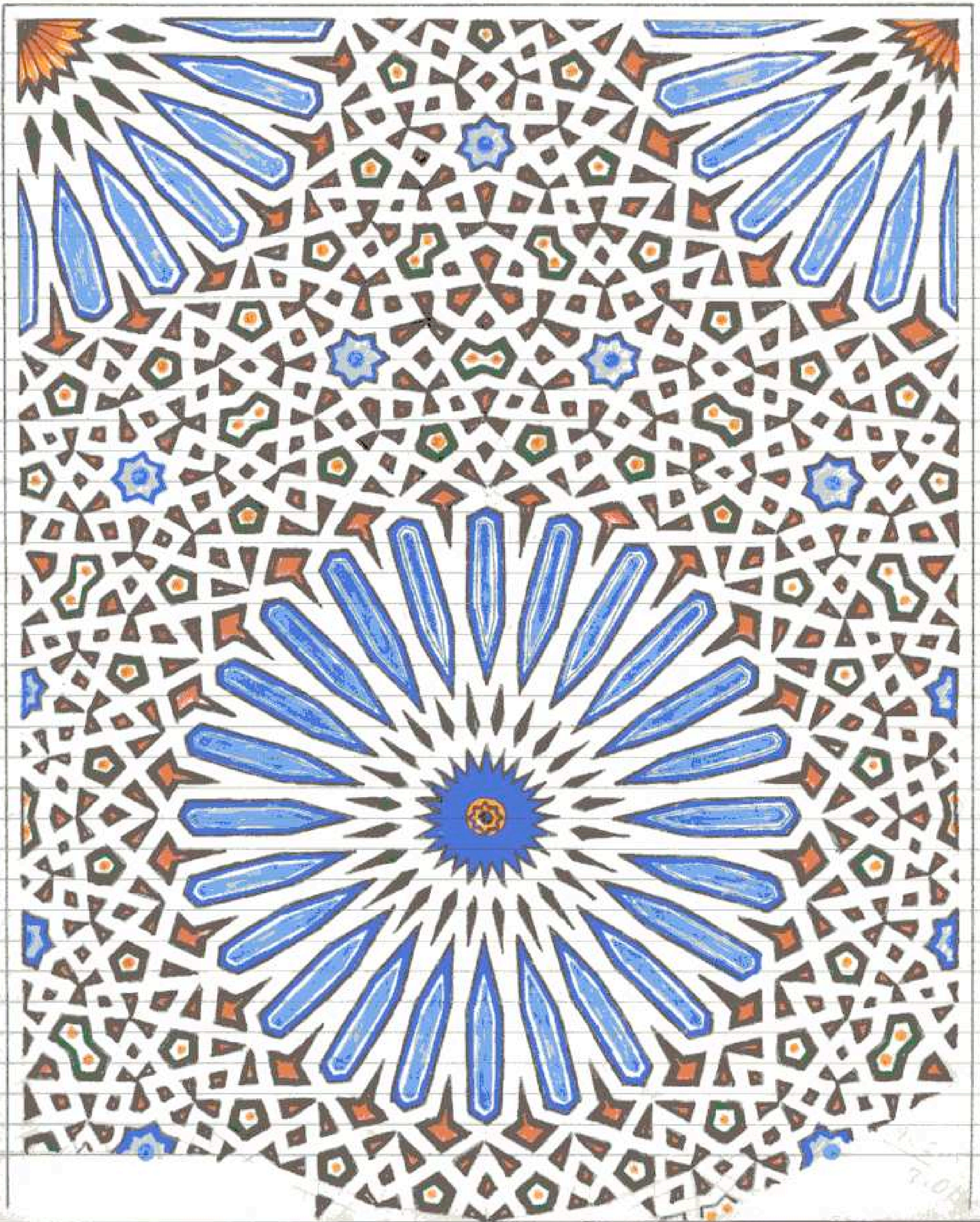
A faience mosaic version on the mihrab of the Eshrefoglu mosque, Beyşehir makes the small hexagon surrounding the triad a regular hexagon

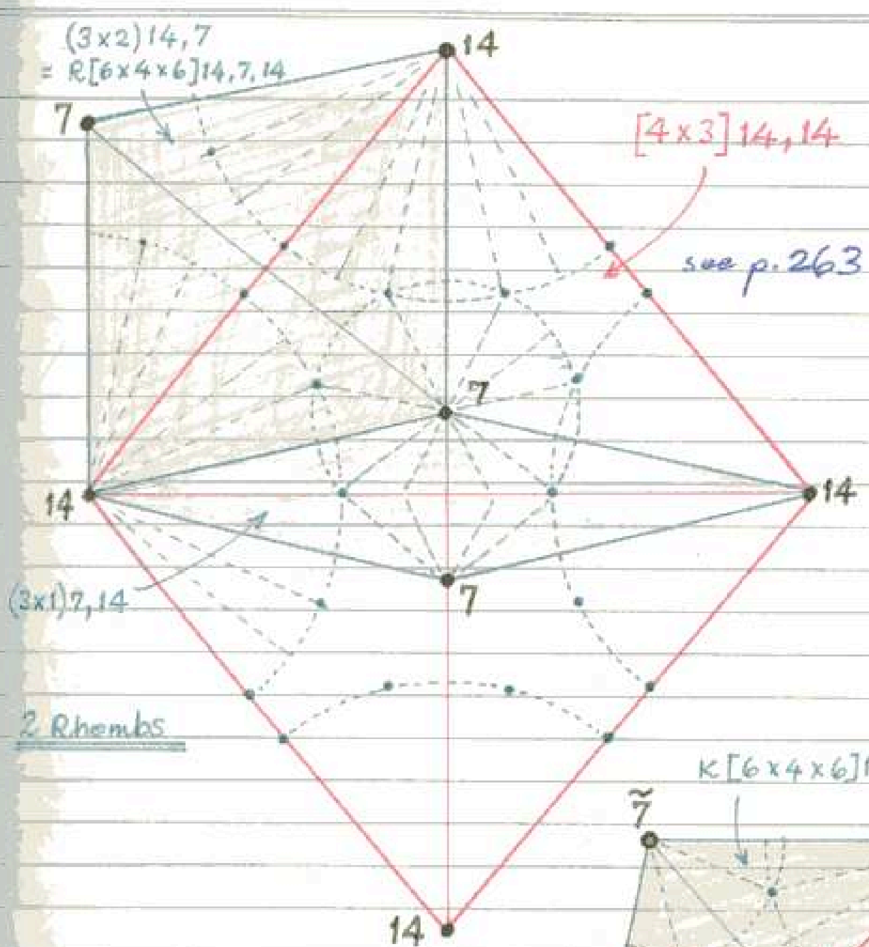
Note the formation of a 24-gon at this level. Possible for the n-centre of (3×2) threads when n > m.

Fri 22 March 1985 W.74 The approximate rhomb (3×2) 7, 24 / Link Type 7

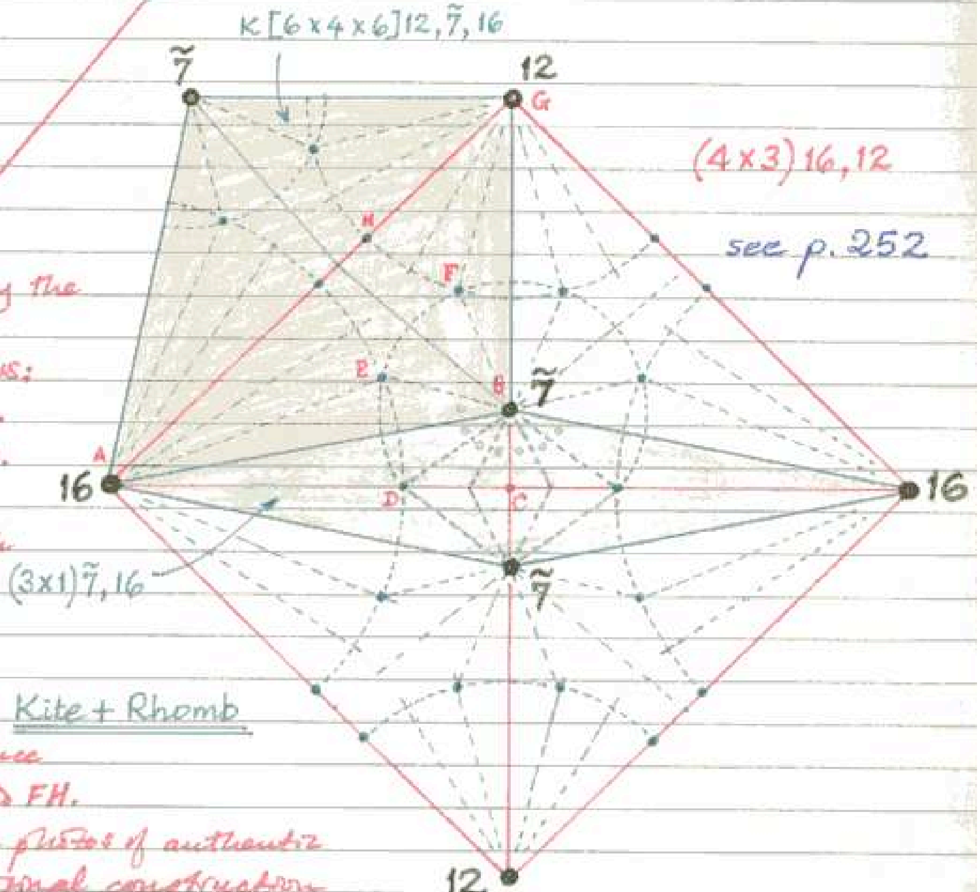
Constructions in Turkey differ slightly. KITE + RHOMB - see page 96

Turkey, Seljuk; carved stone or faience mosaic. Colours below are arbitrary.





As a 2-rhomb pattern this is H2 AAB 14,7/VI (see p. 263, and Bourgoin, 1879 Plate 168). But on the 14s alone it may be regarded as Rp1(4x3)14,14/VI-2. Transferring this pattern to other rhombs in the (4x3) series, we find that the subsidiary rhombus on the edge of the (4x3) rhomb becomes a kite – see below. After (4x3)16,12 the next choice would be (4x3)12,18 which might give a satisfactory pattern – see p. 286.



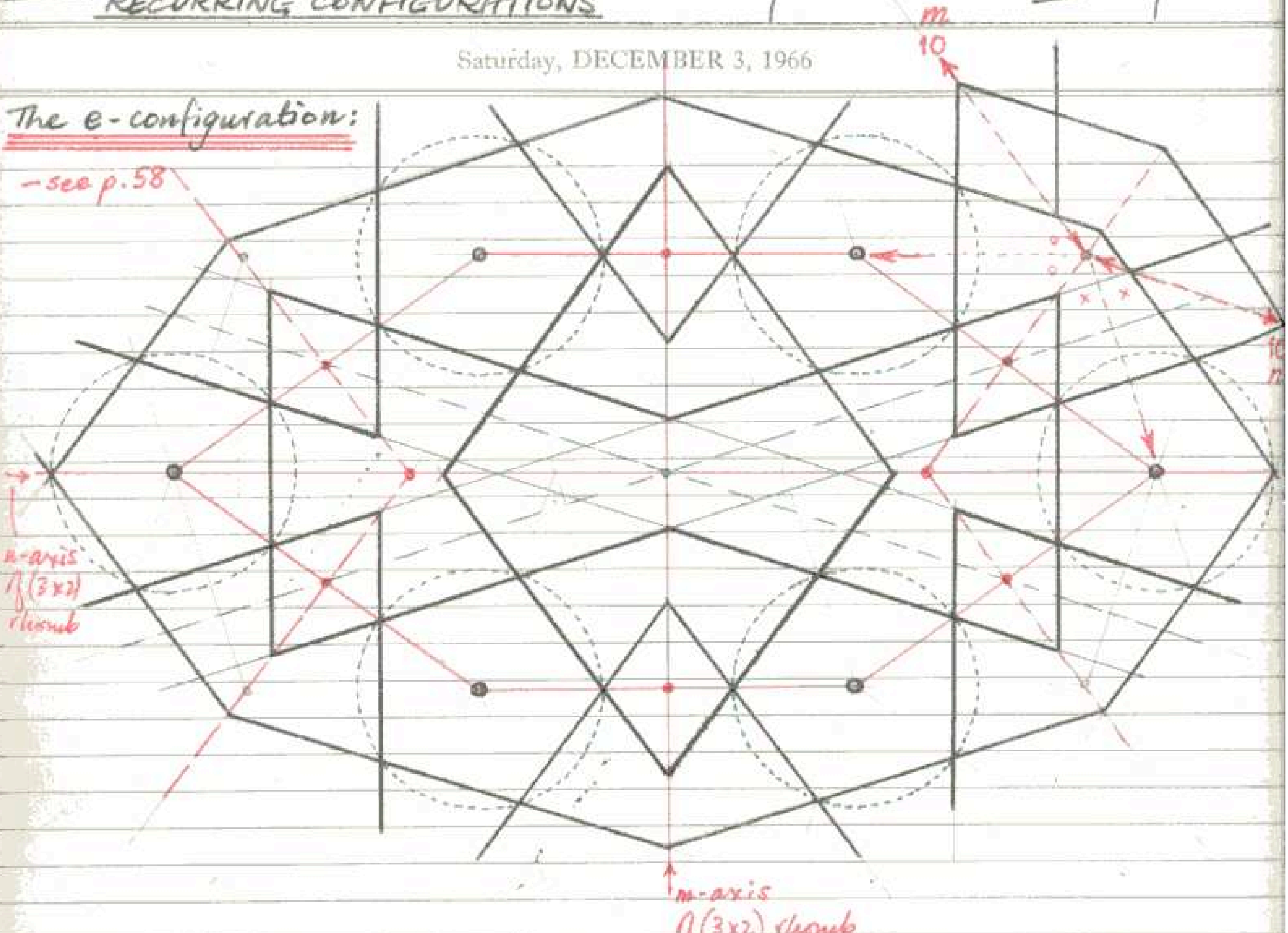
A suggested construction for finding the peripheral pentagon centres in (4x3)16,12/VI-2A is as follows: $\widehat{BAC} = \pi/16$, which determines B. $\widehat{ABD} = \frac{1}{3}\widehat{ABC}$, which determines D. Circles on A and B through D determine points E and F, both of which lie on radii from A and G respectively. Bisecting $\widehat{EBF}$ produces the remaining radius of the $\tilde{7}$-star. On this construction the distance DE is slightly longer than EF and FH. Measurements on tracings from photos of authentic versions suggest that the original construction was similar at least (e.g. the version on p. 252).

285 TYPE VI PATTERNS
RECURRING CONFIGURATIONS

Sat 30 March 1985

Saturday, DECEMBER 3, 1966

The e-configuration:



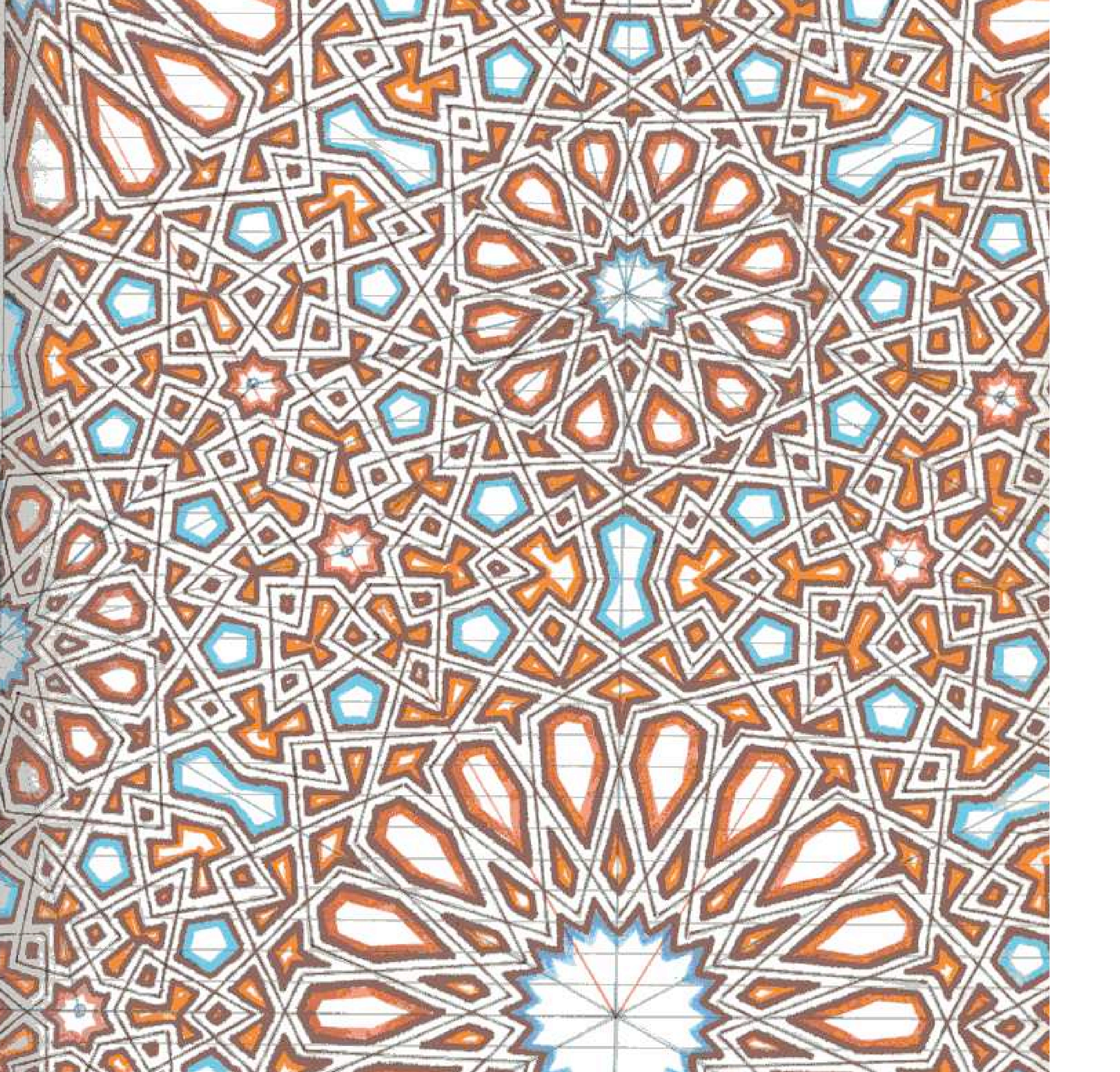
The e-configuration is characteristic of the centre of the (3×2) m, n/ I and VI – 2A rhomb (see p. 60, fig. A and p. 66 – i.d'). The parent rhomb, (3×2) 10,10/I – 2A and –/VI–2A presents no difficulties of construction, but coordination of this configuration with different sized star-motifs simultaneously entails a number of compromises. Patterns in which the two perpendicular mirror axes m-m and n-n of the configuration are preserved are also fairly straightforward (e.g. see Sp1(3×2)12,8/VI–2A on p. 288), but coordination becomes difficult when one of the axes is distorted (fig. 290).

Sunday, DECEMBER 4, 1966

Wed 27 March 1985

H1(4×3)12,18/VI-2A

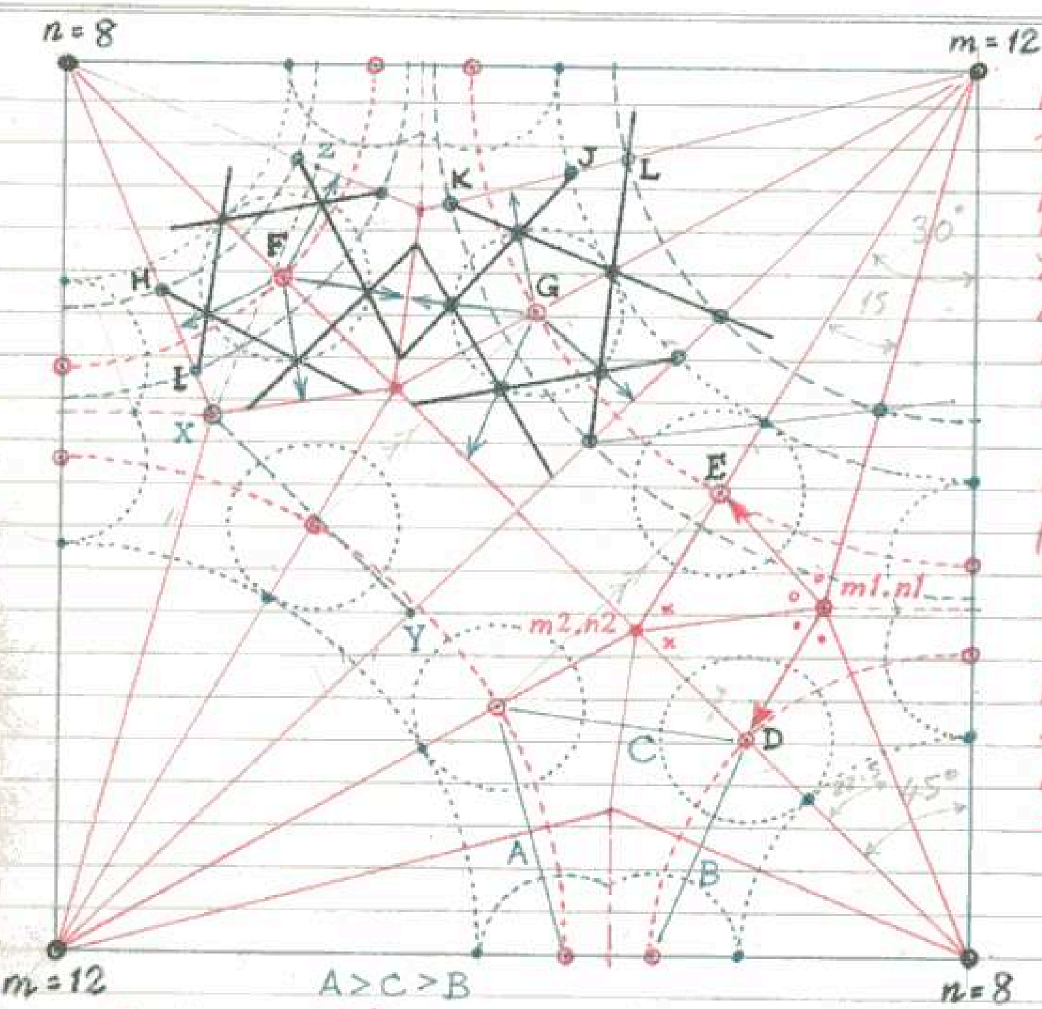
Construction as given on lower half of p. 284. Not yet known to occur as authentic pattern



H1(4×3)12,8/II is of course authentic, e.g. in Cairo, and is shown in Bourgoin (1879) Plate 137.

287 | TYPE VI PATTERNS
1- COORDINATION, PENTAGONS REGULAR

Fri 29 March 1985



Points D, E (which are the centre of the peripheral pentagons) are obtained by bisecting the angles at m1.n1. These points are then repeated symmetrically round both m- and n-centres. Radius of circles which circumscribe these peripheral pentagons is chosen as 3/8 distance B, since this is the smallest of A, B & C. Pentagons are drawn regularly within these circles, as shown at F and G

Circles centred on m, n through points H, I, J, K and L ~~which~~ will enable the pentagons to be drawn more rapidly than having to measure each one out separately (and similarly for other points – if all such points are located first for two accurately drawn pentagons, like those above, then repeated all over the pattern, by measurement or drawing circles, this will enable a ~~much~~ more rapid completion of the outlines of the whole pattern).

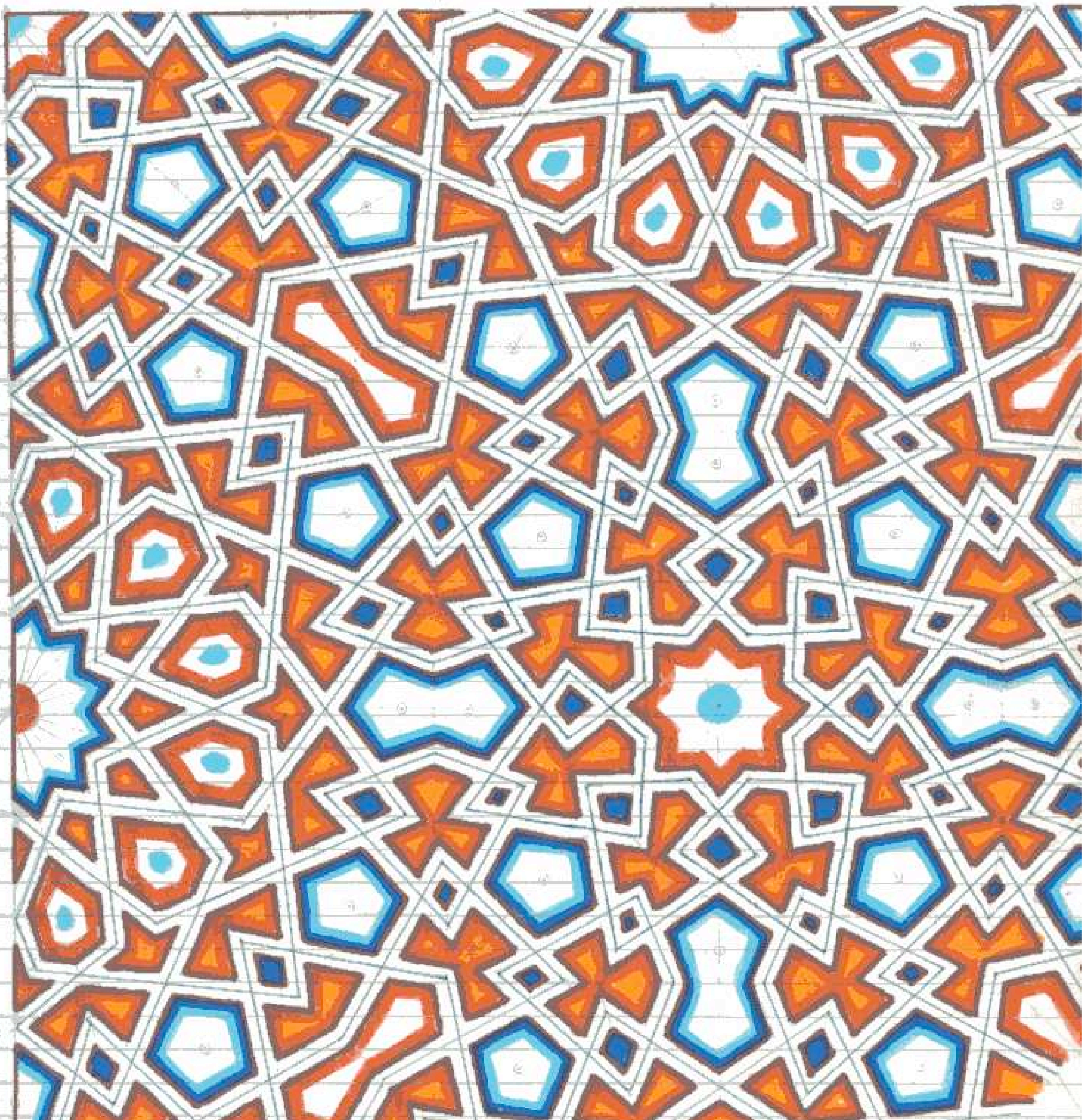
Bourgoin's construction (1879, Plate 117) insofar as I can understand him, seems to locate the centres of the peripheral pentagons round the m-star by dropping perpendicular XY from point X (= m1.n1) to the m-diagonal. He does not mention location of pentagon centres round the 8-star, but presumably this would be by dropping perpendicular XZ. Bourgoin's pentagons are only "à peu près reguliers" since he makes collinear links between adjacent pentagons, but this was emphatically not the authentic method in Cairo. The above pattern in fact has 1-coordination, so pentagons can be regular.

Fri 29 March 1985

TYPE VI PATTERNS
1-COORDINATION
PENTAGONS REGULAR

Sp1 (3x2) 12,8/VI-2A

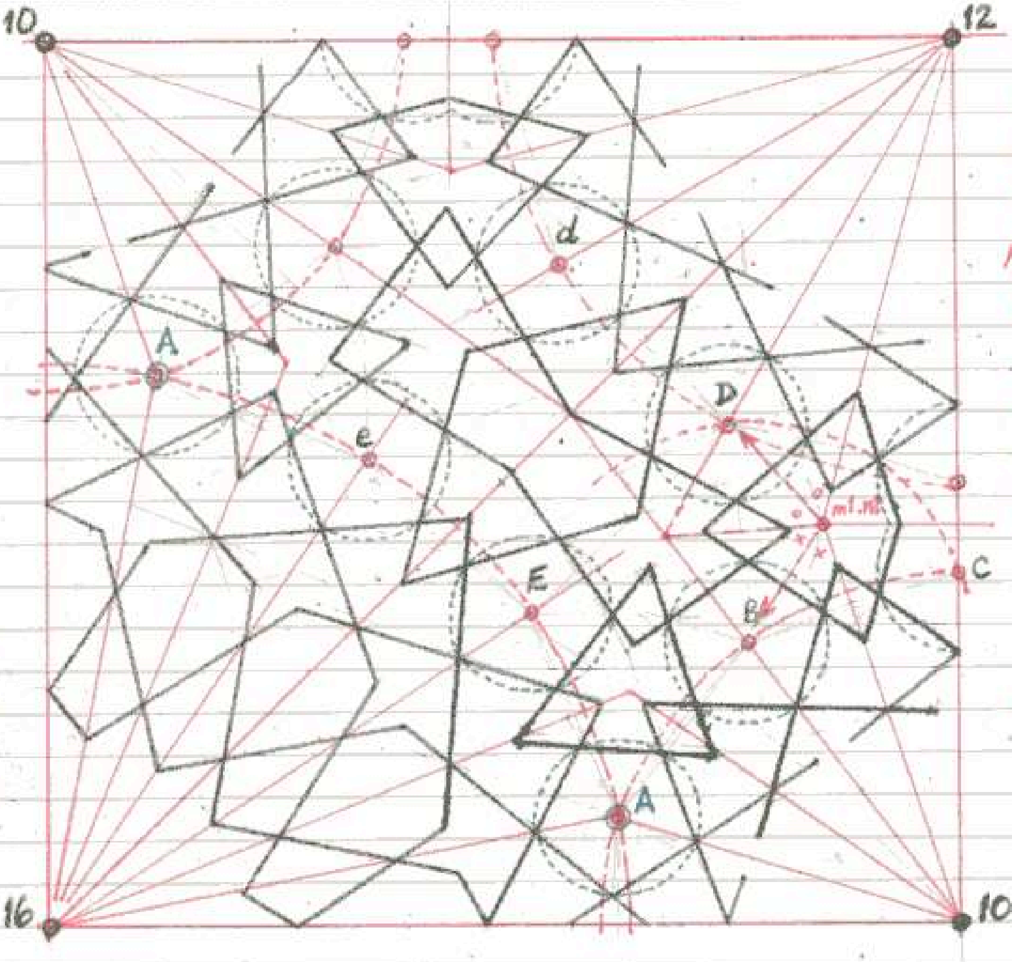
e-CONFIGURATION with 2 AXES of SYMMETRY



Sp1(3x2)12,8/VI-2A. 1-coordination throughout, so all pentagons can be made regular. Authentic, e.g. Cairo - see Al-Wahhab 1946 Vol II fig. 191. Bourgoin (1879) shows it on his Plate 117.

289 TYPE VI PATTERNS

e-Configuration with 1 axis of symmetry Fri 29 March 1985



The panel on the left is my own preliminary attempt at rationalization of the pattern. An authentic version (see Al-Wahhab, 1946 Vol II fig. 210) differs from this in a number of important features, and a comparison with this version is of great interest.

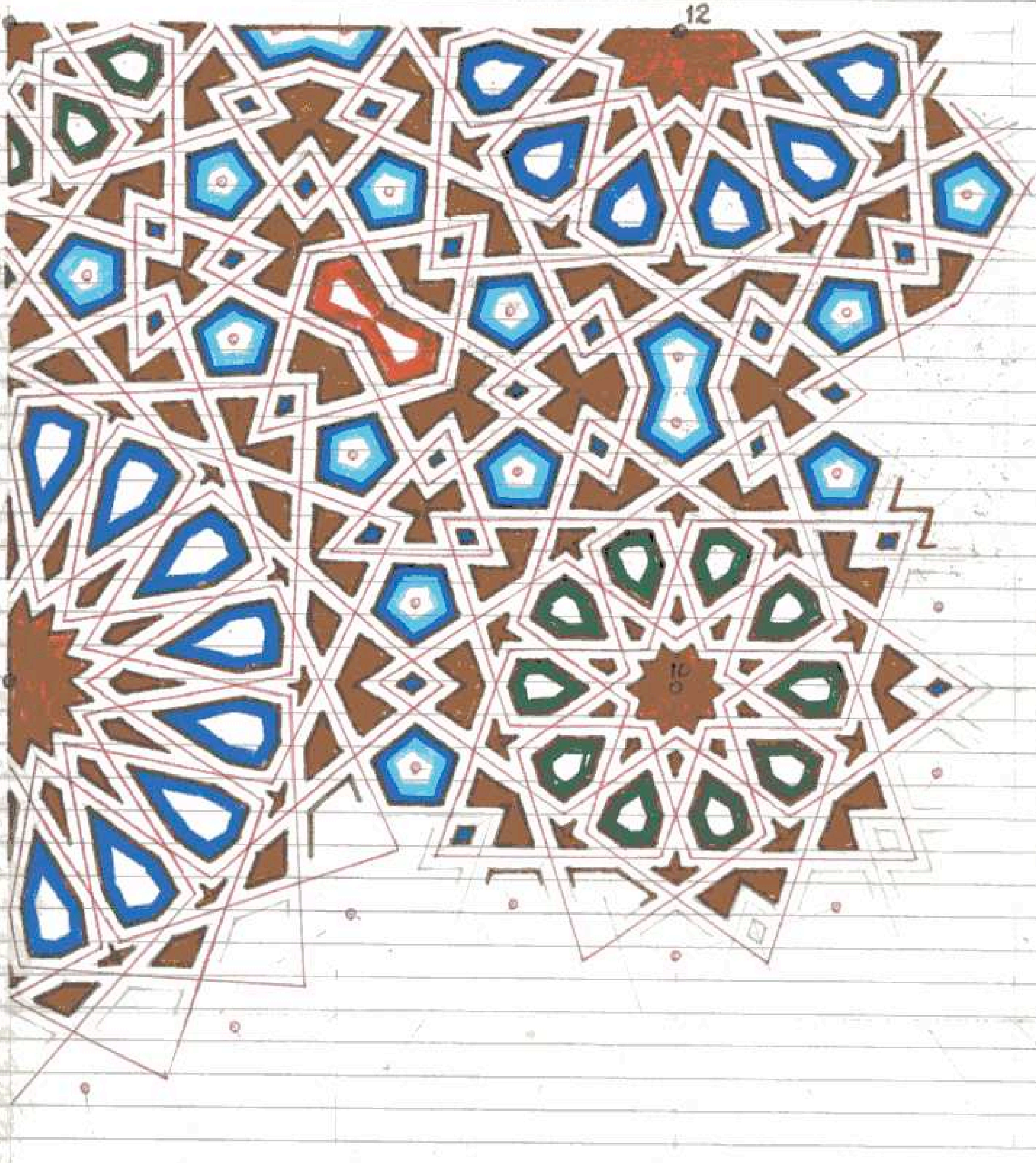
My version makes BD = BC; the authentic example has BD > BC and Dd = Ee

After Fri 29 March 1985

TYPE II PATTERNS

e-CONFIGURATION WITH 1 AXIS OF SYMMETRY

see p. 94



p. 137

Cairo: minbar of Ibn Alāla mosque (Al-Wahhab 1946 II fig. 210 p.

d-CONFIGURATION with 1 AXIS of SYMMETRY

Sun 31 March 1985

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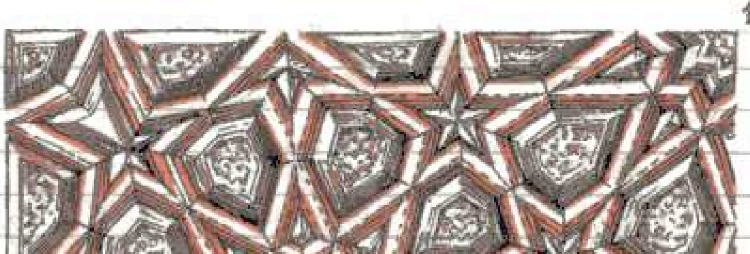


10 12

K[6x5x2]12,10,4/II-2A with interstitial 9̃

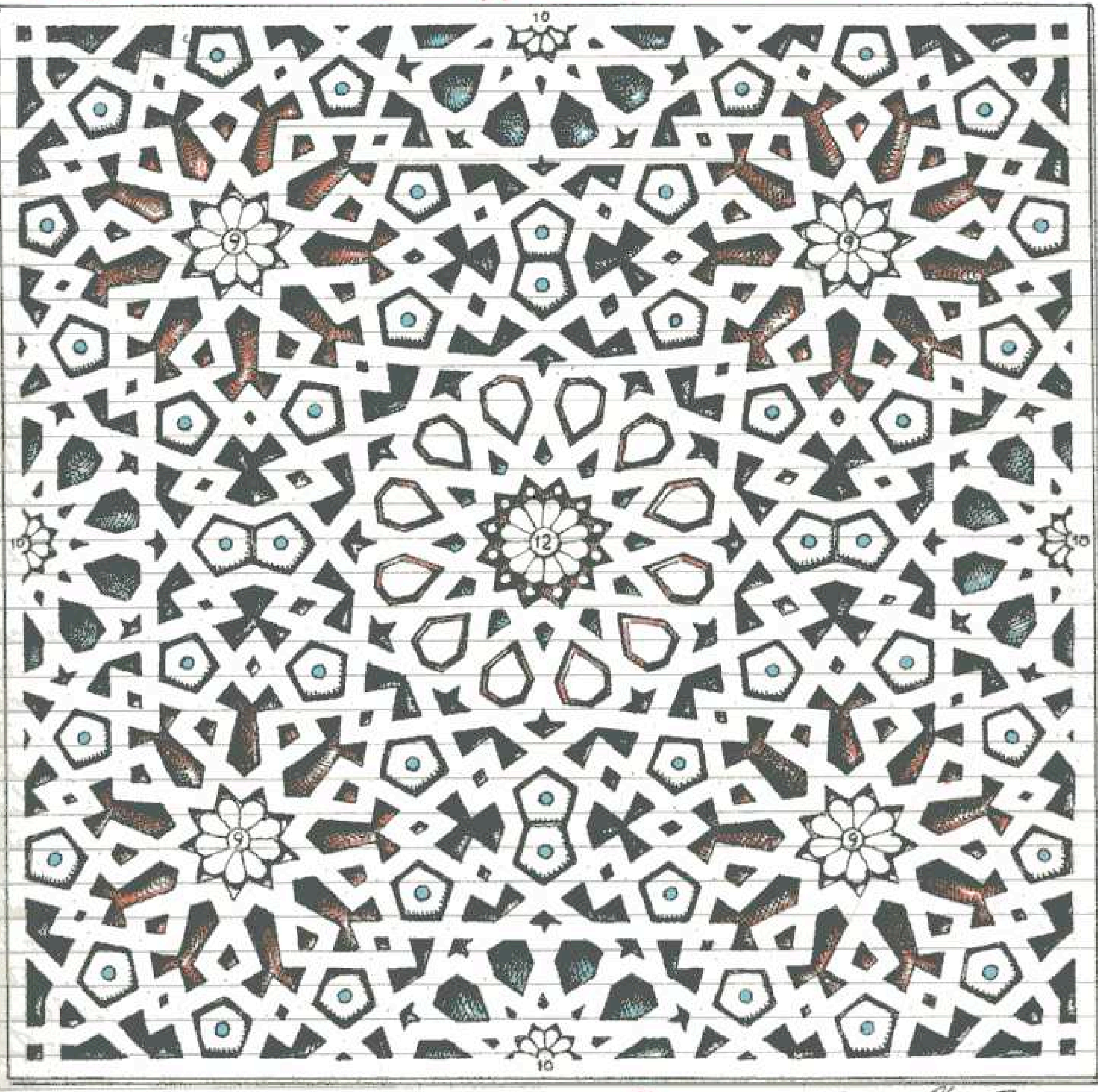
I have already (p. 94) classified the basis of the patterns shown here under "semi-symmetrical" kites, and the present pattern would then become K[6x5x2]12,10,4 (but see the note at the foot of p. 292, opposite). Patterns on this basis occur in Turkey and also in Iran, and appeared at the Great Mosque in Damascus (see Bourgoin, 1879, Plate V; see also his line plate 159). In Turkey the type II version (as in Bourgoin's Plate 159) occurs as carved stone on the entrances to the Agzikara Han (1236-46) and the Cifte mausoleum (c.1270) at Kayseri. The same type of pattern occurs on the side of the wooden minbar in the Ulu Jami, Bursa (c.1399), similar to the example drawn on this page as wooden inlay. A version on the same basis is illustrated in J. Shafā'ī (19--?) on decoration in Persian architecture and wood carving. This is apparently taken from an example of Persian wooden lattice, and consists of polygons in contact with contained stars, similar to the polygons in my type IX pattern illustrated on p. 52 of this notebook (Plate 58 of Shafā'ī's book). No other authentic variations on this basis are known to me, but a satisfactory type I is possible, and the beautiful type VI version is shown opposite on p. 292. The construction of this presents no difficulties, since none of the motifs is greatly different from the parent 10-star, and the fit is quite close. In view of the fact that the kite K[2x5x4]4,9,10 within the pattern is topologically identical to a (3x2) rhomb, other (3x2) pattern types may well suit this pattern.

Sun 31 March 1985

d - CONFIGURATION with 1 AXIS of SYMMETRY

Monday, DECEMBER 12, 1966

Best regarded as a K_1K_2 (2-kite) pattern, as noted below.



K[6×5×2]12,10,4/VI-2A with interstitial $\tilde{9}$. (original) Sun 31 March 85

This designation takes care of the whole pattern, but it is important to note that the pattern contains the kite $K_1[6\times4\times6]10,\tilde{9},12$ which is topologically identical to a (3×2) rhombus. If we recognize this kite as a constituent of the pattern then the design as a whole becomes a 2-kite pattern, the second kite being $K_2[2\times5\times4]4,\tilde{9},10$.

note the odd number